

A Concise Course  
in University Physics

大学物理简明教程

第一册

力学      热学

主    编    王安安  
编写人员    王安安    陶亚琴

重庆大学出版社

A Concise Course  
in University Physics  
大学物理简明教程

第一册

力学 热学

主 编 王安安

责任编辑:黄世芳 版式设计:黄世芳

责任校对:任卓惠 责任印制:秦 梅

\*

重庆大学出版社出版发行

出版人:张鸽盛

社址:重庆市沙坪坝正街174号重庆大学(A区)内

邮编:400030

电话:(023) 65102378 65105781

传真:(023) 65103686 65105565

网址:<http://www.cqup.com.cn>

邮箱:[fxk@cqup.com.cn](mailto:fxk@cqup.com.cn) (市场营销部)

全国新华书店经销

四川外语学院印刷厂印刷

\*

开本:787×1092 1/16 印张:15.25 字数:374千

1998年6月第1版 2005年8月第3次印刷

印数:5 001—7 000

ISBN 7-5624-1554-4/O·159 定价:18.00元

---

本书如有印刷、装订等质量问题,本社负责调换

版权所有,请勿擅自翻印和用本书

制作各类出版物及配套用书,违者必究。

# 前 言

我们正处在一个高新技术飞速发展,科技信息量激增,知识更新加快,国际交流日益广泛的时代。我国的进一步改革开放,社会主义市场经济的建立都要求高校毕业生有更强的适应能力,在人才市场上,有较强的外语应用能力、交流型、综合型的毕业生供不应求。在这种形势下,我们的高等教育正向着重视素质教育的方向转变,而素质与能力是密切相关的,素质的培养要以一定的知识和能力为基础,其中包括独立获取知识的能力。毋庸置疑,直接用外语为工具获取知识、进行交流的能力是人才素质的一个重要方面。

然而,由于历史原因、文化背景、经济基础、外语教育模式和各类师资外语水平等诸多因素的影响,我们在应用外语进行教学方面的基础性工作十分薄弱。在普通高校本科生教育中,教材和教学过程基本上只使用中文这一单一语种,实际上已经制约了学生应用外语(主要是英语)获取知识能力的发展。为了改变这种现状,跟上时代的步伐,试用英文教材,使用英语进行教学的改革便应运而生了。

本教材的编写是编者主持的“试用英文物理教材”教改试点工作的继续,也是编者大学物理教学经验的总结。从国外引进的教材,虽有诸多优点,但在系统上与我国的大学物理教学基本要求不完全对应,为了满足师生对英文物理教材的需要,编写一套根据我国工科物理教学基本要求,顺应工科物理教学改革形势,反映编者在多年物理教学实践中总结出来的教学方法与经验,与我们的学生在一、二年级的英文水平相适应的英文“简明物理学教程”的计划就提到日程上来了,这就是我组织编写这套教材的初衷。这套英文物理教材是1996年经国家教委批准列入正式出版计划的。本教材可供普通高等工科院校本科生物理课130~140学时使用。编者力求突出以下特点:

基本理论讲述方法简明、准确、易学易教。

为使学生读后达到基本训练的要求,配备了适量的例题、思考题和习题。除自编的题目及我校物理教研室常用的题目外还吸收了一部分英文教材中的优秀题目。

每册最后编入一个阅读材料,选自优秀的英文原版教材供学生自学,内容力求反映物理学的新发展。不另编写绪论,用第一册的阅读材料“物理学是什么”代之。

除物理学名词外,绝大多数英文词汇包含在大学英语四级词表内,全部词汇几乎都在英语六级词表内。通过三册书的教学,不但能达到教学基本要求,而且学生直接使用英文基础课教材及专业课教材学习的能力将大大提高。

在第三册的部分插图中使用了编者在教学、科研中积累的素材。

全书21章,具体编写分工如下:

第一册:力学部分(第1~5章)由王安安编写,分子运动论和热力学基础(第6、7章)由陶亚琴和王安安编写。

第二册:电学及稳恒电流的磁场(第8~10章)由吴光敏编写,磁介质和电磁感应(第11、12章)由樊则宾编写,麦克斯威方程组(第13章)由王安安编写。

第三册:机械振动(第14章)由伏云昌编写,机械波(第15章)由王安安和陈劲波编写。

电磁振荡与电磁波(第 16 章)由吴光敏编写,波动光学部分(第 17 章)中干涉与衍射由陈劲波编写,光的偏振由樊则宾编写。近代物理部分(第 18 ~21 章)由伏云昌编写。

第一册绝大部分插图由刘富华用计算机绘制,其余插图由李俊昌教授绘制,封面也由李俊昌设计,谨此致以诚挚谢意。

编写大学英文物理教材是一种大胆的尝试,由于编者水平有限,错误疏漏之处在所难免,希望同行和读者批评指正。我们相信这本教材的出版将对物理教学的现代化和物理教学与国际接轨作出有益的贡献。

