双语教学辅导

数　学

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内 容 提 要

本丛书是中学双语教学材料，由上海中学双语教材编写组从多种原版教材中精选精编而成。原讲义已在上海中学多次试用。考虑到目前双语教学发展的不平衡态势，每个单元由词汇、课文和习题三部分内容组成，以适合初、中、高不同层次的需要。词汇部分概括了本章的要点，课文和习题反映了本章的知识。丛书包括数学、理化和环境科学三册。

各篇课文全部选自原版教材，内容上不增加新的知识点。表述为典范科技英文，文字规范简洁，为了方便教学，作者根据教学经验对于语言上的难点略作了注释。

本书主要讲述中学数学的基本知识，包括集合、数、式、函数、三角、几何、排列组合和概率等，书后附有典型的国外数学试卷。

前　　言

21 世纪是高新科学技术迅猛发展的时代。信息通讯技术的高速发展和快速普及使不同国家、不同地区的人们瞬息之间即可实现交流，这种交流也已从原来的日常生活信息逐步深入到学科、专业领域。要在信息交流中游刃有余，除了掌握网语之外，英语的使用是至关重要的。因为网上信息中，最有价值的内容，绝大部分是用英语发布的，如果想获得科技、经济等各方面最新的第一手资料；如果想让外界了解你的情况，你的想法与见解；如果想在你的学习中得到来自世界各地的帮助，那么，英语，特别是学科英语的掌握与熟练就是必不可少的。

面对当前高中程度学科英语教材奇缺的现状，我们从高中学生的英语水平与学科水平这两方面的实际出发，编写了这套学科英语教材。在编写时我们既考虑学科的交流性，又顾及了学生的可接受性，所有内容全部选自英语国家的原版教材，以确保原汁原味。在选材时，对那些既不是教材核心部分，英语要求又过高的内容，则作了删节，以适于我国学生学习。我们还特别选用了许多各学科的近现代内容，弥补本地教材在学科知识更新上的滞后，使学生可以对各学科的最新科技成果有所了解，对各学科国际上较为常见的教学思路有所体会。书中还有不少与生活实际紧密相连的内容，既可提高学生的学习兴趣，又可开阔视野、增长知识。所有这些都为中学生以英语作为媒体学习学科知识，提供了合适的载体与切入口。

上海中学的双语教学已经有 8 年的历史。自从 1993 年成立国际部以来，学校陆续开出了数学、物理、化学、历史、生物、经济、心理、地理、计算机等各学科英语系列的课程，覆盖了 4～12 年级不同水平和层次的内容。在 8 年的教学实践中，上海中学积累了教材选用、课程设置、教学方式及评价方法等各方面的丰富经验。本书的部分内容从 2000 年春开始，在高一和高二两个年级进行了试教。经过近两年的实践，收到了良好的教学效果。学生们普遍反应学科英语水平有所提高，对国外的学科教学内容有所了解，对国外的学科试题类型有所适应，感受到了英语作为语言工具的作用，体验到能够运用英语进行学习的乐趣。

本丛书的编者对学科英语教学有着较为丰富的经验，在专业素养方面有较高的造诣又有很好的英语水平，对国外的学科教材有十分深入的理解，又熟悉我国的学科教材，了解中国学生的学科知识结构与英语水平，编写时他们在取长补短、深入浅出方面下了许多功夫。因此本教材具有针对性强、现代气息浓、易读易学的特点。

本套丛书既可以作为中学学科英语教学的课堂教材，也可以作为课外辅导材料，还可用作各学科教师的教学参考用书。对学科双语教材感兴趣的其他读者，也可以把它用作为入门读物。我们恳切希望，本丛书的出版可以对上海乃至全国的双语教学起到一定的促进作用。

