国家出版基金项目
NATIONAL PUBLICATION FOUNDATION

The Complete Works of Wu Wen-Tsun

Mathematics Mechanization I

吴文俊全集

◆数学机械化卷I

——Mathematics Mechanization: Mechanical Geometry Theorem-Proving, Mechanical Geometry Problem-Solving and Polynomial Equations-Solving

吴文俊/著 高小山/编订

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内 容 简 介

本卷收录了吴文俊的 *Mathematics Mechanization: Mechanical Geometry Theorem-Proving, Mechanical Geometry Problem-Solving and Polynomial Equations-Solving* 一书. 本书是围绕作者命名的"数学机械化"这一中心议题而陆续发表的一系列论文的综述, 试图以构造性与算法化的方式来研究数学, 使数学推理机械化以至于自动化, 由此减轻繁琐的脑力劳动.

全书分成三个部分: 第一部分考虑数学机械化的发展历史, 特别强调在古代中国的发展历史. 第二部分给出求解多项式方程组所依据的基本原理与特征列方法. 作为这一方法的基础, 本书还论述了构造性代数几何中的若干问题. 第三部分给出了特征列方法在几何定理证明与发现、机器人、天体力学、全局优化和计算机辅助设计等领域中的应用.

本书可供数学工作者、数学及计算机专业高年级大学生和研究生以及有关工程人员参阅.

图书在版编目(CIP)数据

吴文俊全集. 数学机械化卷. Ⅰ/吴文俊著; 高小山编订. —北京: 龙门书局, 2019.5

国家出版基金项目

ISBN 978-7-5088-5550-9

Ⅰ. ①吴… Ⅱ. ①吴… ②高… Ⅲ. ①数学理论-文集 Ⅳ. ①O1-53

中国版本图书馆 CIP 数据核字(2019) 第 074591 号

责任编辑: 赵彦超 李 欣 / 责任校对: 邹慧卿

责任印制: 肖 兴 / 封面设计: 无极书装

科学出版社 出版

北京东黄城根北街 16 号

邮政编码: 100717

http://www.sciencep.com

北京通州皇家印刷厂印刷

科学出版社发行 各地新华书店经销

*

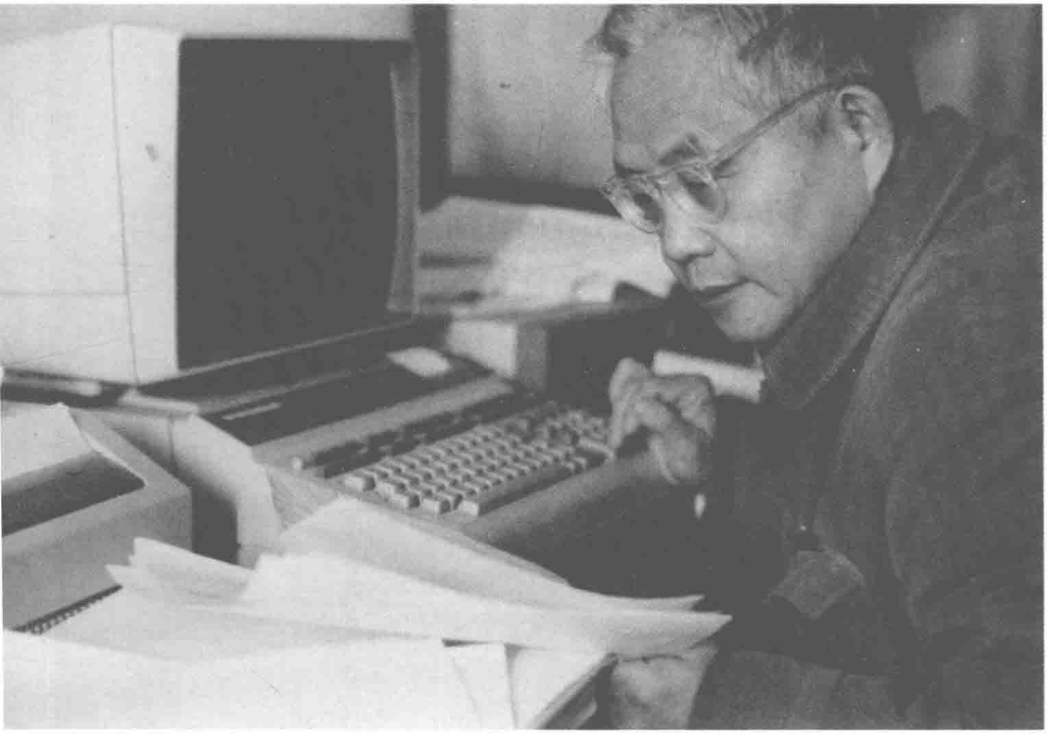
2019 年 5 月第 一 版 开本: 720 × 1000 1/16

2019 年 5 月第一次印刷 印张: 26 3/4

字数: 540 000

定价: 188.00 元

(如有印装质量问题, 我社负责调换)