编 者

1997 年 5 月于昆明

# Contents

## Part One Mechanics

Chapter 1	Kinematics	1
§ 1-1	Frame of Reference Particle	1
§ 1-2	Position Vector and Displacement	3
§ 1-3	Velocity and Acceleration	4
§ 1-4	Two Types of Problems in Kinematics Sample Problems	9
§ 1-5	Circular Motion	14
§ 1-6	Relative Motion	19
	Summary	20
	Questions	23
	Problems	25
Chapter 2	Dynamics—Newton's Laws of Motion	29
§ 2-1	Newton's Laws Force Inertial Reference Frame	29
§ 2-2	General Properties of Forces in Mechanics	34
§ 2-3	The Fundamental Forces of Nature	38
§ 2-4	Fundamental Quantities and Units Dimensions	41
§ 2-5	Applying Newton's Laws of Motion	43
	Summary	51
	Questions	53
	Problems	55
Chapter 3	Work and Energy	59
§ 3-1	Work	59
§ 3-2	Kinetic Energy The Law of Kinetic Energy	63
§ 3-3	Conservative Force and Potential Energy of Weight	65
§ 3-4	Elastic Potential Energy and Universal Gravitational Potential Energy	68
§ 3-5	Conservation of Mechanical Energy	69
§ 3-6	The Conservation of Energy	72
	Summary	76
	Questions	78

Problems .....	79
<b>Chapter 4 Momentum .....</b>	<b>83</b>
§ 4-1 Linear Momentum and Linear Impulse Momentum Theorem .....	83
§ 4-2 Conservation of Momentum .....	86
§ 4-3 Collision .....	89
§ 4-4 Conservation of Angular Momentum of a Particle .....	93
Summary .....	97
Questions .....	99
Problems .....	100
<b>Chapter 5 Rotation of a Rigid Body .....</b>	<b>104</b>
§ 5-1 Motion of a Rigid Body .....	104
§ 5-2 Torque The Law of Rotation Rotational Inertia .....	107
§ 5-3 Applying the Law of Rotation .....	113
§ 5-4 Kinetic Energy and Work in Rotational Motion .....	117
§ 5-5 Angular Momentum of a Rigid Body Conservation of Angular Momentum .....	119
Summary .....	126
Questions .....	128
Problems .....	128

## Part Two Thermodynamics

<b>Chapter 6 The Kinetic Theory of Gases .....</b>	<b>133</b>
§ 6-1 Essential Concepts of the Kinetic Theory of Gases .....	133
§ 6-2 State Parameters Equilibrium State Ideal Gas Law .....	135
§ 6-3 The Representation of Pressure for Ideal Gas .....	138
§ 6-4 The Average Translational Kinetic Energy and Temperature .....	142
§ 6-5 The Equipartition Theorem of Energy .....	144
§ 6-6 The Mean Free Path .....	148
§ 6-7 The Maxwell Speed Distribution .....	151
§ 6-8 The Boltzmann Distribution .....	155
Summary .....	158
Questions .....	162
Problems .....	163
<b>Chapter 7 Fundamentals of Thermodynamics .....</b>	<b>166</b>
§ 7-1 Internal Energy Heat and Work .....	166
§ 7-2 The First Law of Thermodynamics .....	167
§ 7-3 Application of the First Law to Isochoric, Isobaric and Isothermal Processes .....	169

§ 7-4	The Heat Capacities of an Ideal Gas .....	175
§ 7-5	Application of the First Law of Thermodynamics to Adiabatic Processes .....	179
§ 7-6	Cyclical Processes Thermal Efficiency .....	182
§ 7-7	Carnot Cycle Reverse Cycle .....	186
§ 7-8	The Second Law of Thermodynamics .....	189
§ 7-9	Reversible and Irreversible Process .....	191
§ 7-10	Statistical Meaning of the Second Law of Thermodynamics .....	194
* § 7-11	Entropy .....	197
Summary	.....	202
Questions	.....	206
Problems	.....	208
Reading Material	.....	212
Words	.....	217
Appendices	.....	222
. The International System of Units ( SI) *	.....	222
. Some Fundamental Constants of Physics	.....	224
Answers for Problems	.....	225

Part One

**Mechanics**

# Chapter 1

## Kinematics

Kinematics is the study of the geometry of motion: it deals with the mathematical description of motion in terms of position, velocity, and acceleration. Kinematics serves as a prelude to dynamics, which studies force as the cause of changes in motion.

### § 1-1 Frame of Reference Particle

#### 1. Frame of reference coordinate system

The world we live in is made of matter, from the largest bodies, such as the earth, one of the nine planets in solar system, the galaxies and the entire universe, to the smallest particles, such as molecules, atoms and subatomic particles: electrons and nucleus composed of protons and neutrons, each proton and each neutron is made of two kinds of quarks, called up quark and down quark. Although the objects above differ in size by a factor of more than  $10^{40}$ , they have a universality—being in endless motion, and from this point of view, we say that the motion is absolute.

