

Xiaoyan Lei

High Speed Railway Track Dynamics

Models, Algorithms and Applications

高速铁路轨道动力学——模型、算法与应用



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Foreword

In recent years, the construction and operation of Chinese high-speed railways has scored remarkable achievements, making great contribution to the sound and fast development of national economy. It has been widely acclaimed by the whole society and become another example of the world high-speed railway development. By the end of 2015, China had over 17,000 km newly built high-speed railway lines in operation, ranking the first in the world. In field of high-speed railway system, China is credited with the most comprehensive technologies, the strongest integration capability, the fastest running speed, the longest operation mileage, and the largest scale construction in the world.

The high-speed railway construction started late in China, so it has not accumulated sufficient theoretical and practical experience. Undoubtedly, it has been faced with a series of technical problems and challenges during the process of high-speed railway construction and operation, which requires the strong academic and technical support. The high-speed railway track structure consists of the rail, the rail pads and fastenings, the track slab, the cement asphalt mortar filling layer, the concrete basement, the subgrade, and the ground (or bridge). But there are great differences in the material properties and the structures, many technological difficulties, and complicated service environments. Besides, the train speed is over 200–300km/h. All these issues are closely related to the wheel-rail dynamics. Thus, high-speed railway track dynamics is the key basic discipline to ensure the safe operation of trains. When the train speed increases, the interactions between the locomotive, rolling stock, and track structure significantly increase. The environment of dynamic system becomes deteriorated, and the safety and stability of high-speed train and the reliability of track structure are thus faced with severe tests. When the train is running at high-speed, the influence of the plane and profile conditions of lines and track irregularities on passengers' comfort and the influence of the running train on the surrounding environment will be magnified, and the consequences of train accidents will become more hazardous. All of these must be explored based on high-speed railway track dynamics. Therefore, the in-depth study of high-speed trains and track interaction mechanism will provide the necessary

theoretical guidance for the design of the high-speed rail system and the operation of high-speed trains.

The book, entitled *High-Speed Railway Track Dynamics*, is a summary of the theoretical and practical research findings on high-speed railway track dynamics made by the author and his research team. It systematically introduces the latest research findings of the author and his research team, focusing on the high-speed railway track dynamics theories, models, algorithms, and engineering applications. The book is mainly featured by the frontier research contents, complete theoretical system, correct and effective model and algorithms, and rich engineering applications. What are particularly worth mentioning in the book are the author's researches in aspects of track element and vehicle element model and algorithm, moving element method for dynamic analysis of the vehicle and track coupling system, cross-iteration algorithm for dynamic analysis of the vehicle and track coupling system, dynamic behavior analysis for the track transition between ballast track and ballastless track, and environmental vibration analysis induced by overlapping subways, which are of great innovation and application value. The series of research findings have been published in the related famous international academic journals, such as *Journal of Vibration and Control*, *Journal of Transportation Engineering*, *Journal of Rail and Rapid Transit*, and *Journal of Sound and Vibration*, which are widely recognized and cited by the fellow scholars at home and abroad.

I strongly believe that the publication of this book will provide an important theoretical guidance for Chinese high-speed railway track design, construction, and operation, making great contributions to promoting the scientific development of railway engineering discipline and the railway technological progress and realizing the sustainable development of Chinese high-speed railways.

January 2016

Mengshu Wang
Academician of Chinese Academy of Engineering

Preface

Since the world's first high-speed railway has been constructed and put into operation, the high-speed railway is highly competitive compared to other means of transportation for its distinct advantages in speed, convenience, safety, comfort, environment-friendly design, large capacity, low energy consumption, all-weather transportation, etc. According to the statistics from the International Union of railways (UIC), up to November 1, 2013, the total operation mileage of the high-speed railways of other countries and regions in the world is 11605 km. 4883-km high-speed railways are under construction, and 12570-km high-speed railways are planned to be constructed. The total operation mileage of high-speed railways in China are 11028 km, and 12000-km lines are under construction. The total operation mileage of Chinese high-speed railways accounts for half of that of the world's high-speed railways. China has become a country with the longest operation mileage and the largest scale construction in the world in high-speed railway. Meanwhile, 18 cities in China (including Hong Kong) possess urban rail transit with a total mileage of 2100 km, and 2700-km lines are under construction, ranking the first in the world. The high-speed railways and rail transit system have become the important engines to drive the socioeconomic development. The independent innovation of high-speed railway technology has become the major national strategic demand.