上海中学校长

唐盛昌

2002 年 1 月

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Ch. 1　Sets of Numbers

Glossary

set　集合
finite set　有限集
infinite set　无限集
subset　子集
single-element set　单元素集合
union set　并集
intersection set　交集
complement set　补集
empty set　空集
universal set　全集
operation　运算
correspondence　对应
one-to-one correspondence　1-1 对应
converse operaton　逆运算
add(addition)　加
multiply(multiplication)　乘
substract(difference)　减
divide(division)　除
number　数
natural number　自然数
rational number　有理数
irrational number　无理数
real number　实数
positive integer　正整数
negative integer　负整数
whole number　非负整数
proper fraction　真分数
improper fraction　假分数
consecutive integer　连续整数
decimal number system　十进制数系
decimal point　小数点
prime number　素数,质数
composite number　合数
random number　随机数
recurring decimal　循环小数
infinite recurring decimal　无限循环小数
infinite irrecurring decimal　无限不循环小数
terminating decimal　有限小数
number line　数轴
highest common factor　最大公约数
least common multiple　最小公倍数
field　域
group　群
inverse　逆元
identity　单位元
commutative law　交换律
associative law　结合律
distributive law　分配律
binary operation　两元运算
reflexive property　反射性
symmetric property　对称性
transitive property　传递性
substitution property　替代性
contemporary mathematics　近代数学

§ 1 Number system

Consider the set of numbers $\{1,2,3,4,5,...\}$. If we select any two numbers from this set, say 4 and 6, when they are added we obtain another number in this set, i. e., $4+6=10$. Similarly, when we multiply these two numbers, namely $4\times 6=24$, again we end up with a number that belongs to this set. Such a set is called a closed set under addition and multiplication. But what happens when we divide 6 by 4, does the result belong to this set? This basic question proved to be quite an issue in the early development of mathematics. We will briefly look at the evolution of the number system which enabled mathematics to develop the different number systems that we so readily use (and take for granted) today.

The set so far considered, i. e., $\{1,2,3,4,...\}$ is known as the set of natural numbers and is denoted by the letter **N**, i. e., the set of **Natural numbers** is $\mathbf{N}=\{1,2,3,...\}$ which is also referred to as the set of positive integers. Now, although this appears as if we were born with the ability to possess such "trivial knowledge", it was only in the nineteenth century that the Italian mathematician Peano (1858—1932) and others like him, were successful in describing the set **N** in a way which brought about (in a mathematical way) the nature of the basic properties of **N**. One of the properties that he formularized became the basis of the so called "Principle of Mathematical Induction".

What happens when two numbers are subtracted? If we choose 6 and 4, then $6-4=2$, which is still a natural number, but $4-6=-2$. What about $4-4(=0)$, where did this fit in this particular number system? The question then naturally arose (as it did when we considered $6\div 4$), where do these numbers belong?

Because of questions like these, it was necessary to expand the number system to sets that included negative numbers, zero and fractions.

The **set of integers** is defined as $\mathbf{Z}=\{...,-3,-2,-1,0,1,2,3,...\}$. This now enabled mathematicians to solve problems like, find x where $x+6=4(x=-2)$. This also meant that we now had two sets, one of which was wholly contained within the other. That is, we now had that $\mathbf{N} \subset \mathbf{Z}$.

We still had to deal with fractions. To this end, the set $\mathbf{Z}$ was extended to the set of rational numbers, $\mathbf{Q}$, where $\mathbf{Q}=\left\{\frac{m}{n}: m \in \mathbf{Z}, n \in \mathbf{Z}, n \neq 0\right\}$. Notice that the restriction on this set is that division by zero is not allowed. For example, we could now solve equations of the form $3x+1=5$ $\left(\Leftrightarrow x=\frac{4}{3}\right)$. We now had the relationship that $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q}$.

At this stage all seems to be in order——until we try to solve a problem of the form $x^2=2$, or determine the area of a circle (given by πr^2). Numbers generated from such problems could not be found in any of the sets found so far. This is where the set of **irrational numbers** came into play. Irrational numbers became the set of numbers that did not belong to the set $\mathbf{Q}$ and it was denoted by $\overline{\mathbf{Q}}$. Numbers that belong to this set are $\pi, e, \sqrt{2}$ (i. e., surds) and all numbers that cannot be expressed as a fraction.

Sometimes we run into numbers like, 0.33333..., what type of number is this? It appears as if it is an irrational number, but in fact, it is a rational number. This can be shown as follows: Let $x=0.33333\ldots$, $\therefore 10x=3.33333\ldots$.

