奥数题库

第1届国际中小学生数学能力检测（初中组）试题解答

朱华伟　孙文先　编著

科学出版社

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内 容 简 介

本书收录了第1届国际中小学数学能力检测(2011-2012),2012年国际数学竞赛(初中组),2012年国际数学竞赛台湾选拔赛(初中组),2011年澳大利亚数学能力检测中学初级卷、中级卷的全部试题,对每一道试题均给出了详细解答,有些题还给出了多种解法,目的是加深读者对问题的理解,从中得到有益的启发.

本书可供初中数学资优生、准备初中升高中数学考试的考生、准备参加各类初中数学竞赛的选手、中学数学教师、高等师范院校数学教育专业大学生、数学爱好者及数学研究工作者参考.

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张景中谈奥数

华伟教授认为，竞赛数学是教育数学的一部分. 这个看法是言之成理的. 数学要解题，要发现问题、创造方法. 年复一年进行的数学竞赛活动，不断地为数学问题的宝库注入新鲜血液，常常把学术形态的数学成果转化为可能用于教学的形态. 早期的国际数学奥林匹克试题，有不少进入了数学教材，成为例题和习题. 竞赛数学与教育数学的关系，于此可见一斑.

写到这里，忍不住要为数学竞赛说几句话. 有一阵子，媒体上面出现不少讨伐数学竞赛的声音，有的教育专家甚至认为数学竞赛之害甚于黄、赌、毒. 我看了有关报道后第一个想法是，中国现在值得反对的事情不少，论轻重缓急还远远轮不到反对数学竞赛吧. 再仔细读这些反对数学竞赛的意见，可以看出来，他们反对的实际上是某些为牟利而又误人子弟的数学竞赛培训. 就数学竞赛本身而言，是面向青少年中很小一部分数学爱好者而组织的活动. 这些热心参与数学竞赛的数学爱好者（还有不少数学爱好者参与其他活动，例如青少年创新发明活动、数学建模活动、近年来设立的丘成桐中学数学奖），估计不超过约两亿中小学生的百分之五. 从一方面讲，数学竞赛培训活动过热产生的消极影响，和升学考试体制以及教育资源分配过分集中等多种因素有关，这笔账不能算在数学竞赛头上；从另一方面看，大学招生和数学竞赛挂钩，也正说明了数学竞赛活动的成功因而得到认可. 对于

青少年的课外兴趣活动，积极的对策不应当是限制堵塞，而是开源分流. 发展多种课外活动，让更多的青少年各得其所，把各种活动都办得像数学竞赛这样成功并且被认可，数学竞赛培训活动过热的问题自然就化解或缓解了.

摘自《走进教育数学》丛书总序

前　言

国际中小学数学能力检测(International Mathematics Assessments for Schools, IMAS)是一项为小学中年级、高年级与初中学生所举办的世界级数学检测,旨在以国际宏观的尺度衡量学生在三项认知层面——知识、应用与推理上的表现,为参加者提供在不同领域、不同难度、每年一次的公开数学检测,测试学生的数学能力,挑战学生开阔数学视野的能力,评估学校、国家与地区之间的数学水平,给予聪颖与能力强的学生能透过大众认可的检测评价他们在数学上的能力,支持资优教育的发展.

IMAS 针对小学中年级(即三、四年级)、高年级(即五、六年级)与初中(即七、八年级)程度的学生分别命题进行检测,参加者可以根据自己的能力,自行选择检测项目的内容与难度,即学生可以跳级或降级参加检测,即小学三年级学生可以参加小学高年级组检测,九年级学生可以参加初中组检测.

IMAS 分第一轮检测和第二轮检测. IMAS 第一轮检测——包括 25 道题,答题时间为 75 分钟. 前 20 题为选择题,最后 5 题答案为 000 至 999 整数的填空题. 题目的难度等级分布情况为:第1~10 题为容易题,每题3 分;第 11~20 题为适中题,每题4 分;第21~25 题为难题,每题 6 分. 总分为 100 分. 考试时间 75 分钟. 学生将根据其所在考区同年级学生 IMAS 第一轮检测的成绩给予以下奖项:一等奖——成绩在前 10% 者;二等奖——成绩在前 11% 至前

25%之间者;三等奖——成绩在前26%至前50%之间者.每位学生都可得到一张个人的详细成绩报告,报告列出整体平均标准及个人成绩的百分等级,提供学生在不同层次技能的表现,检测所显示出的强项与弱点可督促学生的学习.这些珍贵的信息将对各地区的数学教育研究有很大的帮助.