In the remainder of this chapter we shall discuss the position and the speed of various objects. To do this scientifically we must first answer the question “ position with respect to what? ” and “ velocity with respect to what ”? If we choose different objects as the reference to describe the motion of a given body, the indications will be different. For example, if you stand on the ground in a train station and let a ball drop freely from your hand, the motion of the ball seems to be along straight line by you, but along a parabola of trajectory projected horizontally by the observer seating in a moving coach passing through the station. From this point of view, we say that motion is a relative concept and it must always be referred to a particular body that serves as a reference chosen by the observer. Since different observers may use different frames of reference, it is important to know how

---

The nine planets of solar system include the inner planets: Mercury, Venus, Earth and Mars; the outer planets: Jupiter, Saturn, Uranus, Neptune and Pluto. The sun is in the center of solar system.

There are so many galaxies in the universe that there are about  $2.3 \times 10^5$  galaxies in an angular patch of the sky,  $40^\circ$  by  $40^\circ$ . They are in at a middle distance, at about  $1.4 \times 10^9$  light-years from the Earth. Our sun is in a spiral galaxy: the Galaxy of the Milky Way. The total number of stars in our own galaxy is about  $10^{11}$ . The sun is in one of the spiral arms.

The quarks within protons and neutrons are of two kinds, called up and down. The proton consists of two up quarks and one down quark bound together; the neutron consists of one up quark and two down quarks bound together. Besides these two kinds of quarks, physicists have discovered four other kinds, called by quaint names Strange, Charmed, Top and Bottom. These quaint quarks do not occur in pieces of ordinary matter, they can only be manufactured under exceptional circumstances in high-energy collisions between subatomic particles.

observations made by different observers are related. For example, when we discuss motions on the surface of the earth, this is the most cases in our course, it is convenient to take the earth 's surface as our frame of reference. For the motion of the earth or other planets, a particular set of stars called fixed stars, for instance, sun is a good choice, whereas for the motion of the electrons in an atom, the nucleus of the atom is preferred. You are free to choose the frame of reference, but in all cases it is necessary to specify what reference frame is being used and you must always be aware of your choice and be careful to make all your measurements with respect to it.

In physics a frame of reference is usually pictured in terms of a coordinate system, consisting of three mutually perpendicular axes, called the X, Y and Z axes, relative to which position in space, velocity, acceleration and orbit can be specified. These three axes intersect at the origin ( O ) of the coordinate system. In Fig. 1-1, let us consider two observers, one of them on the sun and the other on the earth, both observers are studying the same motion of an artificial satellite of the earth. To the observer using frame X Y Z , the satellite appears to describe an almost circular path around the earth. To the solar observer using frame XYZ, the satellite 's orbit appears as a wavy line.

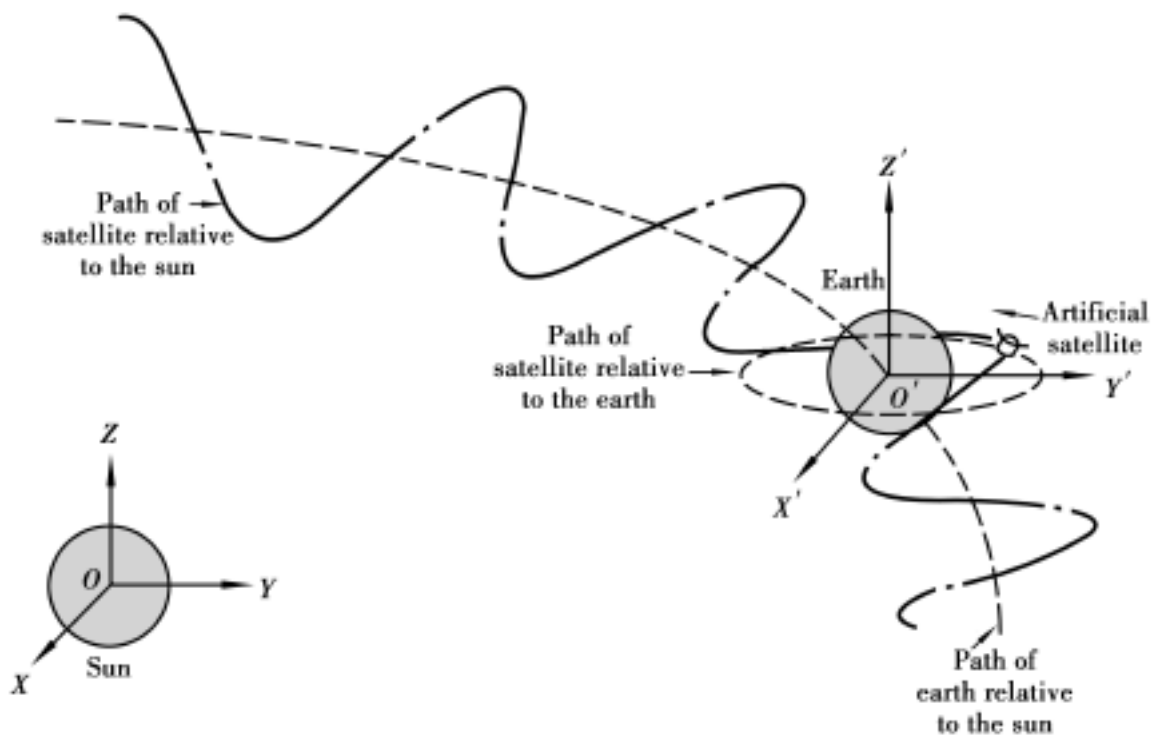


Figure 1-1 Orbit of a satellite relative to the earth and the sun.

