ADVANCES IN ECONOMETRICS AND QUANTITATIVE ECONOMICS

Edited by
G. S. MADDALA,
PETER C. B. PHILLIPS, and
T. N. SRINIVASAN

Advances in Econometrics and Quantitative Economics

Essays in honor of Professor C. R. Rao

Edited by

G. S. Maddala,
Peter C. B. Phillips,
and
T. N. Srinivasan

BLACKWELL
Oxford UK & Cambridge USA

First published 1995

Blackwell Publishers Inc.
238 Main Street
Cambridge, Massachusetts 02142, USA

Blackwell Publishers Ltd
108 Cowley Road
Oxford OX4 1JF
UK

Library of Congress Cataloging in Publication Data

Advances in econometrics and quantitative economics: essays in honor of C. R. Rao / edited by G. S. Maddala, Peter C. B. Phillips and T. N. Srinivasan.
p. cm.
Includes bibliographical references and index.
ISBN 1-55786-382-2 (acid-free paper)
1. Econometrics. 2. Economics, Mathematical. I. Rao, C. Radhakrishna (Calyampudi Radhakrishna), 1920– . II. Maddala, G. S. III. Phillips, P. C. B. IV. Srinivasan, T. N., 1933– .
HB139.S688 1995 94-48513
330'.01'5195—dc20 CIP

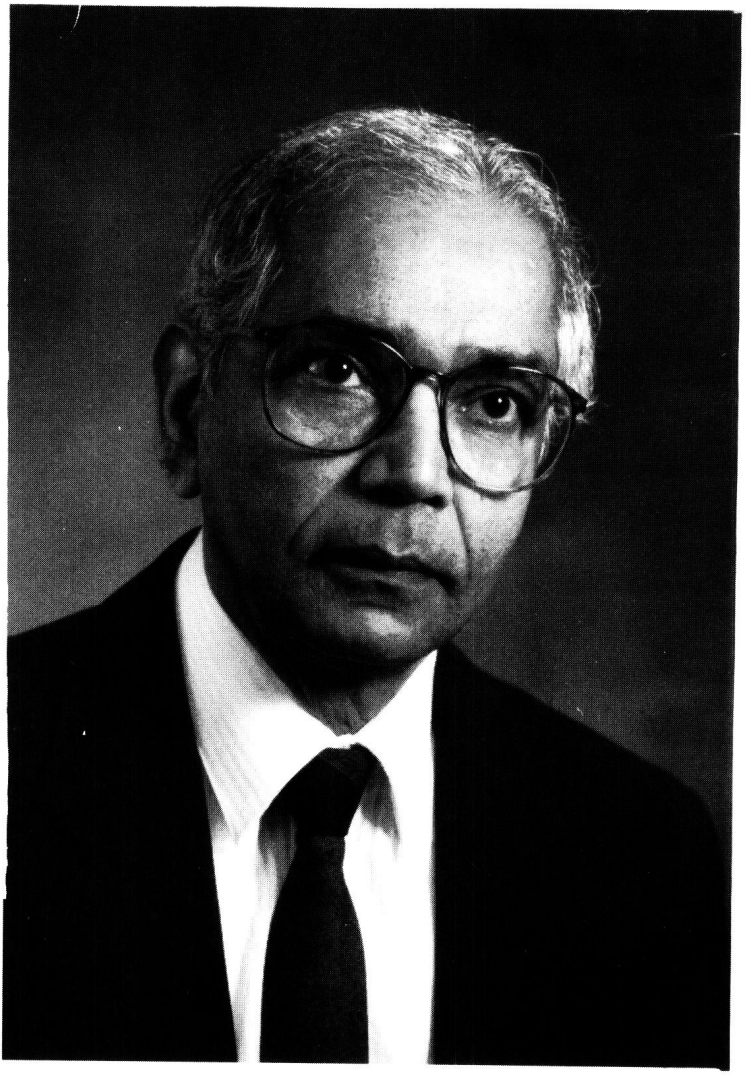
British Library Cataloguing in Publication Data

A CIP catalogue record for this book is available from the British Library.

Typeset in 10 on 12 pt Times
by Best-set Typesetter Ltd., Hong Kong
Printed in Great Britain by Hartnolls Limited, Bodmin, Cornwall

This book is printed on acid-free paper.