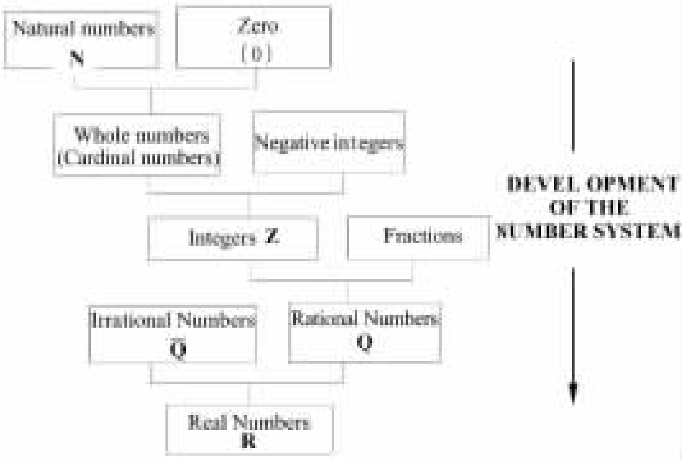
That is, we have that $\quad 10x=3+0.33333\ldots$

But, $x=0.33333\ldots$, so, $\quad 10x=3+x$

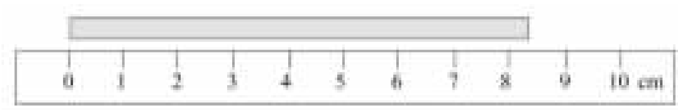
Therefore, $\quad 9x=3 \Leftrightarrow x=\frac{3}{9}\left(=\frac{1}{3}\right)$

Therefore, we have that $0.33333\ldots=\frac{1}{3}$, which is a rational number.

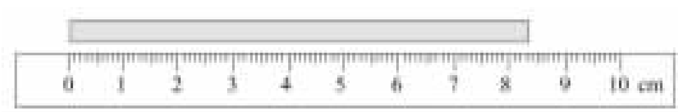
The **union of** the **rational set** and the **irrational set** produce the **set of real numbers.** That is, $\mathbf{Q} \cup \overline{\mathbf{Q}}=\mathbf{R}$. The set of real numbers contains every number that can be thought up (excluding some numbers that belong to a set known as **complex numbers**). Also, as the number system has developed over many hundreds of years, it might very well be the case that a new set of numbers, which contains new numbers that cannot be accounted for using our present system is yet to be developed! We now provide a diagram highlighting the relationship between the sets covered.



Whenever a quantity is measured, there is some error in the measurement. The amount of error usually depends on the quality of the device used to make the measurement. Here are two attempts to measure the length of a rod:



This ruler is only marked with centimetre graduations and suggests that the length of the rod is 8cm.



This ruler is marked with millimetre graduations and allows us to estimate the length of the rod as 8.3cm.

In the first case, the length is given as 8cm which means that the length of the rod is somewhere between 7.5 and 8.5cm. In the second case, when using the more accurate ruler, giving the length as 8.3cm means that the true length is between 8.25cm and 8.35cm. In summary:

A measurement of 8cm means that the true value is in the range 7.5cm to 8.5cm and is said to be accurate to **1 significant figure.**

A measurement of 8.3cm means that the true value is in the range 8.25cm to 8.35cm and is said to be accurate to **2 significant figures.**

In most cases, the number of significant figures in a

culations. In this case, the data is accurate to 2 significant figures and so the answer should be rounded to this level of accuracy. Rounding the answer to 2 significant figures gives 6800 square metres.

In this case, we should note that the largest possible values of the length and width are 91.5 and 75.5 metres giving a largest possible value for the area of 6 908.25m^2. Similarly, the smallest possible area is 90.5 by 74.5 = 6742.25m^2. From these figures, it is evident that we are not able to calculate the area to a greater level of accuracy than 2 significant figures.

2. Area $=\pi\times 2.33^2\approx 17.05539cm^2$. It is appropriate to perform all calculations to a high level of accuracy. In this case, for example, you should use the "π" key and not any less accurate approximation such as 3.14. The data is accurate to 3 significant figures and so the answer should be rounded to a similar level of accuracy. In this case the fourth figure is a 5 and so the result must be rounded up to 17.1cm^2.

Exercises

1. Analyze each statement. If the statement is true, explain why. If the statement is false, state a counter-example.

(a) Every integer is a rational number.

(b) The rational numbers are closed under multiplication.

(c) The irrational numbers are closed under addition.

(d) e is a real number.