The wiggles in the satellite 's orbit are greatly exaggerated here.

## 2. Particles

The moving objects that we might examine are among countless possibilities. We shall restrict our concentration on a simple case—translational motion of a particle first, which is defined as the change of position of the particle as a function of time. In the case of an ideal particle—a body with mass, but with no size and no shape, therefore position as a function of time gives a complete description. We can represent an object as a particle ( that is as a mass point) if every small part of the object moves in exactly the same way. The concept of particle is an ideal model, the motion of objects are usually more complicated. In some circumstances we are not interested in the size, orientation, and internal structure of a body, we can treat the body as a particle, concentrate on its

translational motion and ignore all the other motions. For example, we can describe the motion of a ship sailing down a river or a car traveling on a street as a particle motion—for most purposes it is sufficient to know the position of the center of the ship or the car as a function of time.

You must be aware that an object can be treated as a particle in one situation but not in another. The earth behaves pretty much like a particle if we are interested only in its orbital motion around the sun. If we study the rotation of the earth (revolve on its own axis), however, the earth is not a particle at all.

It is a very useful method in physics to simplify an object as an ideal model which helps us to solve the major problem in a subject. You will use more ideal models in the other parts of this course.

### 3. Time interval and time

It is necessary to distinguish two concepts, time interval and time. When we say time in physics, we mean a give instant. For example, some scheduled flight takes off at 8 00 am from Beijing, lands at 11 00 am on Kunming, 8 o'clock is an instant and so is 11 o'clock. The 3 hours that the whole flying lasts is a time interval. The position of a moving particle is corresponding to a given instant labeled with  $t$  while the distance it passed is corresponding to a given time interval labeled with  $\Delta t$ .

## § 1-2 Position Vector and Displacement

### 1. Position vector

When we describe the motion of a particle, the first question is: "Where is it?". In three dimensional world, we need a vector to answer this question. We locate a particle by a vector  $r$ , extending from the origin of the coordinate system to the particle's position as in Fig. 1-2. Thus,

$$r = xi + yj + zk \quad (1-1)$$

in which,  $i$ ,  $j$  and  $k$  are unit vectors and  $x$ ,  $y$  and  $z$  are the components of the vector  $r$ . The components can be positive, negative or zero.

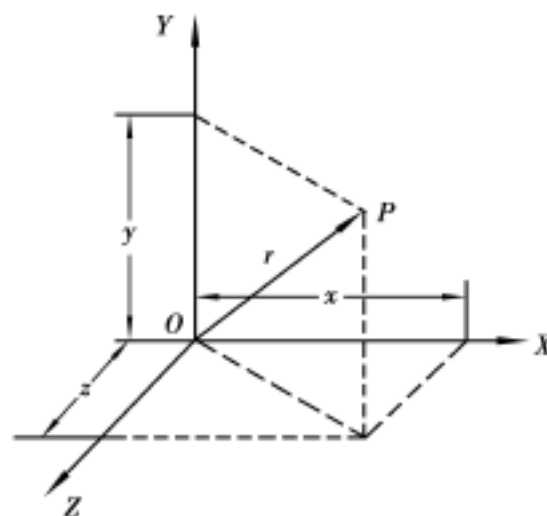


Figure 1-2 Position vector

We shall define position, displacement, velocity and acceleration for the general case of three dimensions. To simplify the figures, we shall illustrate them in two dimensions in the rest of this chapter.

### 2. Position function

Mechanical motion is defined as the process of change in position with time. In principle, the position vector can be correlated with the time by means of a vector function

$$\mathbf{r} = \mathbf{r}(t) \quad (1-2a)$$

Its three components are written by the following scalar functions

$$x = x(t), \quad y = y(t), \quad z = z(t) \quad (1-2b)$$

Eq. (1-2a) or (1-2b) is defined as the position function that determines the location of a particle at any given time. Combining Eq. (1-1) and (1-2b), we have

$$\mathbf{r} = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k} \quad (1-3)$$

which is equivalent with Eq. (1-2a) and (1-2b).

The path equation can be obtained by eliminating  $t$  from Eq. (1-2b):

$$f(x, y, z) = 0$$

If the path of a particle is a straight line, the motion is called as a rectilinear motion; if the path is a curve, the motion is called as a curvilinear motion.