IMAS第二轮检测——邀请各考区参加IMAS第一轮检测成绩在前10%的学生参加.试题共有15题,其中第1~5题为选择题,每题4分;第6~13题为填空题,每题5分;第14~15题为详答题(必须写出详细演算过程),每题20分,部分答对给予部分分数.总分为100分.IMAS第二轮检测成绩达各考区前5%者颁发金牌,成绩达前6%~15%者颁发银牌,成绩达前16%~30%者颁发铜牌,以及相应的证书.

根据各考区IMAS组织工作、参加检测人数及第二轮检测成绩综合评估居前的考区获邀派队参加国际数学竞赛(International Mathematics Competition,IMC),中国可以派中学、小学各四个队参赛,每个队由四名学生、两名教师组成,IMC组委会免费提供受邀学生、教师在比赛国家期间的所有食宿,参赛队自行负担出国的往返交通费用.

本书收录了2011年IMAS(初中组)、2012年IMC(初中组)和2012IMC台湾选拔赛(初中组)的全部试题,对每一道试题均给出了详细解答,有些题还给出了多种解法,目的是使读者加深对问题的理解,可以从中得到有益的启发.

"他山之石,可以攻玉",为了帮助读者了解国外同级别数学能力检测的信息,我们给出了2011年澳大利亚数学能力检测初级卷(7、8年级)、中级卷(9、10年级)试题与解答.

参加第一届 IMAS 命题工作的有：Andy Liu（加拿大亚伯达大学）、朱华伟、孙文先（台湾财团法人台北市九章数学教育基金会）、Simon Chua（菲律宾数学研究会）、郑振初（香港教育学院）、郑焕、付云皓、邹宇、李涛、张传军、杨姗、周弋林等.

在本书的编校过程中，广州大学软件所博士生付云皓同学提供了很大的帮助，在此向他表示真诚的感谢. 对于本书存在的问题，热忱希望读者不吝赐教.

朱华伟　孙文先

2012 年 12 月于广州大学城

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第 1 章　第 1 届国际中小学生数学能力检测(IMAS)试题(初中组)

1.1　第一轮英文试题

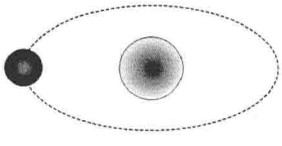
1. What is $2011+1102\times(1-3)$?

(A)193　　(B)4215　　(C)6226　　(D)−193　　(E)−6226

2. Which number is the largest?

(A)3.14　　(B)π　　(C)$\frac{22}{7}$　　(D)3.135　　(E)304%

3. The temperature on the shady side of a certain planet is −253℃. The temperature on its sunny side is only −223℃. Which of the following statement is an accurate description of the relation between the temperatures on the shady side and on the sunny side?



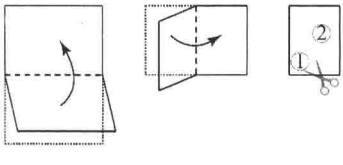
(A) The temperature of its sunny side is 30℃ higher than its shady side;

(B) The temperature of its sunny side is 30℃ lower than its shady side;

(C) The temperature of its sunny side is 476℃ higher than its shady side;

(D) The temperature of its sunny side is 476℃ lower than its shady side;

(E) The temperature of its sunny side is the same as its shady side.



4. The given diagram shows a rectangular piece of paper folded in quarters along two perpendicular folds. If a cut is made around the corner marked 1, which of the following **cannot** possibly be the shape of the resulting hole in the piece of paper?

