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大学预科数学

PRE-U PURE MATHEMATICS

张 弘
（Zhang Hong）

【罗马尼亚】保罗·乔治斯库
（Paul Georgescu）

主编

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主编

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PREFACE

Pre-U Pure Mathematics is a textbook which is specifically aimed at the international students in the pre-university foundation school of Jiangsu University. It has been developed following an extensive period of research and consultation with a large number of teachers and students, several of its chapters being already tried and tested in the foundation school, and it is intended to be usable both as a classroom resource and as a textbook to support self-motivated learners away for classroom.

This textbook covers all the requirements for Pure Mathematics from the latest pre-university level specifications and course requirements for the international students studying in China. From the very beginning, it has been conceived to be as clear and helpful as possible. It uses a concrete approach, introducing new concepts or revisiting older ones via extensive sets of worked examples and clear figures and diagrams, aiming at diminishing the discomfort which is often caused at this level by a more abstract approach and at developing a solid conceptual basis on which deeper understanding can be built upon.

The presentation builds on a minimal amount of required knowledge, attempting to provide comprehensive explanations of all topics covered in a friendly, conversational style. The presentation is split into smaller units or modules, clearly labeled as such, each dealing with a single aspect, in clear progression. This makes for easier knowledge assimilation, one step at a time, and helps the reader proceed at a steady pace, while isolating the possible difficulties.

Each chapter contains a comprehensive summary, which lets students review the key concepts and methods, and numerous exercises to reinforce understanding via problem-solving. Remarks and margin notes provide supporting commentary on key definitions, topics and formulae, aiming at clarifying difficult aspects and sometimes at providing mnemonics for remembering formulae and key steps of approaches.

Mathematical knowledge can be delivered in various manners, but ultimately under-standing comes from within the learner, when the individual participates, explores, discovers and makes connections. We hope that this textbook will not only deliver content, but will also be a worthy companion and a helpful guide on this road to understanding.

The Authors

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Chapter 1 Algebra I

1.1 Sets and numbers

1.1.1 Set theory

Definition 1.1.1—Set. A **set** is simply a collection of objects.

There are two ways of describing the members of a set. One way is by **intensional definition**, using a rule or a semantic description.

■ **Example 1.1** (1) A is the set whose members are positive integers that is less than 10.

(2) B is the set of colors of the flag of China.

■

The second way is by **extensional definition**, enclosing the list of members in curly brackets.

■ **Example 1.2** (1) $\{1,2,3\}$ is the set consisting of the objects 1,2,3.

(2) {Nike, Adidas, Reebok, Li Ning, Under Armour} is the set consisting of the objects Nike, Adidas, Reebok, Li Ning, Under Armour.

(3) $\{\{1\},2\}$ is the set consisting of two objects, one being the set $\{1\}$ and the other being the number 2.

■

® The order in which the objects are written doesn't matter. For instance, the set

{Nike, Adidas, Reebok, Li Ning, Under Armour}

is the same set as

{Reebok, Li Ning, Nike, Under Armour, Adidas}.

Often, however, this is not a convenient way to describe our set. For example, the set consisting of all the people who live in China is, for most purposes, best described by precisely this phrase. Describing this set in name list form is unlikely to be useful. As another example, the set of all real numbers whose square is greater than 10 is neatly described by the notation

$$\{x \mid x \text{ is a real number and } x^2 > 10\}.$$

This is to be read "the set of all x such that x is a real number and $x^2 > 10$". The symbol (vertical bar) "|" is the "such that" part of the phrase.

Definition 1.1.2—Element. The objects that are in the set are known as **elements or members** of the set.

Notation 1.1 If x is an element of a set A, we then write $x \in A$. This can also be read as "x is a member of a set A" or "x belongs to A" or "x is in A". The notation $x \notin A$ means that x is not a member of A.

® The objects that make up a set can be anything: numbers, people, letters of the alphabet, other sets, and so on.

■ **Example 1.3**

$1 \in \{1,2,3\}$, but $4 \notin \{1,2,3\}$.

■

■ **Example 1.4**

$\{1,3,5\}$ is the set consisting the objects 1,3 and 5.

■

■ **Example 1.5**

{Fred,dog,1.45} is the set consisting the objects Fred,dog and 1.45.

■

■ **Example 1.6** If $S=\{x \mid x$ a real number, $0 \leqslant x \leqslant 1\}$, then $1 \in S$ but Fred $\notin S$.

■

® Two sets are defined to be equal when they consist of exactly the same elements.

■ **Example 1.7**

$\{1,3,5\}=\{3,5,1\}=\{5,1,3\}$.