However, with the rising running speed of trains, the increased transportation density, and the increased transportation loads, the interaction between train and track has become intensified. To adapt to this change in railway development, the countries all over the world have strengthened the railway technological innovation and widely adopted new technologies, designs, materials, processes, and modern management methods in railway engineering. Hence, the concept of modern railway track has emerged. Modern railway track is developed with the advent of high-speed railway and heavy-haul railway, which must meet the fast and high-speed demand of passenger transport and the fast and heavy-haul demand of freight transport. Compared with the conventional railway track, the modern track has the following characteristics: ① high-standard subgrade; ② new subrail

foundation; ③ heavy and super-long continuous welded rail; ④ the scientific management of track maintenance; and ⑤ the organization of safe operations and coordination of railway and environment. Obviously, the traditional track mechanics and structural analysis methods cannot satisfy the needs of modern railway track analysis and design. Owing to the rapid development of computers and numerical methods in recent decades, new theories and methods are applied to the track mechanics and track engineering, making it possible to solve many problems which seemed impossibly hard to solve in the past and those complex problems emerging in the process of railway modernization.

This book is a systematic summary of the related theoretical and applied research findings on high-speed railway track dynamics made by the author and his research team over a decade. In the past decades, sponsored by many projects, such as the Natural Science Foundation of China (50268001, 50568002, 50978099, U1134107, 51478184), International Cooperation and Exchange Project (2010DFA82340), Natural Science Foundation of Jiangxi Province (0250034, 0450012), Sino-Austria, Sino-Japan, Sino-Britain, and Sino-America Scientific Cooperation Projects, "Sponsor Program for Key Teachers of Colleges and Universities" of the Ministry of Education (GG-823-10404-1001), Science and Technology Development Plan of China Railway Bureau (98G33A), Outstanding Science & Technology Innovation Team of Jiangxi Province (20133BCB24007), and the Training Program of the Leading Personnel of Key Disciplines and Technologies of Jiangxi Province (020001), the author and his research team have conducted the in-depth and systematic researches on high-speed railway track dynamics, environmental vibration and noise induced by railway traffic, and engineering applications of track dynamics and have achieved fruitful research findings, most of which belong to the frontier issues of the track dynamics theory. Participants in the research team are as follows: Xiaoyan Lei, Liu Linya, Feng Qingsong, Zhang Pengfei, Liu Qingjie, Luo Kun, Luo Wenjun, Fang Jian, Zhang Bin, Xu Bin, Wang Jian, Tu Qinming, Zeng Qine, Lai Jianfei, Wu Shenhua, Sun Maotang, and Xiong Chaozhua.

The draft of this book has been used as the textbook for postgraduate students majoring in road and railway engineering in East China Jiaotong University for several years. The book falls into 15 Chapters: 1. Track Dynamics Research Contents and Related Standards; 2. Analytic Method for Dynamic Analysis of the Track Structure; 3. Fourier Transform Method for Dynamic Analysis of the Track Structure; 4. Analysis of Vibration Behavior of the Elevated Track Structure; 5. Track Irregularity Power Spectrum and Numerical Simulation; 6. Model for Vertical Dynamic Analysis of the Vehicle-Track Coupling System; 7. A Cross-Iteration Algorithm for Vehicle-Track Coupling Vibration Analysis; 8. Moving Element Model and Its Algorithm; 9. Model and Algorithm for Track Element and Vehicle Element; 10. Dynamic Analysis of the Vehicle-Track Coupling System with Finite Elements in a Moving Frame of Reference; 11. Model for Vertical Dynamic Analysis of the Vehicle-Track-Subgrade-Ground Coupling System; 12. Analysis of Dynamic Behavior of the Train-Ballast Track-Subgrade

Coupling System; 13. Analysis of Dynamic Behavior of the Train-Slab Track-Subgrade Coupling System; 14. Analysis of Dynamic Behavior of the Transition Section Between Ballast Track and Ballastless Track; and 15. Environmental Vibration Analysis Induced by Overlapping Subways.