Advances in Econometrics and Quantitative Economics



C. R. Rao

Contributors

T. W. Anderson	Stanford University, USA
R. N. Bhattacharya	Indiana University, USA
C. L. Cavanagh	Columbia University, USA
J. G. Cragg	University of British Columbia, Canada
S. G. Donald	Boston University, USA
L. Gardiol	University of Lausanne, Switzerland
C. Gourieroux	CREST – CEPREMAP, Paris, France
A. Holly	University of Lausanne, Switzerland
U. Jensen	University of Kiel, Germany
O. Linton	Oxford University, UK
M. Loretan	Federal Reserve Board, Washington DC, USA
G. S. Maddala	The Ohio State University, USA
A. Monfort	CREST – Paris, France
M. Nerlove	University of Maryland, USA
W. K. Newey	Massachusetts Institute of Technology, USA
P. C. B. Phillips	Yale University, USA
C. A. P. Pinske	University of British Columbia, Canada
M. L. Puri	Indiana University, USA
E. Renault	GREMAQ – University of Toulouse, France
P. M. Robinson	London School of Economics, UK
T. J. Rotherberg	University of California, Berkeley, USA
T. Schuermann	AT&T Bell Labs, USA
K. Tanaka	Hitotsubashi University, Japan
P. K. Trivedi	Indiana University, USA
J. M. Wooldridge	Michigan State University, USA

Preface

Professor C. R. (Calyampudi Radhakrishna) Rao will be seventy-five on September 10, 1995. This collection of essays celebrates both Professor Rao's many fundamental contributions to statistical foundations of econometrics and his seventy-fifth birthday.

An interesting account of how Professor Rao started to work in statistics can be found in his autobiographical essay (Rao, 1991). He joined the Indian Statistical Institute in 1941, after receiving an MA with a first class and first rank in mathematics from the Andhra University in 1940. He obtained an MA degree in statistics from Calcutta University in 1943 with a first class, first rank and a gold medal. Subsequently he went to Cambridge England to work under R. A. Fisher and was awarded his doctorate in 1948.

A steady stream of original contributions from him began in the early 1940s and continues to flow in good measure to this day. A very early contribution, later to become famous as the Cramér–Rao inequality, was derived by him in 1943 (in a hard night's work in response to the question of a student, V. M. Dandekar, who later became one of India's leading economists). Because of the suspension of the publication of journals in India during the Second World War, this paper (Rao, 1945) was published only in 1945. It contains many other results, including what was later known as the Rao–Blackwell theorem. This paper is also notable for its introduction of differential geometric methods in statistical inference for the first time. It is also perhaps the first attempt to discuss the theme of estimation in small samples in terms of actual variance and methods of obtaining best unbiased estimates.

Econometricians extensively use what they call the LM (Lagrangean Multiplier) test following the terminology introduced by Silvey. In fact, this test is none other than the "score" test introduced in 1947 by Professor Rao ten years before Silvey. Besides the score test, many other tests and techniques in econometrics can be traced to Professor Rao's pioneering work. For example, a very popular test in econometrics that is known as the Hausman test follows from a result (on page 317) of Professor Rao's book *Linear Statistical Inference* (1973). This book has been translated into six major languages of the world. It is the one most likely to be on the bookshelf of almost every econometrician because it contains an extraordinarily valuable compendium of results in matrix algebra as well as statistical theory. According to the Social Science Citation Index, it is the most cited book in statistics in econometric work.

As Director of the Research and Training School at the Indian Statistical Institute (ISI), Professor C. R. Rao developed a variety of educational programs in the theory and applications of statistics at various levels. Under his leadership, the ISI won international recognition as an outstanding center for theoretical disciplines. His colleagues, students and others who have worked with him at the ISI and elsewhere over the years have experienced his wonderful hospitality and charm and his characteristic modesty. *The Times of India* (December 31, 1988) chose Professor C. R. Rao to be among the top ten greatest contributors of all time to Indian science.

Professor Rao's many contributions to theoretical statistics in general and multivariate analysis in particular arose from his involvement in the analysis of large data sets at the ISI, such as the Anthropometric Survey in 1941 in the United Provinces of India. He believed with R. A. Fisher that the development of statistical methodology must be firmly rooted in the problems of inference thrown up by actual data. His interest in careful collection and analysis of socio-economic data led to his founding the Indian Econometric Society to encourage econometric work in India. With his enthusiastic support, the Indian Econometric Society has held seven conferences since 1972 on the data base of the Indian economy and has so far published five volumes of conference papers.