### 3. Displacement

Displacement is the change in position during a given time interval. In Fig. 1-3, at time  $t$  the particle is at point A, given by position vector  $\mathbf{r} = \mathbf{OA}$ . At a later time  $t_1$ , the particle will be at B with  $\mathbf{r}_1 = \mathbf{OB}$ . Although the particle has moved along the arc  $AB = S$ , the displacement is the vector given by

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

or

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r} \quad (1-4)$$

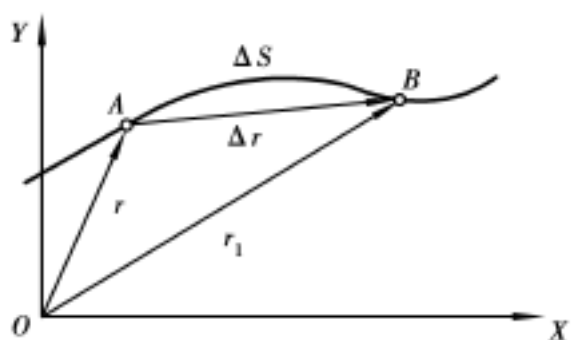


Figure 1-3 Displacement  $\mathbf{r}$  during time interval  $t$ .

Note that displacement indicates the change in position, not the path length over the same time interval. Displacement is a vector, its magnitude  $|\mathbf{r}|$  is the length of the chord AB; path is a scalar  $s = \Delta S$ , the length of the arc AB. In most cases,  $|\mathbf{r}| < s$  (Fig. 1-3), only in the limiting case  $t \rightarrow 0$ ,  $|\mathbf{r}|$  can be regarded equal to  $s$ .

For example, a man walks from point A along the rim of a circle of radius  $R$  for half a round, his displacement is  $2R$ , but path is  $\pi R$ . A particle moves back and forth in  $x$  axis for one period, its displacement is zero, but path equals to  $2A$  ( $A$  is the amplitude). You should also be aware of the difference between  $|\mathbf{r}|$  and  $s$ .

## § 1-3 Velocity and Acceleration

### 1. Velocity

The second question to describe the motion of a particle is "How fast is the change of position?". If  $\mathbf{r}$  is the displacement that occurs during the time interval  $t$ , its average velocity for this interval is defined as

$$\bar{v} = \frac{\bar{r}}{t}$$

The direction of average velocity points in the same direction of displacement ( Fig. 1-3 ); the magnitude of it equals  $|\bar{r}|/t$ . Obviously, average velocity is related to the specified time interval  $t$ , and it takes into account only the net displacement in the time interval  $t$ , ignores the details of the motion, and gives no credits for back and forth motion or the length of the path.

To describe the motion of a particle at a given time  $t$  or at a given point, we must make  $t$  very small. The instantaneous velocity at time  $t$  is obtained by evaluating  $\bar{r}/t$  in the limit that  $t$  approaches zero

$$v = \lim_{t \rightarrow 0} \frac{r}{t} = \frac{dr}{dt} \quad (1-5)$$

Thus, the instantaneous velocity is defined as the time derivation of the position vector.

Direction of instantaneous velocity:

To determine the direction of  $v$  at  $A$ , let us see Fig. 1-4, When  $t$  approaches 0, point  $B$  approaches point  $A$ , as indicated by  $B, B', \dots$ . With the vector  $AB$  changing continuously in both magnitude and direction, in the limit when  $B$  is very close to  $A$ ,  $AB = r$  coincides in direction with the tangent at  $A$ , therefore, the instantaneous velocity is a vector tangent to the path, and points to the advance direction.

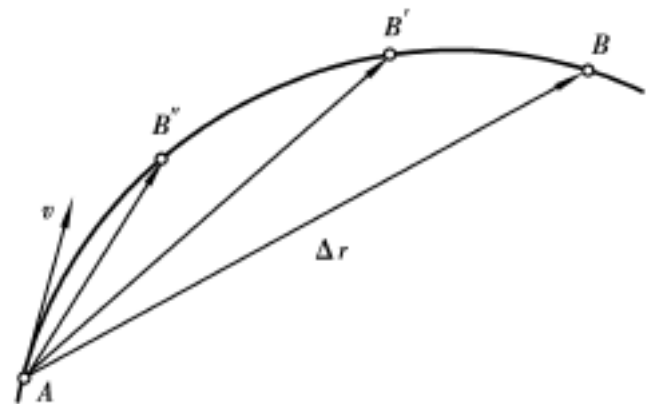


Figure 1-4 The velocity is tangent to the path at  $A$ .

Magnitude of instantaneous velocity:

Substituting  $r$  from Eq. (1-3) into Eq. (1-5) gives

$$v = \frac{d}{dt}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k} \quad (1-6)$$

or 
$$v = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k} \quad (1-7)$$

As we see, the three components of the velocity vector are given by

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt} \quad (1-8)$$

and the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (1-9)$$

For the case of the motion in a plane, angle  $\theta$  formed between  $v$  and  $+x$  direction is determined by  $\tan \theta = v_y/v_x$  as shown in Fig. 1-5, usually used to indicate the direction of velocity.