Compared with similar studies, the contents of the five chapters, that is, the cross iteration algorithm for vehicle-track coupling vibration analysis, model and algorithm for track element and vehicle element, finite elements in a moving frame of reference for dynamic analysis of the vehicle-track coupling system, dynamic behavior analysis for the transition between ballast track and ballastless track, and environmental vibration analysis induced by overlapping subways, have distinct characteristics, which may be considered as the author's original findings.

With original concepts, systematic theories, and advanced algorithms, this book attempts at introducing to the readers the latest research findings and developments at home and abroad in high-speed railway track dynamics. It lays great emphasis on precision and completeness of its content. All the chapters are interrelated yet relatively independent, which makes it possible for the readers to browse through the book or just focus on special topics. It combines theories with practice to provide abundant information in the hope of enlightening and helping readers. Any comments and suggestions concerning the book are welcome and appreciated.

On the occasion of the publication of this book, I would like to express my sincere thanks to institutions and people that have been funding, supporting, and caring about my research work and the publication of this book! I would like to extend my special gratitude to Profs. Du Qinghua and Wang Mengshu, academicians of China Academy of Engineering, Mr. Guan Tianbao, senior engineer of Shanghai Railway Bureau, and Profs. Tong Daxun and Wang Wusheng from Tongji University, for their longtime care, guidance, and help. Particularly gratefully, academician Wang takes the trouble to write the foreword for this book. I am also deeply indebted to the translation team, Tang Bin, Lu Xiuying, Yang Zuhua, Liu Qingxue, Li Xing, and Huang Qunhui, from School of Foreign Languages in ECJTU for their translation work. Last but not least, my heartfelt thanks go to my colleagues, postgraduate students, and Mr. Wei Yingjie, editor of Science Press. The publication of this book is the result of their joint efforts.

Kongmu Lakeside
2016

Xiaoyan Lei

About the Book

High-speed railway is highly competitive compared to other means of transportation for its distinct advantages in speed, convenience, safety, comfort, environment-friendly design, large capacity, low energy consumption, all-weather transportation, etc. In recent years, there has been a boom in the construction of high-speed railways throughout the world. To adapt to the fast development of the design and construction of high-speed railways, it is essential to further theoretical studies in high-speed railway track dynamics. As a systematic summary of the relevant research findings on high-speed railway track dynamics made by the author and his research team over a decade, this book explores the frontier issues concerning the basic theory of high-speed railway, covering the dynamics theories, models, algorithms, and engineering applications of the high-speed train-track coupling system.

Characterized by original concepts, systematic theories, and advanced algorithms, this book lays great emphasis on precision and completeness of its content. All chapters are interrelated yet relatively independent, which makes it possible for the readers to browse through the book or just focus on special topics. It combines theories with practice to provide abundant information in the hope of introducing to the readers the latest research findings and developments in high-speed railway track dynamics. The book can be used both as textbook or reference book for undergraduates, postgraduates, teachers, engineers, and technicians involved in civil engineering, transportation, highway, and railway engineering.

About the Author



Xiaoyan Lei is a professor and doctoral supervisor of East China Jiaotong University, chair professor of Jiangxi Jinggang Scholars, and director of Railway Environmental Vibration and Noise Engineering Research Center of the Ministry of Education.

He received his Ph.D. degree in solid mechanics from Tsinghua University in 1989. He served as a visiting scholar at University of Innsbruck, Austria, during 1991–1994, a visiting professor at Kyushu Institute of Technology, Japan, in 2001, and a senior research fellow at the University of Kentucky, USA, in 2007. He has been awarded many academic titles

including the first- and second-rank talents of the National Talents Project, the leading personnel of key disciplines and technologies of Jiangxi Province and the leading talent of Jiangxi Ganpo Talent Candidates 555 Program. His academic positions include the senior member of American Society of Mechanical Engineers (ASME), executive director of China Communication and Transportation Association, director general of Theoretical and Applied Mechanics Society of Jiangxi Province, and deputy director general of Railway Society of Jiangxi Province. In addition, he works as the editorial member of *Journal of the China Railway Society*, *Journal of Railway Science and Engineering*, *Journal of Urban Mass Transit*, *Journal of Transportation Engineering and Information*, *International Journal of Rail Transportation*, *Journal of Civil Engineering Architecture*, and *Education Research Monthly*.