2. State the number of significant figures in these values:

(a) 34.52	(b) 5673.7	(c) 1200
(d) 4.001	(e) 0.00452	(f) 0.00340
(g) 784520	(h) 0.450	(i) 4503450
(j) 0.00452	(k) 67.4500	(l) 0.56204

3. Round the following values to the number of significant figures given:

(a) 2.526[2 S. F]	(b) 24650[3 S. F]
(c) 0.347[2 S. F]	(d) 45627 [4 S. F]

(e) 0.4523[2 S. F]　　(f) 3.624[1 S. F]

(g) 56720[2 S. F]　　(h) 0.04537[3 S. F]

(i) 0.0045[2 S. F]　　(j) 345620[3 S. F]

(k) 0.0453[2 S. F]　　(l) 89000[1 S. F]

4. A square has an area of 67cm^2. Find the length of one of the sides, giving the answer to an appropriate number of significant figures.

5. A rectangle has an area of 56cm^2 and a length of 5.1cm. Find the width of the rectangle, giving the answer to an appropriate number of significant figures.

6. The angle of elevation of a building is 34° when measured from a distance of 65 metres. Find the height of the building giving the answer to an appropriate number of significant figures.

7. A painting is 782 mm wide and 679 mm high. What is the length of the diagonal(to an appropriate level of accuracy)?

8. Consider the series of fractions $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\cdots$ How many terms of this series will you need to add before the total is equal to 3, correct to three significant figures?

9. The fraction $\frac{22}{7}$ is often used as an approximation to π. To how many significant figures is this accurate?

10. The expression $\left(1+\frac{1}{n}\right)^n$ is equal to the irrational number e(press $e^{x\wedge}$ 1 on a calculator). What is the smallest value of n that will give e correct to 2 significant figures?

§2 Operations of Real Numbers

The Greeks contributed greatly to the development of mathematics in general and of geometry in particular. In an attempt to use logic to describe phenomena in the natural world, Greek thinkers provided the foundation on which much of contemporary mathematics was built. But even the Greeks could not foresee the extent of their contributions. Their studies of the relationships that exist in geometric planes and solids would be extended to include abstractions that they could not comprehend.

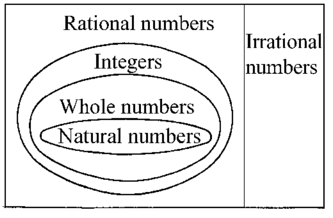
One important aspect of the Greeks' study of geometry is the technique of geometric construction. This is the practice of creating or duplicating a figure using only a compass and a straightedge. Geometric constructions can be directly applied in areas such as drafting. But they can also be used to illustrate abstract concepts about the real number system.

It is important to have an understanding of the properties of the real number system. The set of **real numbers R** is the union of the set of rational numbers **Q** and the set of irrational numbers. The set of rational numbers has several subsets which can be used to describe different real world phenomena.

Natural numbers	$\boldsymbol{N}=\{1,2,3,4,\ldots\}$
Whole numbers	$\boldsymbol{W}=\{0,1,2,3,4,\ldots\}$
Integers	$\boldsymbol{Z}=\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$

Rational numbers are defined as numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b\neq0$. Each rational number can be written as a terminating or repeating decimal. Numbers such as $\frac{3}{4}$, $\frac{11}{2}$, -0.05, 2.3, and $0.272727\ldots$ are all rational.

The Real Number System



Irrational numbers are real numbers that are not rational numbers. The set of irrational numbers contains numbers such as $\sqrt{2}$, $\sqrt[3]{5}$, π, and e. The sets of rational and irrational numbers have no elements in common and are therefore mutually exclusive.

A calculator performs operations with rational numbers. The calculator manipulates irrational numbers by converting them to very accurate decimal approximations. To reduce rounding errors in your calculations, remember to confine your rounding to the last step and to store decimal approximations in the calculator's memory when necessary. Also, for greater accuracy, use the calculator keys designated for irrational numbers, such as π or e.

Modeling

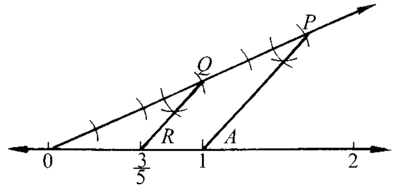
How can you use geometric constructions to represent real numbers on a number line?

Just as modeling can be used to make a real world problem more understandable, it can also be used to make an abstract concept more concrete. For example, the techniques of geometric construction can be used to illustrate the idea that every element in the set of real numbers can be represented on a number line. Draw a line with a straightedge and choose a point on it to represent 0. You can then choose some unit length and use a compass to locate points for the integers. Geometric constructions can also be used to find points on the number line that represent rational numbers and points that represent irrational numbers.