■

■ **Example 1.8**

$\{x \mid x^2-1=0\}=\{-1,1\}$.

■

Definition 1.1.3—Subset. The set B is called a **subset** of the set A if and only if $x \in B$ implies $x \in A$.

Notation 1.2 To indicate that B is a subset of A, we write $B \subseteq A$ and say "B is contained in A" or "A contains B".

Definition 1.1.4—Proper subset. If the set B is a subset of the set A, but B is not equal to A, then B is also a **proper** (or **strict**) **subset** of A.

Notation 1.3 To indicate that B is a proper subset of A, we write $B \subset A$.

■ **Example 1.9** If $S=\{$pig,$\{$dog$\}$,cat$\}$, then$\{$cat$\} \subset S$, $\{$pig,$\{$dog$\}\} \subset S$, $\{\{$dog$\}\} \subset S$.

■

Definition 1.1.5—Intersection. The set of elements common to two sets A and B is called the **intersection** of A and B (see Figure 1.1, called a **Venn diagram**).

Notation 1.4 To indicate the intersection of A and B, we write $A \cap B$. Consequently, $x \in A \cap B$ if and only if $x \in A$ and $x \in B$.

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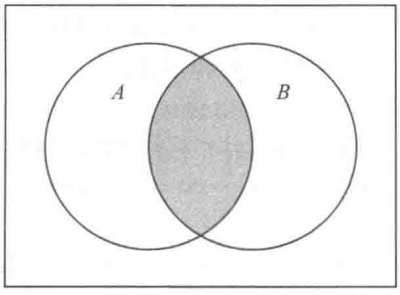


Figure 1.1 The intersection of two sets (shaded region).

■ **Example 1.10** Let A be the set $\{2,3,4,7\}$ and B be the set $\{3,4,5,6,7\}$

$$A \cap B = \{3,4,7\}.$$

■

■ **Example 1.11** Let

$$A = \{\text{pentagon, diamond, square, rectangle}\}$$

and

$$B = \{\text{five pointed star, square, triangle, pentagon}\}.$$

Then

$$A \cap B = \{\text{pentagon, square}\} \text{ (see Figure 1.2).}$$

■

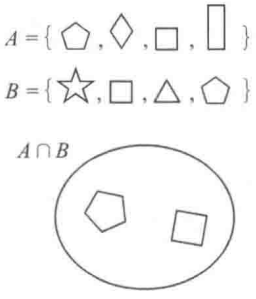


Figure 1.2 The intrtsection of *A* and *B*.

Definition 1.1.6—Union. The **union** of two sets A and B is the set of elements that are either in A or in B (see Figure 1.3).

Notation 1.5 To indicate the union of A and B, we write $A \cap B$.

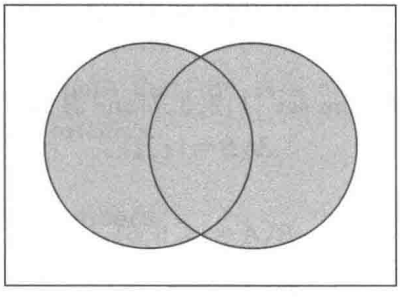


Figure 1.3 The union of two sets (shaded region).

This does not exclude objects that are elements of both A and B.

■ **Example 1.12** Let A be the set $\{2,3,4,7\}$ and B be the set $\{3,4,5,6,7\}$. Then

$$A\cup B=\{2,3,4,5,6,7\}.$$

■

■ **Example 1.13** Let A be the set $\{x|x<1\}$ and B be the set $\{x|x\geqslant 0\}$. Then

$$A\cup B=\{x|-\infty<x<+\infty\}.$$

■

Definition 1.1.7—Empty set. As a convention, we define the **empty set** to be set consisting of no elements at all.

Notation 1.6 The empty set is denoted by $\varnothing$. If the sets A and B have no elements in common, we say A and B are **disjoint** and write $A\cap B=\varnothing$.

® By convention, $\varnothing$ is a subset of every set.

■ **Example 1.14** $1\notin\varnothing$, since $\varnothing$ has no elements.

■

■ **Example 1.15** Let A be the set $\{1,2,4,7\}$ and B be the set $\{3,5,8,9\}$. Then

$$A\cap B=\varnothing.$$

■

■ **Example 1.16** Let A be the set $\{x|x<1\}$ and B be the set $\{x|x\geqslant 0\}$. Then

$$A\cap B=\{x|0\leqslant x<1\}.$$

■

■ **Example 1.17** The subsets of $\{1,2\}$ are $\{1,2\},\{1\},\{2\}$ and $\varnothing$.

■

® The number of subsets of a given set A is 2^n, in which n is the number of elements of A.