We believe that econometricians would find many contributions of Professor Rao useful in their work, and have therefore appended his complete bibliography to this volume. It is hard to do full justice to the many facets of Professor Rao's work in a single volume. The present

volume is just a modest contribution by some econometricians in tribute to his work.

G. S. Maddala
P. C. B. Phillips
T. N. Srinivasan

References

Rao, C. R. (1945) Information and accuracy attainable in the estimation of statistical parameters. *Bulletin of the Calcutta Mathematical Society*, 37, 81–91 (also published in *Breakthroughs in Statistics, volume 1: Foundations and Basic Theory*. New York: Springer-Verlag, pp. 235–48).

Rao, C. R. (1947) Large sample tests of statistical hypotheses concerning several parameters with applications to problems of estimation. *Proceedings of the Cambridge Philosophical Society*, 44, 50–7.

Rao, C. R. (1973) *Linear Statistical Inference and Its Applications*, 2nd edn. New York: Wiley.

Rao, C. R. (1991) *Statistics as a Last Resort: an Autobiographical Account of Research Work*. Published by T.D. Dwivedi, Concordia University, Montreal.

A Conversation with C. R. Rao

Morris H. DeGroot

Calyampudi Radhakrishna Rao was born on September 10, 1920, in Hadagali, Karnataka State, India. He received an MA in mathematics from Andhra University in 1940; an MA in statistics from Calcutta University in 1943; a PhD from Cambridge University in 1948, with a thesis entitled "Statistical problems of biological classification"; and an ScD from Cambridge in 1965 on the basis of his published work in statistics as a whole. He joined the Indian Statistical Institute as a statistician in 1944 and became a professor in 1949, the director of the Research and Training School in 1964, the secretary and director of the Institute in 1972, and the Jawaharlal Nehru Professor in 1976. In 1979 he was appointed a University Professor in the Department of Mathematics and Statistics at the University of Pittsburgh. He has been the President of the International Statistical Institute, the Institute of Mathematical Statistics, the International Biometric Society, and the Indian Econometric Society; and since 1964, an editor of *Sankhyā*. He is a Fellow of the Royal Society (FRS) of the United Kingdom; a Foreign Honorary Member of the American Academy of Arts and Science; an Honorary Member of the International Statistical Institute; an Honorary Fellow of the Royal Statistical Society; an Honorary Life Member of the Biometric Society; a Fellow of the Indian National Science Academy and Indian Academy of Sciences; an Honorary Fellow of King's College, Cambridge; and a Founder Fellow of the Third World Academy of Sciences, Trieste. He has received honorary degrees from Andhra University, India, 1967; Leningrad University, USSR, 1970; Delhi University, India, 1973; Athens University, Greece, 1976; Osmania University, India, 1977; Ohio State University, USA, 1979; Universidad Nacional de San Marcos,

Peru, 1982; University of the Philippines, 1983; and University of Tampere, Finland, 1985.

The following conversation took place in his office at the University of Pittsburgh one morning in November 1985.

DeGroot: To start at the beginning, tell me a little about your childhood in India, what your home life was like, and how you came to get interested in statistics.

Rao: Well, I come from a family of a kind of landed aristocracy. They had a lot of property and never cared to go and study and try to make a living on the knowledge that they acquired in their studies. However, at one stage they were interested in the legal profession and they all became lawyers and judges and so on.

DeGroot: Who do you mean by "all"?

Rao: My ancestors, my relations. I was, so to say, the first one to get interested in science. I started off with mathematics, but then the war broke out when I was just finishing my master's degree in mathematics and there were not many opportunities for those with a degree in mathematics to get a job. So I tried to get a job in the military service. I applied to the Department of Survey, which is sort of a party to any military expedition. I was disqualified because I was too young and they didn't want me, but that took me to a place called Calcutta. I belonged to the South but I had gone to Calcutta for the interview to get into the military service. At that time I came to know of the Indian Statistical Institute which had been established by Mahalanobis, so I just casually visited the place and talked to people there. They said that statistics was a new subject and if there are no opportunities for a job with a mathematics background, why not come and study statistics.