He has undertaken more than 60 scientific research projects at all levels, including National Key Basic Research Program of China, Key International Cooperation and Exchange Project, Natural Science Foundation of China, Natural Science Foundation of Jiangxi Province, Key International Bidding Project, Projects from the Ministry of Education, China Railway Corporation and Science and Technology Development Plan of Jiangxi Province, and Enterprise Commissioned Cooperation Projects. His publications cover over 200 academic

papers and 6 monographs on railway engineering. He is also the author of 6 invention patents and 18 computer software copyrights. There is a long list of his academic awards: the 2nd prize of National Science and Technology Progress Award in 2011, the 1st prize of Natural Science Award of Jiangxi Province in 2005, the 1st prize of Science and Technology Progress Award of Jiangxi Province in 2007, the 1st prize of Railway Science and Technology Award in 2011, the 2nd prize of Natural Science of the Ministry of Education, the 1st prize of Excellent Teaching Award of Jiangxi Province, and the prize of the first batch Three One Hundred Original Natural Science Books sponsored by General Administration of Press and Publication of the People's Republic of China. As a leading expert and distinguished professor, he has presided over the appraisal and evaluation of more than 10 scientific research projects.

His research interests focus mainly on high-speed railway track dynamics and environmental vibration and noise induced by railway traffic.

Contents

1	Track Dynamics Research Contents and Related Standards	1
1.1	A Review of Track Dynamics Research	1
1.2	Track Dynamics Research Contents	6
1.3	Limits for Safety and Riding Quality and Railway Environmental Standards	7
1.3.1	Safety Limit for Regular Trains	7
1.3.2	Riding Quality Limits for Regular Trains	8
1.3.3	Safety and Riding Quality Limit for Rising Speed Trains	10
1.3.4	Railway Noise Standards in China	11
1.3.5	Railway Noise Standards in Foreign Countries	11
1.3.6	Noise Limit for Railway Locomotives and Passenger Trains in China	12
1.3.7	Environmental Vibration Standards in China's Urban Areas	14
1.3.8	Limit for Building Vibration Caused by Urban Mass Transit	18
1.4	Standards of Track Maintenance for High-Speed Railway	19
1.4.1	Standards of Track Maintenance for French High-Speed Railway	20
1.4.2	Standards of Track Maintenance and Management for Japanese Shinkansen High-Speed Railway	20
1.4.3	Standards of Track Maintenance and Management for German High-Speed Railway	21
1.4.4	Standards of Track Maintenance and Management for British High-Speed Railway	23

1.4.5	The Measuring Standards of Track Geometry for Korean High-Speed Railway (Dynamic)	24
1.4.6	Standards of Track Maintenance for Chinese High-Speed Railway	25
1.4.7	The Dominant Frequency Range and Sensitive Wavelength of European High-Speed Train and Track Coupling System	25
1.5	Vibration Standards of Historic Building Structures	28
	References.	33
2	Analytic Method for Dynamic Analysis of the Track Structure.	37
2.1	Studies of Ground Surface Wave and Strong Track Vibration Induced by High-Speed Train	37
2.1.1	The Continuous Elastic Beam Model of Track Structure	38
2.1.2	Track Equivalent Stiffness and Track Foundation Elasticity Modulus.	40
2.1.3	Track Critical Velocity	41
2.1.4	Analysis of Strong Track Vibration.	41
2.2	Effects of Track Stiffness Abrupt Change on Track Vibration.	44
2.2.1	Track Vibration Model in Consideration of Track Irregularity and Stiffness Abrupt Change Under Moving Loads	44
2.2.2	The Reasonable Distribution of the Track Stiffness in Transition	53
	References.	56
3	Fourier Transform Method for Dynamic Analysis of the Track Structure	57
3.1	Model of Single-Layer Continuous Elastic Beam for the Track Structure	57
3.1.1	Fourier Transform	58
3.1.2	Inverse Discrete Fourier Transform	60
3.1.3	Definition of Inverse Discrete Fourier Transform in MATLAB	61
3.2	Model of Double-Layer Continuous Elastic Beam for the Track Structure	62
3.3	Analysis of High-Speed Railway Track Vibration and Track Critical Velocity.	64
3.3.1	Analysis of the Single-Layer Continuous Elastic Beam Model	64
3.3.2	Analysis of Double-Layer Continuous Elastic Beam Model	66