Definition 1.1.8—Complement. Inside a universe U one may define the **complement** of A, also termed the **absolute complement** of A, to be the set of all elements of U not in A.

Notation 1.7 The complement of A is denoted by A^C or $\overline{A}$.

® $A\cup A^C=U$.

Definition 1.1.9—Relative complement. The **relative complement** of A with respect to a set B, also termed the difference of sets B and A is the set of elements in B but not in A.

Notation 1.8 The relative complement of A with respect to B is denoted by $B\backslash A$ or $B-A$ or $B\cap A^C$.

■ **Example 1.18** Let A be the set $\{1,2,3,7\}$and B be the set $\{3,4,5,6,7\}$. Then

$$A\backslash B=\{1,2\}$$

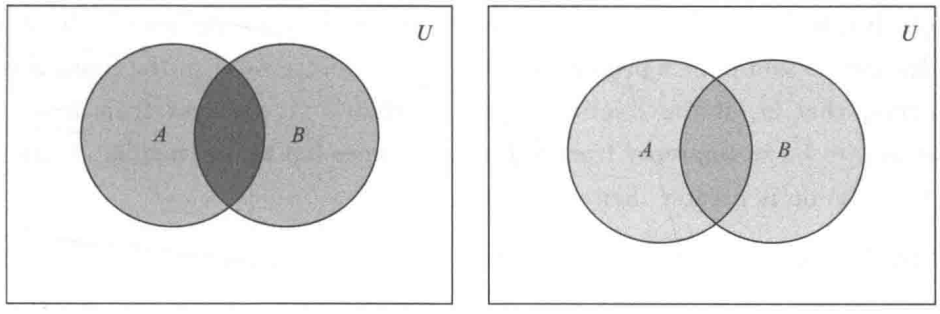
and

$$B\backslash A=\{4,5,6\}.$$

■

Definition 1.1.10—De Morgan's laws. (1) The complement of the union of two sets is the same as the intersection of their complements.

(2) The complement of the intersection of two sets is the same as the union of their complements. See Figure 1.4 for a representation of De Morgan's laws using Venn diagrams.



(a) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (b) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Figure 1.4 De Morgan's law represented with Venn diagrams.

Notation 1.9 In set theory and Boolean algebra, De Morgan's laws are written formally as

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

and

$$\overline{A \cap B} = \overline{A} \cup \overline{B}.$$

Review of the usual sets of numbers

You are already familiar with the following sets of numbers:

Natural numbers. The set of natural numbers is $\{0,1,2,3,4,\cdots\}$ and is denoted by the symbol **N**. They are also called counting numbers since we use them for counting. Note that the set of natural numbers is an infinite set.

Negative integers. The set of negative integers is $\{\cdots,-6,-5,-4,-3,-2,-1\}$ and is denoted by the symbol **Z**. This is also an infinite set.

Integers. The set of integers is $\{\cdots,-3,-2,-1,0,1,2,3,\cdots\}$ and is denoted by the symbol **Z**, which is for the German "Zahlen", meaning numbers. It can be seen that the set of integers is made up of the set of positive and negative whole numbers. Remember that this set include zero. One must note that the sets of natural numbers, and negative numbers are subsets of the set of integers.

1.1.2 Rational and irrational numbers

Rational numbers are numbers which can always be written as a **common fraction**, i.e., as an integer divided by another integer. In particular, they are numbers which can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not zero. The top number, a,

is called the **numerator** and bottom number, b, is called the **denominator.**

Notation 1.10 The set of rational numbers is represented by the symbol **Q**, which stands for "quotient".

■ **Example 1.19** $\frac{1}{3}$, $\frac{-2}{7}$, $\frac{27}{5}$ and $\frac{-9}{7}$ are rational numbers.

■

Simple fractions can be positive or negative, proper or improper. In general, a common fraction is said to be a **proper fraction** if the absolute value of the fraction is strictly less than one, that is, if the fraction is greater than -1 and less than 1. A common fraction is said to be an **improper fraction**, or sometimes **top-heavy fraction**, if the absolute value of the fraction is greater than or equal to 1.

■ **Example 1.20** $\frac{1}{3}$, $\frac{-2}{5}$ and $\frac{3}{7}$ are proper fractions, while $\frac{10}{3}$, $\frac{-12}{5}$ and $\frac{53}{2}$ are improper fractions.

■

The numbers that may be represented in the decimal system are the **decimal fractions**, sometimes called **decimal numbers**, or simply **decimals.** Those are rational numbers that may be expressed as a fraction, the denominator of which is a power of ten.