纪念吴文俊先生诞辰100周年

编 者 序

中国现代数学的崛起，开始于20世纪初，经历了几代人坚苦卓绝的努力. 在这百年奋战中涌现出来的数学家中，吴文俊是最杰出的代表之一. 他早年留学法国，留学期间就已在拓扑学方面做出了杰出贡献，提出了后来以他的名字命名的“吴公式”和“吴示性类”. 回国后提出了“吴示嵌类”等拓扑不变量，发展了统一的嵌入理论. 他关于示性类与示嵌类的研究，已成为20世纪拓扑学的经典，至今还在前沿研究中使用. 20世纪70年代以来，吴文俊院士在汲取中国古代数学精髓的基础上，开创了崭新的现代数学领域——数学机械化. 他发明的被国际上誉为“吴方法”的数学机械化方法，改变了国际自动推理的面貌，形成了自动推理的中国学派，已使中国在数学机械化领域处于国际领先地位. 上述工作无疑属于 20 世纪中国数学赶超国际先进水平的标志性成果，而吴文俊院士博大精深的科学研究，除了拓扑学与数学机械化以外，还跨越了代数几何、博弈论、中国数学史、计算图论、人工智能等众多领域，并在每个领域都留下了这位多能数学家的重要贡献.

吴文俊先生是一位具有强烈爱国精神的数学家. 自 1950 年谢绝法国师友的挽留回到祖国后，半个世纪如一日，为在他深爱的中华故土发展数学事业而鞠躬尽瘁. 除了第一流的科研成果，吴文俊先生长期身处中国数学界领导地位，在团结带领整个中国数学界赶超世界先进水平方面，也做出了不可磨灭的贡献. 特别是，吴文俊先生在担任中国数学会理事长期间，领导中国数学最终成功地加入了国际数学联盟，此举大大提高了我国数学界的国际地位，同时也为我国成功举办 2002 年国际数学家大会铺平了道路.

吴文俊治学严谨，学术思想活跃，无论获得多么高的声誉，他总是勤奋地在科研第一线工作，一生积极进取，锲而不舍，不断取得新的成就. 在开始从事机器证明时，他已近花甲之年，从零开始学习编写计算机程序，每天十多个小时在机房连续工作，终于在几何定理机器证明这一难题上取得成功.

吴文俊先生为中国现代数学的发展建立了丰功伟绩，而他本人却始终淡泊、谦逊. 他处事公正豁达，待人充满善意，受过他帮助的人可以说不计其数. 正因如此，这位有着崇高国际声望而平易近人的学者，受到了每一个认识他的人格外的爱戴与尊敬.

2019 年 5 月 12 日是吴文俊先生百年诞辰. 为了纪念这个特殊的日子，我们编辑出版了《吴文俊全集》，通过系统地收录、整理吴文俊先生的学术著作和论文，纪

念吴先生的学术思想及学术成就. 全集共计 13 卷, 包括拓扑学 4 卷、数学机械化 5 卷以及数学史、博弈论与代数几何、数学思想各 1 卷; 同时, 全集还设有附卷, 收录吴文俊先生的同事、学生和其他社会各界人士发表过的与吴先生有关的各类文献资料.

最后, 我们对在全集编辑中给予帮助的各位同事表示衷心感谢; 感谢国家出版基金对于全集出版的资助; 感谢科学出版社编辑人员在出版全集时认真细致的专业精神; 感谢相关出版与新闻机构在版权方面提供的帮助.

李邦河　高小山　李文林

2019 年 3 月

Preface

The present book is actually a collection of essays of the writer since 1977 centered around the subject which has been baptized the name of *mathematics mechanization.* The subject tries to deal with mathematics in a constructive and algorithmic manner so that the reasonings become mechanical, automated, and as much as possible to be intelligence-lacking, with the result of lessening the painstaking heavy brain-labor.