(A) Octagon　　(B) Quadrilateral

(C) Hexagon　　(D) Triangle　　(E) Circle

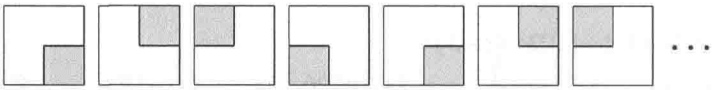
5. Around 550 BC, the Greek mathematician Pythagoras discovered and proved a theorem which now bears his name. To celebrate this achievement, he had 100 cows killed for a feast. Thus the result is also known as the One Hundred Cows Theorem. What is the anniversary of this result in 2011? (There is no Year 0.)

(A)2562 (B)2560 (C)2561 (D)1460 (E)1461

6. A rectangle is 6 cm by 8 cm. It is revolved about an axis on the rectangle itself. What is the number of different cylinders that may be obtained in this way?

(A)2 (B)4 (C)6 (D)8 (E)Infinity

7. There is a pattern to the given sequence of figures:



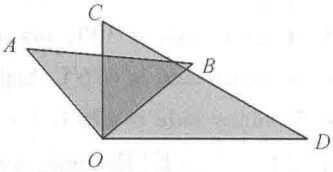
(1) (2) (3) (4) (5) (6) (7)

Which of the following will be the 2011-th figure of the sequence?

(A) (B) (C) (D) (E)

8. The given diagram shows two overlapping right triangles having a common vertex O. If $\angle AOD = 123°$, what is the measure, in degrees, of $\angle BOC$?

(A)33 (B)53 (C)57 (D)60 (E)66

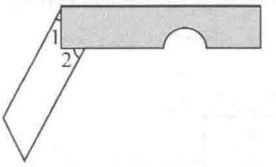


9. A greengrocer is having an apple sale. The price is \$ 6 per kilogram. If the total purchase exceeds 3 kilograms, a 20% discount is applied to the portion over 3 kilograms. There is no discount if the total purchase does not exceed 3 kilograms. If Leith buys 8 kilograms of apples from this greengrocer, how much does he pay?

(A) \$ 32 (B) \$ 36 (C) \$ 42 (D) \$ 44 (E) \$ 21

10. The given diagram shows a pocket knife. The shaded part is a rectangle with a small semicircular indentation. The two edges of the blade are parallel, forming

angles 1 and 2 with the shaft as shown. What is the measure, in degrees, of $\angle 1 + \angle 2$?



(A)30　　(B)45　　(C)60

(D)90　　(E)could not be determined

11. The given diagram shows the projected sale and actual sale of a certain toy company for the fourth quarter of the year. The achievement percentage is equal to $\frac{\text{actual sale}}{\text{projected sale}} \times 100\%$. What is this achievement percentage?

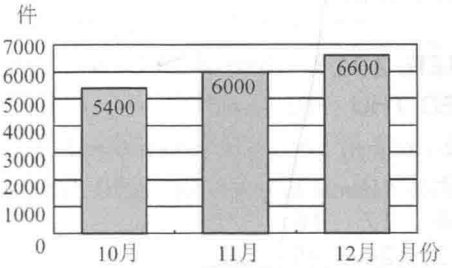


Fig. A　Projected sale

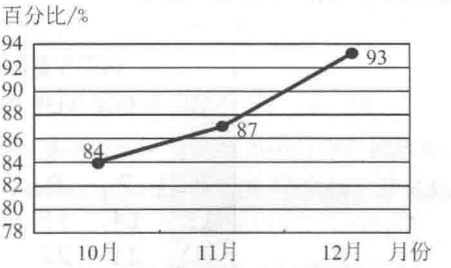
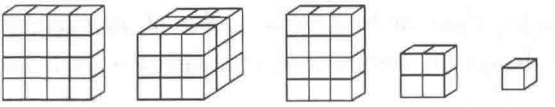


Fig. B　Actual sale

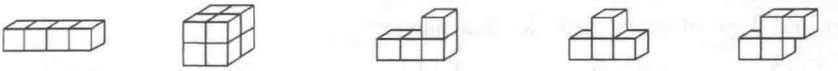
(A)86%　　(B)88.3%　　(C)88%　　(D)86.3%　　(E)90.3%