■ **Example 1.21** 0.6, -0.157 and 8.976 are decimal numbers, since

$$0.6=\frac{6}{10},\ -0.157=-\frac{157}{1\,000},\ 8.976=\frac{8\,976}{1\,000}.$$

■

The decimal system has been extended to infinite decimals, to represent any real number by using an infinite sequence of digits after the decimal separator. In this context, the usual decimals are sometimes called **terminating decimals.** A **repeating decimal**, sometimes called a **recurring decimal**, is an infinite decimal that, after some place, repeats indefinitely the same sequence of digits. For example, $0.080\,808\,080\,8=0.\dot{0}\dot{8}$ (or, in another notation, $0.(08)$). An infinite decimal represents a rational number if and only if it is a repeating decimal.

In mathematics, the **irrational numbers** are all real numbers which are not rational numbers.

Notation 1.11 The set of irrational numbers is represented by the symbol **H**, or sometimes by **R\Q.**

■ **Example 1.22** $\sqrt{2}$ is an irrational number. Another well known irrational number is π. You cannot find an exact value for $\pi=3.141\,592\,653\,598\cdots$. Also, the square root of a prime number is an irrational number. Examples include $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$ and $\sqrt{13}$. Also, any rational multiple of an irrational number is an irrational number. For example, $3\sqrt{2}$, $5\sqrt{3}$, $17\sqrt{5}$ are all irrational numbers.

■

® The mere presence of a square root or radical sign does not mean that the number is

irrational. For example $\sqrt{4}$, $\sqrt{9}$ and $\sqrt{36}$ are all rational numbers. Also, the product of irrational numbers is not necessary irrational, even if those numbers are different from each other. For instance, $\sqrt{2}$ and $2\sqrt{2}$ are irrational, but their product is 4, which is rational.

Exercise 1.1 Sort out the following numbers into rational and irrational, giving reasons.

(1) 0.325;

(2) 0.666 6 ⋯;

(3) 0.090 909 ⋯;

(4) 0.732 334 345 5 ⋯.

■

Solution

(1) 0.325 Rational (terminating decimal number).

(2) 0.666 6 ⋯ Rational (non-terminating recurring decimal number).

(3) 0.090 909 ⋯ Rational (non-terminating recurring decimal number).

(4) 0.732 334 345 5 ⋯ Irrational (non-terminating non-recurring decimal).

■

Exercise 1.2 Which of the following sets contain rational numbers only?

(1) $P=\{2, \sqrt{3}, 7\}$;

(2) $Q=\{15, \frac{1}{6}, \sqrt{36}\}$;

(3) $R=\{x \mid -3 \leqslant x \leqslant 8\}$.

■

Solution

(1) The set P is not a set of rational numbers since $\sqrt{3}$ is an irrational number.

(2) The set Q is a set of rational numbers because $\frac{1}{6}$ is a common fraction and $\sqrt{36}=6$.

(3) Since x can be either rational or irrational in the given interval, the set R is a set of real numbers, and not a set of rational numbers.

■

Operations with rational numbers

Addition and subtraction of fractions

Adding and subtracting fractions are easy when the denominators are the same. When fractions have different denominators, there are a few steps you have to follow to find a common denominator so the fractions can be added to or subtracted from each other.

® In mathematics, the lowest common denominator or least common denominator (abbreviated LCD) is the least common multiple of the denominators of a set of fractions. It simplifies adding, subtracting, and comparing fractions.

Exercise 1.3 Perform the following operations:

(1) $\frac{1}{2}+\frac{2}{5}$;

(2) $3\frac{1}{2}+2\frac{2}{3}$;

(3) $\frac{3}{4}-\frac{1}{3}$;

(4) $5\frac{1}{3}-1\frac{11}{12}$.

■

Solution

(1) $\frac{1}{2}+\frac{2}{5}=\frac{5+4}{10}$ (the LCD of 2 and 5 is 10)

$=\frac{9}{10}$.

(2) $3\frac{1}{2}+2\frac{2}{3}=\frac{7}{2}+\frac{8}{3}$ (changing mixed fractions into improper fractions)

$=\frac{21+16}{6}$ (the LCD of 2 and 3 is 6)

$=\frac{37}{6}=6\frac{1}{6}$.

(3) $\frac{3}{4}-\frac{1}{3}=\frac{9-4}{12}$ (the LCD of 4 and 3 is 12)

$=\frac{5}{12}$.

(4) $5\frac{1}{3}-1\frac{11}{12}=\frac{16}{3}-\frac{23}{12}$ (changing mixed fractions into improper fractions)

$=\frac{64-23}{12}$ (the LCD of 3 and 12 is 12)

$=\frac{41}{12}=3\frac{5}{12}$.

■

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