The desire of getting mathematics mechanized or more generally of getting all kinds of reasonings automated, may be traced back to G.W. Leibniz (+1646, +1716). However, the real founding of mathematics mechanization, or more generally automated reasoning, should be, in the writer's opinion, attributed to D.Hilbert (+1862, +1943). The great merit of Hilbert on mathematics mechanization seems to be long time overshadowed by his great success on his existence proof of finite basis theorem and his works on mathematics axiomatization, as well as his controversies with L.Kronecker (+1823, +1891), who may be considered as one of the banner-holders of mathematics mechanization. In spite of these, Hilbert is well-deserved to be the real founder of mathematics mechanization, as reflected by his other kinds of great achievements or influences, for which some examples are listed below:

Example 1. In Hilbert's famous speech on mathematical problems in 1900 the Problem 10 asks for an automated method of diophantine equations-solving. Though the Problem has been solved in the negative in 1970 by Matijasevic, the positive aspects of the negative solution have been shown to be able to bear extremely important consequences to modern pure mathematics. See e.g. the paper of M.Davis, et al in *Proc.Symp. in Pure Math.*, vol.28, (1976).

Example 2. In Hilbert's classic *Grunlagen der Geometrie* in 1899, there is some obscure passage, which, when reinterpreted, is equivalent to an automated method of proving a whole class of geometry theorems of some special type. The well-known theorem of Tarski in 1950 and the latter developments of the writer on mechanical geometry theorem-proving are nothing else but continuation of this method of Hilbert in the narrow realm of elementary geometry. For this reason the writer has baptized the above result of Hilbert, formulated as Theorem 62 in latter editions of

Grundlagen, *Hilbert's Mechanization Theorem.*

Example 3. Hilbert's program about consistency of mathematics implies a program of automated proving of *all* theorems in mathematics by a universal method. The discovery of Goedel has declared the collapse of Hilbert's program in being too ambitious to be realizable. Nonetheless Hilbert's efforts have given rise to a new discipline mathematical logic and show a new way of proving theorems or solving problems class by class under the name of decidability of which Hilbert's Mechanization Theorem furnishes a simple but extremely deep and instructive example. It opens a new way of dealing with mathematics: mechanization of mathematics.

Besides these we may note that there are papers of Hilbert with theorems proved in a constructive and algorithmic manner. As an example, we may cite the proof of the theorem concerning what we call nowadays Hilbert function of an ideal. We may even note that the proof of his finite basis theorem is achieved in forming the finite basis in a somewhat constructive and algorithmic manner.

The present book may be considered as a primary development of mathematics mechanization along the line of thought of Hilbert.

The whole book is divided into three parts. Part I, consisting of Chapters 1 and 2, concerns historical developments of mathematics mechanization, with emphasis on the developments in ancient China. This is quite natural. In fact, in contrast to the modern development of pure mathematics in accordance to the Euclidean axiomatic pattern of ancient Greece, the ancient mathematics of China is highly constructive, algorithmic and computational in character, with most of the results expressed in the form of algorithms. Even in geometry, the ancient Chinese mathematicians paid little attention to geometry theorem-proving. On the contrary, they devoted themselves rather on geometry problem-solving which leads naturally to the solving of polynomial equations. Accordingly, the problem of polynomial equations-solving occupies a central position throughout thousands of years of development of mathematics in ancient China. In present book we follow closely the tradition of our ancient mathematics in focusing on polynomial equations-solving with geometry theorem-proving and geometry problem-solving as its two main applications. This attitude of treatment is well-reflected in the subtitles of the present book.

The Part II, which contains the three chapters 3, 4, and 5, bears the title of Principles and Methods. It aims at the description of the underlying Principle of polynomial equations-solving, with polynomial-coefficients in fields restricted to case of characteristic 0. Based on the general Principle, we show how general Methods of solving such arbitrary polynomial systems may be found. In order to achieve this we shall use the naive notion of *zero-sets* of polynomial sets in forsaking the usual notions of ideals. An analysis of the zero-set of a polynomial set, under the guidelines of the Principle give rise to the fundamental notion of *characteristic set* (abbr. *char-set*) and the further Principles under the names: Well-Ordering Principle, Zero-Decomposition Theorems, and Variety-Decomposition Theorem. These Principles form the basis of actual polynomial equations-solving.