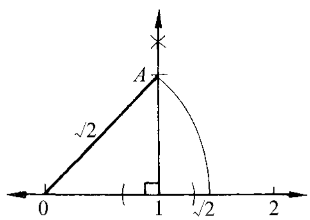
Example 1 Locate each point on the number line.

a. $\frac{3}{5}$ b. $\sqrt{2}$

Solutions



a. Draw a ray through zero on the number line. Mark off five equal lengths on the ray and label the end of the third length Q and the fifth length P. Draw $\overline{PA}$. Copy $\angle P$ at Q. Since $\overline{QR}$ is parallel to $\overline{PA}$, the coordinate of R is $\frac{3}{5}$.



b. Construct a one-unit segment perpendicular to the number line at 1. Draw $\overline{OA}$, making an isosceles right triangle. By the Pythagorean theorem, the length of the hypotenuse $\overline{OA}$ is $\sqrt{2}$. Use a compass to locate this value on the number line.

Theoretically, any real number can be located on the number line. Geometric constructions provide an excellent model of the one-to-one correspondence between the set of real numbers and the set of points on the number line.

A binary operation sets up a correspondence between any two elements in a set and another element in the set. Addition, subtraction, multiplication, and division are familiar binary operations on the set of real numbers. The set of real numbers and the two binary operations of addition and multiplication form an important mathematical structure called a **field** that has the properties listed below. The operations of addition and multiplication are related by the distributive property.

Field Properties of the Set of Real Numbers

Let a, b, and c be real numbers.

	Addition Properties	***Multiplication Properties***
Closure	$a+b$ is a unique real number.	ab is a unique real number.
Commutative	$a+b=b+a$	$ab=ba$
Associative	$(a+b)+c=a+(b+c)$	$(ab)c=a(bc)$
Identity	There exists a unique real number zero, 0, such that $a+0=0+a=a$.	There exists a unique real number one, 1, such that $a\cdot 1=1\cdot a=a$.
Inverse	For each real number a, there is a unique real number $-a$ such that $a+(-a)=(-a)+a=0$.	For each nonzero real number a, there is a unique real number $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right)=\left(\frac{1}{a}\right)a=1$.
Distributive	$a(b+c)=ab+ac$	

Example 2 Name the field property of the real number system illustrated in each statement.

a. $2\pi r=2r\pi$

b. $0.625(1.6)=1$

c. $1+0$ is a real number

d. $2[3.27+(-3.27)]=2(0)$

Solutions

a. commutative, multiplication

b. multiplicative inverse

c. closure, addition

d. additive inverse

The set of real numbers with the operations of addition and multiplication is only one example of a field. A field is any set, finite or infinite, with two binary operations having all the properties listed above. For a finite set, the field properties can be verified by checking all possible combinations of all elements. Of course, if a property does not hold for any combination of elements, then a field is not formed.

Example 3 The operations $\oplus$ and $\otimes$ are defined on the set

$S=\{0,1,2,3\}$ in the tables below. Determine if this set forms a field under these operations.

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\otimes$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Solution

Test to determine if the operation of $\oplus$ is commutative.

$2 \oplus 3=1$ *Find the intersection of the row for 2 and the column for 3.*

$3 \oplus 2=1$ *Find the intersection of the row for 3 and the column for 2.*

A test of all the elements shows that for all elements a and b in S,

$$a \oplus b=b \oplus a$$

Therefore, the operation $\oplus$ is commutative. Similarly, it can be shown that all the field properties are satisfied for the operation $\oplus$. The identity element for $\oplus$ is 0.

The identity element for $\otimes$ is 1. Since there is no 1 in the row for 2, the element 2 has no inverse under the operation $\otimes$. Therefore, the set S with the operations $\oplus$ and $\otimes$ does not form a field.

The field properties, along with the properties of equality listed below, can be used to prove theorems about real numbers.

Properties of Equality

$a=a$	**Reflexive property**
If $a=b$, then $b=a$.	**Symmetric property**
If $a=b$ and $b=c$, then $a=c$.	**Transitive property**
If $a=b$, then b may be substituted for a.	**Substitution property**

Example 4 Using the field properties of real numbers and the properties of equality, prove the theorem: If a, b, and c are real numbers and $a=b$, then $ca=cb$.

Proof