Velocity and speed:

On the other hand, the magnitude of velocity vector can be written as

$$v = |v| = \left| \frac{dr}{dt} \right| = \lim_{t \rightarrow 0} \frac{|\bar{r}|}{t} \quad (1-10)$$

Let  $s$  represent the path length over  $t$ , which is given by the length of the arc  $AB$  ( Fig. 1-3 ), and the closer  $B$  is to  $A$ , the closer the magnitude of  $\bar{r}$  is to  $s$ , that is

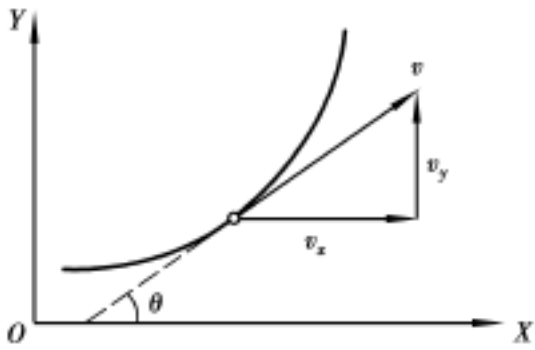


Figure 1-5 Velocity in two dimension

$$\lim_{t \rightarrow 0} \frac{|\mathbf{r}|}{s} = 1$$

therefore 
$$\mathbf{v} = \lim_{t \rightarrow 0} \frac{|\mathbf{r}|}{t} = \lim_{t \rightarrow 0} \frac{s}{t} = \frac{ds}{dt} \quad (1-11)$$

where  $s/t$ , the path length divided by the time taken, is called the average speed, so  $ds/dt$  is the instantaneous speed. Note that speed is a scalar, and Eq. (1-11) means that the magnitude of instantaneous velocity equals instantaneous speed, which can be briefly called as velocity and speed.

The unit of speed is m/s, that is, meter per second in SI system.

Example 1-1 The position of a particle moving in x-y plane is given by  $x = R + R\cos t$ ,  $y = R\sin t$ , here  $R = 1\text{m}$ ,  $\omega = \frac{1}{4}/\text{s}$ .

- Calculate: (1) The path function  $f(x, y) = 0$ ;  
 (2) Velocity at any time;  
 (3) Position vector at  $t = 0$  and  $t = 6\text{s}$ , the displacement  $\mathbf{r}$  and path length  $s$  during this time interval.

Solution

(1) Rearrange the position function as

$$x - R = R\cos t$$

$$y = R\sin t$$

then we have 
$$(x - R)^2 + y^2 = R^2$$

This is the path function of a circle with radius  $R$  and the position of center locates at  $(R, 0)$  as Fig. 1-6 shows.

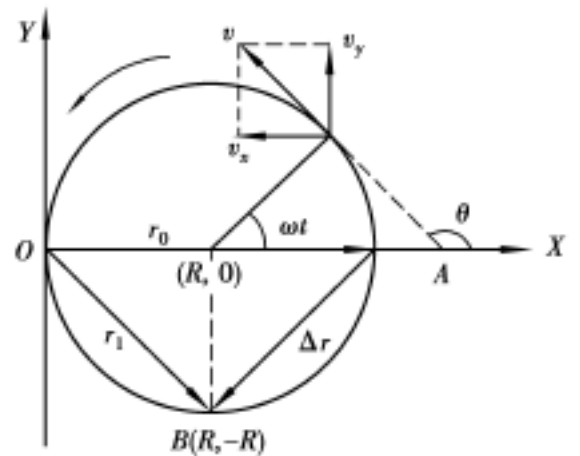


Figure 1-6 for example 1-1

(2) 
$$v_x = \frac{dx}{dt} = -R \sin t \quad v_y = \frac{dy}{dt} = R \cos t$$

$$\mathbf{v} = -R \sin t \mathbf{i} + R \cos t \mathbf{j}$$

$$v = \sqrt{v_x^2 + v_y^2} = R = \frac{1}{4} \text{m/s}$$

which means that the motion is a circular motion with constant speed. The angle between  $\mathbf{v}$  and  $+x$  direction is given by

$$\tan \theta = v_y/v_x = -\cot t$$

By inspection of the signs of  $v_x$  and  $v_y$  at the particular time, you can determine which quadrant the angle is in.

(3) When  $t = 0$ , we have 
$$\mathbf{r}_0 = 2R\mathbf{i}$$

represented by OA, and at  $t = 6\text{s}$

$$\mathbf{r}_1 = \left( R + R\cos \frac{3}{2} \right) \mathbf{i} + R\sin \frac{3}{2} \mathbf{j}$$

that is  $\mathbf{r}_1 = R\mathbf{i} - R\mathbf{j}$

represented by OB. The displacement during  $t = 6\text{ s}$  is

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_0 = -R\mathbf{i} - R\mathbf{j}$$