3.4	Vibration Analysis of Track for Railways with Mixed Passenger and Freight Traffic	86
3.4.1	Three-Layer Continuous Elastic Beam Model of Track Structure	86
3.4.2	Numerical Simulation of Track Random Irregularity.	87
3.4.3	Fourier Transform for Solving Three-Layer Continuous Elastic Beam Model of Track Structure	89
3.5	Vibration Analysis of Ballast Track with Asphalt Trackbed Over Soft Subgrade.	94
3.5.1	Four-Layer Continuous Elastic Beam Model of Track Structure	95
3.5.2	Fourier Transform for Solving Four-Layer Continuous Elastic Beam Model of Track Structure	96
3.5.3	Vibration Analysis of Ballast Track with Asphalt Trackbed Over Soft Subgrade.	99
	References.	105
4	Analysis of Vibration Behavior of the Elevated Track Structure	107
4.1	Basic Concept of Admittance	107
4.1.1	Definition of Admittance.	107
4.1.2	Computational Method of Admittance.	108
4.1.3	Basic Theory of Harmonic Response Analysis	109
4.2	Analysis of Vibration Behavior of the Elevated Bridge Structure.	110
4.2.1	Analytic Beam Model	111
4.2.2	Finite Element Model	115
4.2.3	Comparison Between Analytic Model and Finite Element Model of the Elevated Track-Bridge	116
4.2.4	The Influence of the Bridge Bearing Stiffness	117
4.2.5	The Influence of the Bridge Cross Section Model.	117
4.3	Analysis of Vibration Behavior of the Elevated Track Structure.	120
4.3.1	Analytic Model of the Elevated Track-Bridge System.	120
4.3.2	Finite Element Model	124
4.3.3	Damping of the Bridge Structure.	124
4.3.4	Parameter Analysis of the Elevated Track-Bridge System.	127
4.4	Analysis of Vibration Attenuation Behavior of the Elevated Track Structure	131

4.4.1	The Attenuation Rate of Vibration Transmission	131
4.4.2	Attenuation Coefficient of Rail Vibration	135
References		136
5	Track Irregularity Power Spectrum and Numerical Simulation	137
5.1	Basic Concept of Random Process	138
5.1.1	Stationary Random Process	139
5.1.2	Ergodic	140
5.2	Random Irregularity Power Spectrum of the Track Structure	140
5.2.1	American Track Irregularity Power Spectrum	141
5.2.2	Track Irregularity Power Spectrum for German High-Speed Railways [5].	142
5.2.3	Japanese Track Irregularity Sato Spectrum	143
5.2.4	Chinese Trunk Track Irregularity Spectrum	144
5.2.5	The Track Irregularity Spectrum of Hefei-Wuhan Passenger-Dedicated Line [10]	146
5.2.6	Comparison of the Track Irregularity Power Spectrum Fitting Curves	149
5.3	Numerical Simulation for Random Irregularity of the Track Structure	156
5.4	Trigonometric Series Method	157
5.4.1	Trigonometric Series Method (1).	157
5.4.2	Trigonometric Series Method (2).	158
5.4.3	Trigonometric Series Method (3).	158
5.4.4	Trigonometric Series Method (4).	159
5.4.5	Sample of the Track Structure Random Irregularity	160
References		160
6	Model for Vertical Dynamic Analysis of the Vehicle-Track Coupling System	161
6.1	Fundamental Theory of Dynamic Finite Element Method	162
6.1.1	A Brief Introduction to Dynamic Finite Element Method	162
6.1.2	Beam Element Theory	166
6.2	Finite Element Equation of the Track Structure	172
6.2.1	Basic Assumptions and Computing Model	172
6.2.2	Theory of Generalized Beam Element of Track Structure	173
6.3	Model of Track Dynamics Under Moving Axle Loads.	178
6.4	Vehicle Model of a Single Wheel With Primary Suspension System	180
6.5	Vehicle Model of Half a Car With Primary and Secondary Suspension System	182