So I applied for a course of studies at the Indian Statistical Institute and I was admitted. At that time there were no courses in statistics in the universities. Some universities used to teach statistics as a part of the undergraduate program but not at the postgraduate level. The Indian Statistical Institute was the first place to have what they called a one-year training course in statistics. I took that course, but by the time the course ended Calcutta University had started a master's program in statistics. Mahalanobis said to me, "You join the university and get a formal degree." Since a formal degree is always useful in India as a passport to get jobs, I joined Calcutta University and got a master's degree in statistics. I am one of five first batch students to be awarded an MS degree in statistics by an Indian University. So that's how it all began.

DeGroot: When did you then go to Cambridge and how did that come about?

Rao: After I finished my master's degree, Mahalanobis asked me to work on anthropometric data which had been collected by Dr. D. N. Mazumdar, a well-known anthropologist, in the state of Uttar Pradesh during the 1941 Indian population census. The data were sent to the Indian Statistical Institute for analysis; at that time Mahalanobis had already invented the Mahalanobis distance for studies in anthropology. He assigned the project to me. So my first experience was analyzing data collected on some five thousand individuals, with twenty characters measured on each individual. I produced a report using the Mahalanobis D^2 (distance) in the analysis, and Mahalanobis was very happy with that.

Right at the time when I finished that report, he got a telegram from the anthropological museum in Cambridge saying that a British expedition from Africa had brought a large number of bones and stones and so on. Would he please send one of his students to analyze the data. Mahalanobis replied, "Of course. I have one who is very good in statistics, but he does not know anthropology. So I'll send an anthropologist along with Rao to help you with this project." So we went there and we found a very interesting collection of bones. They had dug out ancient graves, about 1,000 years old, from a place called Jebel Moya in North Africa, and brought the skeletal material to the museum in Cambridge. I was actually employed in the Cambridge museum for a couple of years to work on the skeletal material. Incidentally I joined King's College, where Mahalanobis used to be when he was a student, and I also registered for a PhD degree under R. A. Fisher. But Fisher said I must work on his mice, whether it leads to a thesis or not. So I was doing some experiments on mice for R. A. Fisher which involved the mapping of chromosomes, in addition to my work at the Anthropological Museum.

DeGroot: He actually was treating live mice in his experiments?

Rao: Oh, yes. He had a laboratory where he was breeding mice because he was mapping their chromosomes. Lots of mice, maybe thousands. I think we were some ten people working there at that time. He asked me to study the linkage of four genes on a chromosome and find the distances between them. So I was doing that project for R. A. Fisher in the evenings, and in the daytime I was working in the anthropological museum. After two years we wrote the report on this skeletal material which is published as a book by the Cambridge University Press, *Ancient Inhabitants of Jebel Moya* [with R. K. Mukherji and J. C. Trevor, 1955]. The Cambridge anthropologist and

my Indian colleague who was an anthropologist also got their PhD theses out of that same project. I think my Indian colleague wrote on the anthropological technique and interpretation of results. The Cambridge anthropologist was also a sociologist, so he looked at the artifacts along with the bones. He made his own conclusions and we put everything together.

DeGroot: And your PhD dissertation?

Rao: My PhD dissertation was based on the theoretical work I did in connection with this project, which is partly in the book. But I had some additional results in my thesis. Fisher's discriminant function was used for classification within two categories, and I developed a method for many categories. I also developed another test. We had to do all the multivariate computations by hand at that time. If I had fifty characters it would be impossible to use all of them, so selection of characters was a major problem. I invented a test of whether it is really necessary to take so many characters. I said, "All right, given a set of measurements, is there further information in an additional set of measurements?" So this is the kind of test which I developed and which went into the thesis. R. A. Fisher was my supervisor. Wishart was an examiner.

DeGroot: Was Fisher much involved in your work?

Rao: Well, he thought my test for additional information was very good because it was an extension of a test he developed in discriminant analysis. That is, if somebody specified the discriminant function, he could examine whether that was the true discriminant function or not. This really involved the idea that the given discriminant function carries all the information; given that function, nothing else is needed. So when I extended that test to a given set of measurements, not necessarily one discriminant function, Fisher was happy. I also mentioned to Fisher that I was working on the problem of classification into more than two categories. I used the Bayesian approach to the problem. He asked me to work along a different line, but I used essentially Bayesian techniques. [Laughs] That's the appropriate thing to do in problems of that kind. Later on I described what you should do when you could not know the prior probabilities. I also created a kind of doubtful region where you can take the position that you are unable to say whether an observation belongs to this group or that group, and described how to operate with that region. That is an abstract of what I did for my thesis.