represented by AB in Fig. 1-6,

$$|\mathbf{r}| = \sqrt{(-R)^2 + (-R)^2} = \sqrt{2} R = 1.41\text{ m}$$

while the path length during the same  $t$  is

$$s = \text{arc AOB} = \frac{3}{2} R = 4.71\text{ m}$$

## 2. Acceleration

The path of a particle moving in two or three dimensions is a curve in general, its velocity changes both in magnitude and in direction. The magnitude of the velocity changes when the particle speeds up or slows down. The direction of the velocity changes because the velocity is tangent to the path and the path bend continuously. Fig. 1-6 indicates the velocity  $\mathbf{v}$  at time  $t$ , and  $\mathbf{v}_1$  at  $t_1$ , corresponding to the position A and B, respectively. The change in velocity during the time interval  $t = t_1 - t$ , is represented by  $\Delta\mathbf{v}$  in the vector triangle in which  $\mathbf{v} + \Delta\mathbf{v} = \mathbf{v}_1$ , then  $\Delta\mathbf{v} = \mathbf{v}_1 - \mathbf{v}$ . To describe the average rate of change in velocity for the interval  $t$ , the average acceleration is defined by

$$\mathbf{a} = \frac{\Delta\mathbf{v}}{t}$$

Using the same method as in definition of velocity, the instantaneous acceleration at time  $t$ , referred simply as acceleration is defined by

$$\mathbf{a} = \lim_{t \rightarrow 0} \frac{\Delta\mathbf{v}}{t} = \frac{d\mathbf{v}}{dt} \quad (1-12)$$

which is the time derivation of velocity vector.

Direction of acceleration:

Acceleration vector has the same direction as the limit direction of change in velocity when  $t \rightarrow 0$ , which is always pointing toward the concavity of the curve and, because  $\Delta\mathbf{v}$  is always in the direction in which the curve bends, as shown in Fig. 1-7. Suppose that the direction the acceleration is at an angle of  $\theta$  to the velocity,  $\theta < 90^\circ$ ;  $\theta > 90^\circ$ ; and  $\theta = 90^\circ$  corresponding to the cases of  $|\mathbf{v}_1| > |\mathbf{v}|$ ,  $|\mathbf{v}_1| < |\mathbf{v}|$  and  $|\mathbf{v}_1| = |\mathbf{v}|$ , respectively. It is important to be aware that there is an acceleration whenever the velocity changes either in magnitude or in direction.

Magnitude of acceleration:

From Eq. (1-5), we can also write Eq. (1-12) in the form

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \quad (1-13)$$

Substituting Eq. 1-3 into Eq. 1-13 gives

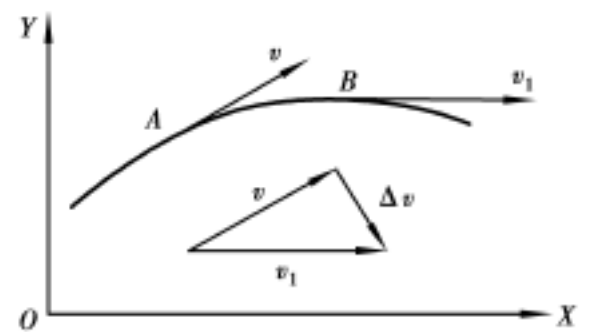


Figure 1-7 Acceleration in curvilinear motion

$$\mathbf{a} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}$$

or 
$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} \quad (1-14)$$

The three components of acceleration are given by

$$a_x = \frac{d^2x}{dt^2}, \quad a_y = \frac{d^2y}{dt^2}, \quad a_z = \frac{d^2z}{dt^2} \quad (1-15)$$

and the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad (1-16)$$

The unit of acceleration is  $\text{m/s}^2$  in SI system.

In the case of a motion in x-y plane, suppose  $\alpha$  is the angle formed by  $\mathbf{a}$  and  $+x$  direction, thus,

$$\text{tg } \alpha = \frac{a_y}{a_x} \quad (1-17)$$

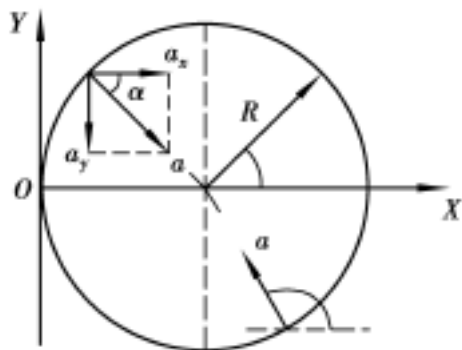


Figure 1-8 for example 1-2

Example 1-2 Suppose the position function is the same as in

example 1-1. Find the acceleration at any time.