12. Leon is given five wooden blocks:



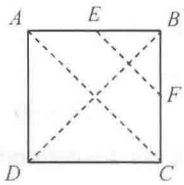
Which of the following blocks should be added so that he can make a 4×4×4 cube? (None of the blocks can be dissected)

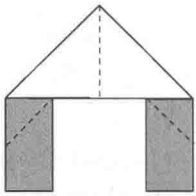
(A)　　(B)　　(C)　　(D)　　(E)



13. The given diagram shows how a square *ABCD* may be dissected into six

pieces by three straight cuts AC, BD and EF, where E and F are the respective midpoints of AB and BC. The pieces are then rearranged to form the given shape. What is the total area, in square centimetres, of the shaded part of the given shape?





(A)200 (B)400 (C)600 (D)800 (E)1000

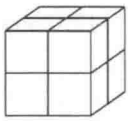
14. The given diagram shows the calendar for the month of November, 2011. Three consecutive numbers from the same column are chosen. Of the following number, which can be the sum of three such numbers?

NOVEMBER 2011

SUN	MON	TUE	WED	THU	FRI	SAT
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

(A)21 (B)37 (C)38 (D)40 (E)54

15. The given diagram shows a large cube formed of eight identical small cubes. The surface area of the large cube is 216 square centimetres less than the total surface areas of the eight small cubes. What is the length, in centimetres, of a side of a small cube?



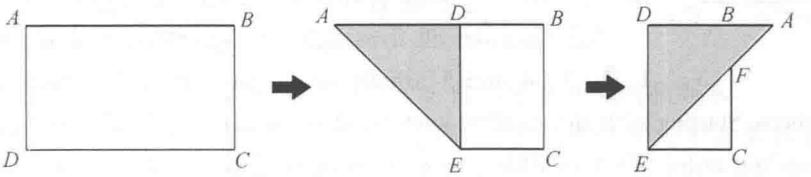
(A)2 (B)3 (C)4 (D)5 (E)6

16. In an NBA basketball game, a player scores 44 points, 5 of which come from 5 foul shots (each shot scores 1 point). He makes more 2-point shots than 3-point shots. Of the following number, which **cannot** possibly be the total number of 2-point and 3-point shots made by this player?

(A)15 (B)16 (C)17 (D)18 (E)19

17. The given diagram shows a rectangle $ABCD$ being folded along a straight

segment AE with E on CD, so that the new position of D is on AB. Triangle ADE is then folded along DE so that the new position of A is on the extension of DB. The new position of AE intersects BC at F. If $AB = 10$ centimetres and $AD = 6$ centimetres, what is the area, in square centimetres, of triangle ABF?



(A)2　　(B)4　　(C)6　　(D)8　　(E)10

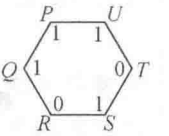
18. A child is operating a remote-controlled car on a flat surface. Starting from the child's feet, the car moves forward 1 metre, makes a 30° turn counterclockwise, moves forward 1 metre, makes a 30° turn counterclockwise, and so on. When the car first time returns to its starting point for the first time, what is the total distance, in metres, that it has covered?

(A)4　　(B)8　　(C)12　　(D)16　　(E)24

19. Each interior angle of a regular convex polygon is greater than 100° and less than 140°. Of the following numbers, which **cannot** possibly be the number of sides of this polygon?

(A)5　　(B)6　　(C)7　　(D)8　　(E)9

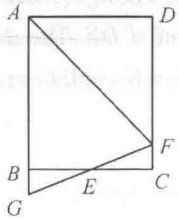
20. In the given diagram, each vertex of the hexagon $PQRSTU$ is labeled with 0 or 1. Starting counterclockwise from a vertex, he multiplies the labels by 3, 7, 15, 31, 63 and 127 respectively and add the six products. If the starting point is P, the final sum is $1\times3+1\times7+0\times15+1\times31+0\times63+1\times127=168$. What is the starting point if the final sum is 180?