DeGroot: With both Fisher and Wishart at Cambridge at that time, there must have been a lot of statistical activity around the university.

Rao: That is true, but Fisher was mostly interested in mathematical genetics. So I took various courses in mathematical genetics. I had enough knowledge when I came back to India to direct research work by my students in mathematical genetics. I actually produced three PhD's in mathematical genetics at that time, very valuable people, but unfortunately they are all working in this country. And Wishart of course was in the school of agriculture. Bartlett was there for a while; then he went to Manchester and Daniels came. About that time the statistical laboratory in Cambridge was established for training statisticians. We had students like Durbin and Bailey and some others. Most of the statisticians at that time were being trained in the statistical laboratory at Cambridge, but it just started in '47 or '48.

DeGroot: Was it unusual for an Indian to be studying at Cambridge at that time?

Rao: No, lots of Indians used to go to Cambridge. It was the place that Indians went for their higher education. But I didn't study in Cambridge in that sense. I was working in Cambridge at the museum, but formally attached to King's College as a research scholar. Later on, King's College honored me by electing me a life fellow. When I was formally enrolled as a research student there, I was not allowed to walk on the lawns in King's College.

DeGroot: I've seen those signs: "Only fellows are allowed to walk on the grass."

Rao: Right. But now I have the privilege of walking on the lawns, and whenever I go there I can stay in the college, have a free dinner, and probably drink as much wine as I want. [Laughs] There are only eleven life fellows at any time. Only if somebody dies do they elect another one.

DeGroot: Some of the most famous results in statistics have your name on them. One example is the Cramér–Rao inequality, as it is commonly known. I know that Cramér and you did not work on this together. Could you tell me how it came about that your names are paired on this result in this way?

Rao: After I completed my master's degree in 1943, I was asked to teach my juniors in the master's course. There was a shortage of teachers. I think R. C. Bose and S. N. Roy were the two main teachers, and there were a few others.

DeGroot: This was at the Indian Statistical Institute?

Rao: At both the Institute and Calcutta University. You see, when Calcutta University started the master's program they had no teachers, so they borrowed all the teachers from the Indian Statistical Institute.

Actually, the lectures were held in the Indian Statistical Institute. Many of our teachers did not know much statistics. They were trying to learn and teach the students. So when I had my formal education, they thought I would be a good teacher and I was asked to teach. I was giving a course on estimation covering all the large-sample properties of maximum likelihood. I proved the asymptotic inequality of R. A. Fisher that the asymptotic variance is not smaller than one divided by the information. Then one student asked, "Why don't you prove it for finite samples?" So I went back home and worked all night. The next day I came up with this inequality, using unbiasedness and the Schwarz inequality. I proved this result in the class in January 1944. Once I started all this, I looked at sufficient statistics. Of course the maximum likelihood estimate is a function of sufficient statistics. Is there an independent small sample justification for sufficient statistics? I thought I must be able to prove some exact properties, and that's how the Rao–Blackwell theorem came up. Which I did two years before Blackwell.

DeGroot: So your work on that type of improvement of an estimator really grew out of your work on inequalities.

Rao: Yes, they are all in the same paper. ["Information and accuracy attainable in the estimation of statistical parameters," *Bull. Calcutta Math. Soc.*, 37 (1945) 81–91.] That's in 1945. Blackwell's work was in 1947. I saw Cramér's work on the information inequality when his book on *Mathematical Methods of Statistics* came out in 1946. That is sort of a basic paper mostly arising out of my teaching that course on estimation. I also developed the differential geometric method. I tried to connect Fisher information with a metric in a Riemann space. So my differential geometric approach to estimation is also in the same paper.

DeGroot: That's quite a paper. This differential geometry approach to estimation is now becoming of interest to many statisticians.