Solution From the result of example 1-1, we have

$$a_x = \frac{dv_x}{dt} = -R^2 \cos t, \quad a_y = \frac{dv_y}{dt} = -R^2 \sin t$$

$$a = \sqrt{a_x^2 + a_y^2} = R^2 = \frac{v^2}{R} \text{m/s}^2 = 0.62 \text{m/s}^2$$

which means that the magnitude of  $\mathbf{a}$  is a constant, the direction of it can be represented by angle between  $\mathbf{a}$  and  $+x$  direction, and

$$\text{tg } \alpha = \frac{a_y}{a_x} = \text{tg } t$$

For example, if  $t=3$ ,  $\text{tg } \alpha = \text{tg } \frac{3}{4} = -1$ , because  $a_x > 0$ ,  $a_y < 0$ , so that  $\alpha = -\pi/4$ , in the fourth quadrant, as shown in Fig. 1-8. On the other hand, we can rewrite  $a_x$  and  $a_y$  as

$$a_x = -\frac{v^2}{R} \frac{x-R}{R}, \quad a_y = -\frac{v^2}{R} \frac{y}{R} \text{ that is } \mathbf{a} = -\frac{v^2}{R^2} [(x-R)\mathbf{i} + y\mathbf{j}]$$

Note that, there is a vector

$$\mathbf{R} = (x-R)\mathbf{i} + y\mathbf{j}$$

which is pointing from the center of the circle to the position of particle in Fig. 1-8, therefore

$$\mathbf{a} = -\frac{v^2}{R^2} \mathbf{R}$$

which means the acceleration is always pointing toward the center of the circle. So it is called centripetal acceleration.

Example 1-3 A particle moves so that its position as a function of time in SI unit is  $\mathbf{r}(t) = 2t\mathbf{i} + (\frac{1}{2}t^2 - 5t + 3)\mathbf{j}$ . Calculate (1)  $\mathbf{r}$ ; (2)  $\mathbf{v}$ ; (3)  $\mathbf{a}$ , when  $t=2\text{s}$ .

Solution

(1)  $\mathbf{r}_2 = 4\mathbf{i} - 5\mathbf{j}$

$$(2) v_x = \frac{dx}{dt} = 2, \quad v_y = \frac{dy}{dt} = t - 5. \quad \text{when } t = 2, \quad v_{2x} = 2, \quad v_{2y} = -3$$

$$\text{so } v_2 = \sqrt{2^2 + 9^2} = \sqrt{13} = 3.6 \text{ m/s}$$

angle is given by  $\text{tg} \theta = -\frac{3}{2}$ . Consider  $v_{2x} > 0, v_{2y} < 0$ , we choose

$$\theta = \text{arctg} -\frac{3}{2} = -56.3^\circ$$

$$(3) a_x = \frac{dv_x}{dt} = 0, \quad a_y = \frac{dv_y}{dt} = 1, \quad a = j, \quad a = |a| = 1 \text{ m/s}^2$$

$a$  is pointing in  $+y$  direction, so that the angle between  $a$  and  $+x$  direction is  $90^\circ$ .

Example 1-4 Someone standing on a cliff, pulls a boat by a pulley, as shown in Fig. 1-9. Suppose that the height of the cliff is  $h$ , the rate of the rope pulled is  $u$ . Find:

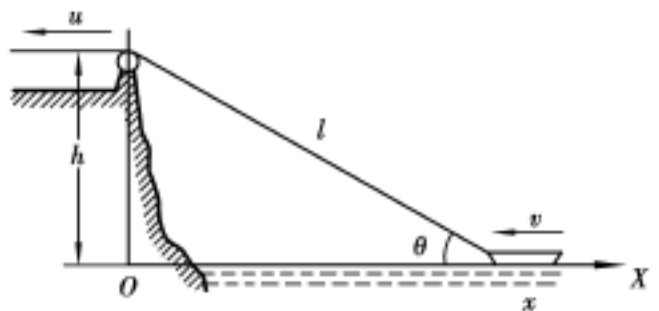


Figure 1-9 for example 1-4

- (1) the velocity of the boat;
- (2) its acceleration.

Solution Because the motion of the boat is in one dimension, set  $x$  axis pointing right, choose the origin at the foot point of the pulley, and let  $l$  representing the variable length of the rope at any time. So that, position vector of the boat is

$$r = xi, \quad \text{Note that } x^2 = l^2 - h^2 \quad \text{or } x = (l^2 - h^2)^{1/2}$$

take time derivation of  $x$ :

$$\frac{dx}{dt} = \frac{1 \frac{dl}{dt}}{\sqrt{l^2 - h^2}} = \frac{-lu}{x}, \quad \text{here } \frac{dl}{dt} = -u \text{ is the rate of rope shortened, then the speed of boat is}$$

$$v = \frac{dx}{dt} = -u \frac{1}{x} = -u \frac{\sqrt{x^2 + h^2}}{x}$$

which is the velocity as a function of coordinate. The negative sign indicates the velocity is in the  $-x$  direction. The acceleration of the boat is then

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = d\left(-u \frac{\sqrt{x^2 + h^2}}{x}\right) / dt = -\frac{u^2 h^2}{x^3}$$

which is the acceleration as a function of coordinate. The results indicate that  $a$  and  $v$  are in the same direction, so the magnitude of  $v$  becomes larger and larger with the value of  $x$  decreased.

## § 1-4 Two Types of Problems in Kinematics Sample Problems

The problems solved by means of vector derivation

From the example 1-1 and 1-2, we can make conclusion that if we know the position vector of the moving particle as a function of time,  $r = r(t)$ , according to Eq. (1-8) and (1-15), by taking