It is to be noted that the underlying Principle of polynomial equations-solving had its origin in ancient Chinese mathematics, notably in the classic *Jade Mirror of Four Elements* in year +1303 of Zhu Szijie. Of course, there are incompleteness and deficiencies in Zhu's work, but uncontestedly it shows the right way of solving arbitrary systems of polynomial equations. To complete the way indicated in Zhu's work we have relied upon the important works of J.F.Ritt as exhibited in his two classics on the algebraic studies of differential equations. Ritt's theory and method are highly constructive, algorithmic, and computational in character which rely in turn heavily on the *early* works of Van der Waerden on algebraic geometry. In contrast to the later developments of modern algebraic geometry, the *early* works of Van der Waerden on algebraic geometry is essentially constructive, algorithmic, and in the main computational. It is based on the very useful notions of *generic point* and specialization, which have unfortunately disappeared in modern algebraic geometry. As a concrete application of Van der Waerden's treatment, we show in Sect. 3 of Chap. 5, how to define the important notions of Chern classes and Chern numbers of algebraic varieties with arbitrary singularities, in basing on the notion of generic points. Though there are diverse alternative definitions for getting such extended notions of Chern classes but not of Chern numbers, our treatment has the peculiarity of being computational, giving for example the remarkable Miyaoka-Yau inequality and its extensions by mere simple computations. We refer this to Sect. 3 of Chap. 5. A further application of this notion of generic pointis is given in Sect. 4 of Chap. 8 in Part III.

Usually polynomial equations are to be solved in complex field. However, to solve in real field is clearly of utmost importance. An example is furnished by optimization problems for which only real solutions are to be considered. The general method given in Chap. 3, which permits to give, theoretically speaking, a complete answer to the problem of polynomial equations-solving in the complex case, furnishes however merely a very incomplete answer with only partial success in the real case. In Sect. 5 of Chap. 5, we point out that, for the important optimization problems, the polynomial case is quite different from the general differential case. In fact, we show that, in the polynomial case of optimization problems, there is a *finite* set of real values, for which the optimal value, supposed exist, is just the optimal value of this finite set. Our method permits also to give partial success for problems involving inequalities. However, the method is far from being satisfactory and much has to be done in the realm of problems in the real field.

Part III, containing chapters 6, 7, and 8, bears the title of Applications and Examples. Chap. 6 concerns applications to polynomial equations-solving. Remark that, though our principles and methods developed in Part II theoretically furnish complete solutions of arbitrary polynomial equations over fields of characteristic 0, the computational complexity is so high that, in actual computations one meets often unsurmountable complications to be able to carry out the computations up to the end. It seems that the only issue is to find some hybrid method of combining our symbolic computations with the usual numerical computations. We give such a hybrid method in Sect. 2 of Chap. 6 with a sound theoretical basis. The writer believes that the method will actually permit us to settle the question for all kinds of problems arising from practical applications but experiments are yet to be done.

Chap. 7 of Part III deals with geometry theorem-proving which is treated as applications of our general principles and methods of polynomial equations-solving. Chap. 8 of Part III deals with diverse problems which, in apparence having nothing to do with polynomial equations, are reduced to some form of polynomial equations-solving and then treated by our general methods.

As a preliminary step toward mathematics mechanization, the present book has to be restricted to the polynomial case for equations-solving, and mainly to elementary geometries for theorem-proving and problem-solving. Only some indications about possible developments in differential geometry and differential equations are slightly touched in the last section, Sect. 5 of Chap. 8. We shall leave the studies of mathematics mechanization in the differential case and in other domains of mathematics in latter occasions.

The writer would like to take this opportunity to express his deep gratitudes to the National Committee of Science and Technology, the National Natural Science

Foundation of China, and the Chinese Academy of Sciences. Without their financial supports and encouragements mathematics mechanization is impossible to be so prosperous in China, today, and also tomorrow.

The writer would like also to express his deep gratitudes to his comrades and colleagues, especially those in Mathematics Mechanization Research Center (MMRC) of Institute of Systems Science, CAS, who have contributed so much to the development of mathematics mechanization, and to the writing of the present book. As they are so numerous and most of their names are scattered throughout the whole book, the writer would like to apologize in mentioning no more their names here in the Preface.

Contents

Chapter 1

Polynomial Equations-Solving in Ancient Times, Mainly in Ancient China

1.1 A Brief Description of History of Ancient China and Mathematics Classics in Ancient China.

Equations-Solving and Theorem-Proving.

There are two main activities of mathematics: theorem-proving and equations-solving. While theorem-proving was originated in ancient Greek mainly in so-called Euclidean geometry, equations-solving was the main concern of mathematics in ancient China. In order to give a brief description of equations-solving of China in ancient times, let us first give a brief chronicle of the history of ancient China.

Brief Chronicle of China.

China turned from tribes dispersed throughout the vast territory into a loosely centralized dynasty in 21st century B.C. and was unified into an empire under the first emperor of Qin in 221B.C. Since that time China was unified and split, split and unified, over and over until the beginning of the 20th century with the collapse of the feudalistic dynasty system. The main periods of ancient China are listed below, the years before and after Christ being indicated by negative and positive numbers respectively.