Rao: Yes. It's so difficult, nobody liked to work on problems of that kind. I am gratified to see that the geodesic distance I defined using the Fisher information metric is now referred to as Rao distance. Now to complete the story, Fisher came out with second-order properties of a maximum likelihood estimator. That's a very difficult concept and Fisher's investigation lacked some details. Since very difficult computations were involved I tried to modify his definition of second-order efficiency and came out with a quantity which characterizes maximum likelihood estimators in a sense if you go to second-order terms by expanding the asymptotic variance. In the second-order

terms, the maximum likelihood estimator has the smallest coefficient. Now that coefficient has been termed the Fisher–Rao inequality by Efron.

DeGroot: That notion of second-order efficiency is very closely associated with you.

Rao: It really started with Fisher. What I did was try to correct Fisher's computations and also modify his criterion of the minimum loss of information because it is very difficult to work with. That's an exact property which does not really hold, but asymptotically to the second order that idea works.

DeGroot: Who do you feel have been the major influences both on your life and on your career?

Rao: I would say Professor Mahalanobis in India. His whole idea was that statistics must have a purpose. So he said, don't try to solve problems in mathematical statistics. First look at the data and see what the problems are, and then try to develop the appropriate methodology for that kind of data. The next is, of course, R. A. Fisher. When I went to work with him, I first told him, "I have studied mathematical statistics; I have also done some research. Would you give me a problem that I could work for my thesis?" – as students do in this country. Fisher said, "No, the problem must be yours. I can help you to solve the problem." [Laughs]

Also, I wrote a paper in mathematical genetics and said that the method I developed was superior to one that somebody else gave, but Fisher said, "I won't read your paper unless you do some computations. Gather the data and compare what your method gives with what the other method gives, and then come with a paper and I shall read it." This is the kind of advice which I think was very useful later in my consultation work and also in my own research in mathematical statistics. I always develop methodology from the data given to me for analysis rather than look at others' work and try to extend it in terms of mathematics.

I used to meet Wishart frequently when I was in Cambridge because we used to discuss lots of problems. That was also useful, but the two people who provided inspiration for me are R. A. Fisher and Mahalanobis. Of course, at that time Neyman's work was becoming popular and everybody had to read that. So I read a lot of work that Neyman did and I think he was also responsible for my getting into the current research at that time in testing of hypotheses.

DeGroot: Did Fisher ever discuss Neyman's work with you?

Rao: Yes. He was critical about Neyman. Whenever I had dis-

cussions and wanted to use Neyman's ideas in solving some of the problems, Fisher always discouraged me from doing that. Actually, I now feel that I could have done some work which is currently popular if I had used Neyman's ideas in those days. I was not reluctant to use Neyman's ideas, but I did not probably because of Fisher's influence.

DeGroot: Was Fisher generally pretty good about reading things that you would give him?

Rao: Oh, yes. He would take great interest. And then he would read and comment on it. He was also a very exciting teacher. Not many could follow what he was teaching; but if one tried to see what he was saying, he was an inspiring teacher. I also used to attend his seminars and they were very good. But most of the time when I was at Cambridge I used to help him in his genetic experiments.

DeGroot: You've published, according to my count, nine books and about 230 papers. Your book *Linear Statistical Inference and Its Applications* is of course widely known [second edition, John Wiley, New York, 1973]. I've always felt that the title was something of a misnomer because that book really covers much more than linear statistical inference. In fact, it covers much of what is in a standard graduate course in statistical theory.

Rao: Actually, I had a different title but the referees suggested this title because it would compete better with other books with similar titles at that time. So I said, "OK. From the commercial point of view, if that title will sell very well then you may choose that title." [Laughs] It had a different title to begin with.

DeGroot: What was your original title?

Rao: Advanced Statistics or something like that. Actually, when I wrote my first book and called it *Advanced Statistical Methods in Biometric Research* [first edition, John Wiley, New York, 1952; reprint, Haffner, New York, 1974], I really summarized what I did in Cambridge on the anthropological material. Most of the examples there are all from anthropometric data. People wondered how I suddenly introduced data on head length and head breadth and so on in between theoretical discussions. But the major emphasis in that book is the applications of statistical methodology to univariate and multivariate data. It's slightly more theoretical than R. A. Fisher's book, but because the title said biometric methods the statisticians were not attracted to it.

DeGroot: Yes, that's another book whose title is much narrower than the content of the book – where you cover much more material.