6.6	Vehicle Model of a Whole Car With Primary and Secondary Suspension System	184
6.7	Parameters for Vehicle and Track Structure	187
6.7.1	Basic Parameters of Locomotives and Vehicles	187
6.7.2	Basic Parameters of the Track Structure	189
	References	198
7	A Cross-Iteration Algorithm for Vehicle-Track Coupling	
	Vibration Analysis	201
7.1	A Cross-Iteration Algorithm for Vehicle-Track Nonlinear Coupling System	201
7.2	Example Validation	207
7.2.1	Verification	207
7.2.2	The Influence of Time Step	210
7.2.3	The Influence of Convergence Precision	211
7.3	Dynamic Analysis of the Train-Track Nonlinear Coupling System	212
7.4	Conclusions	218
	References	220
8	Moving Element Model and Its Algorithm	221
8.1	Moving Element Model	221
8.2	Moving Element Model of a Single Wheel with Primary Suspension System	224
8.3	Moving Element Model of a Single Wheel with Primary and Secondary Suspension Systems	227
8.4	Model and Algorithm for Dynamic Analysis of a Single Wheel Moving on the Bridge	231
	References	234
9	Model and Algorithm for Track Element and Vehicle Element	235
9.1	Ballast Track Element Model	236
9.1.1	Basic Assumptions	236
9.1.2	Three-Layer Ballast Track Element	236
9.2	Slab Track Element Model	239
9.2.1	Basic Assumptions	239
9.2.2	Three-Layer Slab Track Element Model	240
9.2.3	Mass Matrix of the Slab Track Element	241
9.2.4	Stiffness Matrix of the Slab Track Element	242
9.2.5	Damping Matrix of the Slab Track Element	246
9.3	Slab Track-Bridge Element Model	248
9.3.1	Basic Assumptions	248
9.3.2	Three-Layer Slab Track and Bridge Element Model	248
9.3.3	Mass Matrix of the Slab Track-Bridge Element	249
9.3.4	Stiffness Matrix of the Slab Track-Bridge Element	250

9.3.5	Damping Matrix of the Slab Track-Bridge Element . . .	253
9.4	Vehicle Element Model	254
9.4.1	Potential Energy of the Vehicle Element	256
9.4.2	Kinetic Energy of the Vehicle Element	260
9.4.3	Dissipated Energy of the Vehicle Element	260
9.5	Finite Element Equation of the Vehicle-Track Coupling System	261
9.6	Dynamic Analysis of the Train and Track Coupling System . . .	262
	References	269
10	Dynamic Analysis of the Vehicle-Track Coupling System with Finite Elements in a Moving Frame of Reference	271
10.1	Basic Assumptions	272
10.2	Three-Layer Beam Element Model of the Slab Track in a Moving Frame of Reference	272
10.2.1	Governing Equation of the Slab Track	273
10.2.2	Element Mass, Damping, and Stiffness Matrixes of the Slab Track in a Moving Frame of Reference	276
10.3	Vehicle Element Model	289
10.4	Finite Element Equation of the Vehicle-Slab Track Coupling System	289
10.5	Algorithm Verification	290
10.6	Dynamic Analysis of High-Speed Train and Slab Track Coupling System	292
	References	299
11	Model for Vertical Dynamic Analysis of the Vehicle-Track-Subgrade-Ground Coupling System	301
11.1	Model of the Slab Track-Embankment-Ground System Under Moving Loads	301
11.1.1	Dynamic Equation and Its Solution for the Slab Track-Subgrade Bed System	302
11.1.2	Dynamic Equation and Its Solution for the Embankment Body-Ground System	305
11.1.3	Coupling Vibration of the Slab Track-Embankment-Ground System	307
11.2	Model of the Ballast Track-Embankment-Ground System Under Moving Loads	309
11.2.1	Dynamic Equation and Its Solution for the Ballast Track-Subgrade Bed System	310
11.2.2	Coupling Vibration of the Ballast Track-Embankment-Ground System	312
11.3	Analytic Vibration Model of the Moving Vehicle-Track-Subgrade-Ground Coupling System	313