Xia (about (−21c, −16c)).

Yin or Shang (about (−16c, −11c)).

Western Zhou (about −11c, −771).

Spring and Autumn Period (−770, −476).

Warring-States Period (−475, −221).

Qin Dynasty (−221, −207).

Western Han Dynasty (−206, +8).

Eastern Han Dynasty (+25, +220).

Three Kingdoms Period (+220, +265).

Western Jin (+265, +316).

Eastern Jin and 16 Kingdoms (+317, +420).

Southern and Northern Dynasties (+420, +589).

Sui Dynasty (+581, +618).

Tang Dynasty (+618, +907).

Five Dynasties and Ten Kingdoms Period (+907, +960).

Northern Song Dynasty (+960,+1127) and Liao Dynasty (+938, +1125).

Southern Song Dynasty (+1127,+1274) and Jin Dynasty (+1127, +1234).

Yuan Dynasty (+1271, +1368).

Ming Dynasty (+1368, +1644).

Qing Dynasty (+1644, +1911).

Some Characteristic Features of Chinese Ancient Mathematics.

1. Instead of calculations of pencil-paper type, the ancient Chinese made all computations in manipulating rods on counting boards or anything alike to serve the purpose. Counting rods made of bones or bamboo had been excavated in years 1971 and 1975 which date back to Western Han Dynasty in 1c or 2c B.C. See the photograph in Fig. 1.1.

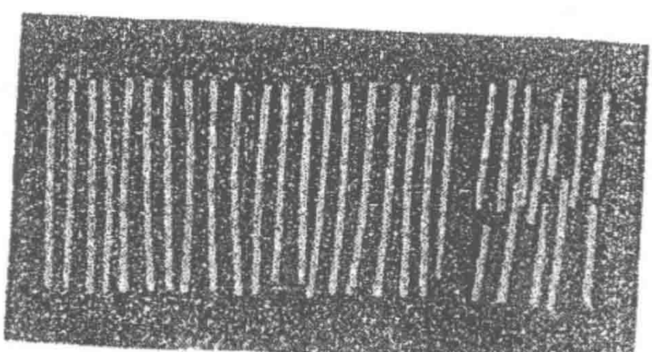


Fig. 1.1.

2. The Chinese already possessed, in very remote times, the most perfect *place-valued* decimal system. This allowed them to represent the integers by properly arranged rods placed in due positions on the board. In particular, the integer 0 in, or as, a decimal integer was just represented by leaving some empty place in the right position. Thus, the integer 22022 will be represented on the board in the form

‖ = = ‖

Highly efficient arithmetic operations should have been well-developed much earlier than the Qin Dynasty. In fact, the word "arithmetic", the usual terminology for "mathematics", was just a literal translation of Chinese characters "Suan Shu" meaning "counting methods."

3. Results were usually expressed in the form of separate problems, each of which was divided into several items, as follows.

(1) Statement of the problem with numerical data.

(2) Numerical answer to the problem.

(3) "Shu", or the method of arriving at the result. It was most often just what we call today the "algorithm", sometimes also just a formula or a theorem. Note that the numerical values in (1) play no role at all in the method, which was so general that any other numerical value of same type could be substituted equally well. Item (1) thus served just as an illustrative example which shows the motivation of considering such kind of problems to be solved by this "Shu".

(4) Sometimes "Zhu", or demonstrations which explained the reason underlying the method in Item (3). It was actually a "proof" when "Shu" was actually a formula or a theorem.

(5) "Cao", meaning drafts, containing details of the calculations for arriving at the final result, appeared first in some classic of Tang Dynasty. It was then widely added in writings of Song and Yuan Dynasties. This was possible since printing measures were well-developed in these days.

4. The main concern of our ancestors was the solving of problems arising in reality or from necessity which naturally led to the solving of equations. Thus instead of devoting to theorem-proving as in the ancient Greek mathematics, equations-solving was the central theme of Chinese ancient mathematics. In fact, equations-solving may be considered as a golden thread permeating the whole of Chinese ancient mathematics which will be explained in more details in the next section.

5. For geometry instead of developing a deductive system for theorem-proving, measuration and geometry problem-solving were emphasized based on some different way of proof-procedure which will be explained in more details in Sect. 3 of Chap. 2.

6. In a word, the ancient Chinese mathematics was highly computational and *mechanical* in character. It seems that it was developed in its own way, indepen-