(A)Q　　(B)R　　(C)S　　(D)T　　(E)U

21. A drunk walks 1 metre east. Then he stops, makes a 90° turn clockwise or counterclockwise and walks 2 metres. Then he stops, makes a 90° turn clockwise or counterclockwise and walks 3 metres. He continues in this pattern, stopping, making 90° turn clockwise or counterclockwise and walks 1 metre more than the preceding segment. What would be the longest distance, in metres, between his initial position

and his position when he makes his seventh stop?

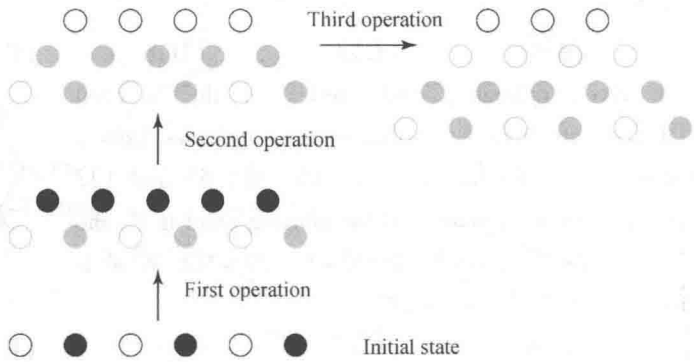


22. In the given diagram, $ABCD$ is a rectangle with $AB = 25$ cm and $BC = 20$ cm. F is a point on CD and G is a point on the extension of AB such that FG passes through the midpoint E of BC. If $\angle AFE = \angle CFE$, what is the length, in cm, of CF?

23. Consider all five-digit numbers using each of the digits 1, 2, 3, 4 and 5 exactly once, possibly with a decimal point somewhere. Starting with the smallest such number, namely, 1.2345, they are listed in ascending order. What is 1000 times the difference of the 150th and the 145th numbers?

24. On a straight line are 6 counters, each either black or white. We operate the row of counters as follow: for two consecutive counters with the same colour, a white counter is placed between them, for two consecutive counters with different colour, a black counter is placed between them. And then the original counters are removed. This is one operation. After a total of three operations we are left with three white counters. What is the number of possible colour patterns of the six counters initially?

An example is attached.



25. Mickey lives in a city with six subway lines. Every two lines have exactly one common stop for changing lines, and no three lines meet at a common stop. His home is not at one of the common stops. One day, Mickey suddenly decides to leave home and travel on the subway, changing trains at least once at each stop before returning home. What is the minimum number of changes he has to make to accomplish this task?

1.2　第一轮中文试题

注意事项

一般规定

1. 进入试场后,未获监考老师许可之前不可翻开此测验题本.

2. 不可以使用计算器、对数表、数学图表、手机与任何电子计算器具. 作答时可使用直尺与圆规,以及两面全空白的草稿纸.

3. 题目所提供之图形只是示意图,不一定精准.

4. 最前 20 题为选择题,每题有五个选项. 最后 5 题要求填入的答案为 000 至 999 的正整数. 题目一般而言是依照越来越难的顺序安排,对于错误的答案不会倒扣分数.

5. 本活动是数学能力检测而不同于学校测验,别期望每道题目都会做.

6. 请依照监考老师指示,谨慎地在答案卡上填写您的基本数据. 若因填写错误或不详所造成之后果由学生自行负责.

7. 须等待监考老师宣布开始作答后,才可以打开题本进行答题.

作答须知

1. 限用 B 或 2B 铅笔填写答案.

2. 请用 B 或 2B 铅笔在答案卡上将您认为正确选项的圆圈涂满(不是在题本上).

3. 您的答案卡将由计算机阅卷,为避免计算机误判,请不要在答案卡上其他任何地方涂划任何记号. 填写答案卡时,若需要修改,可使用软性橡皮小心擦拭,并确定答案卡上无残留痕迹.

特别约定

为确保竞赛之公平性及认证成绩优异学生,IMAS 主办单位保留要求考生重测之权利.

1–10 题,每题 3 分

1. 算式 $2011+1102\times(1-3)$ 的值等于

(A)193　　(B)4215　　(C)6226　　(D)−193　　(E)−6226

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