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工程塑性理论 及其在金属成形 中的应用 (英文版)

**Engineering Plasticity:
Theory and Applications
in Metal Forming**

王仲仁 胡卫龙 苑世剑 王小松 著

高等教育出版社

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JICIZAI JINSHU CHENGXING ZHONG DE YINGYONG

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Preface

With enormous pleasures, I, on behalf of all of the authors of the book, feel deeply honored to contribute our years of attained experience of research and teaching work through the book in English version to the readers who are engaged in the engineering plasticity regarding metal forming.

This book makes detailed introductions of authors' academic contributions: the sequential correspondence law between stress and strain components, the zoning of yield graphics under plane stress states and three-dimensional stress states, the prediction of the dimensional variation tendency of work pieces, the general yield criterion, the graphical description of the anisotropic yield criterion and also shell hydro-forming for manufacturing large vessels.

This book performs mechanical analyses of a couple of special forming technologies, which include, rotary forging, viscous pressure forming, multipoint sandwich forming, and isothermal forging.

For decades, the authors of this book actively took part in the international academic exchanges and published a great number of academic papers. This book systematically summarized a number of scattered papers that were published on various periodicals, on conference proceedings and on the *Journal of Material Processing Technology*, in 2004 published a Special Issue dedicated to Professor Z. R. Wang on the occasion of his issued 70th birthday.

It is quite difficult to get real understanding of many concepts in the theory of plasticity, such as the concept that the values of stress are dependent on the orientation of the plane acted on, that the equivalent stresses and equivalent strains are the extension from the strength theory, that the yielding function is different from the plastic potential, and others. The book is meant to hammer them home.

The ever-increasing requirement to raise the precision of theoretical analysis in engineering practices creates the demands for, apart from the yield function and the plastic potential function, the hardening model able to describe the nonisotropic hardening characteristics of materials. The book discusses this issue in depth.

The chapter and section authors: Chapter 1, W. L. Hu and Z. R. Wang; Chapters 2 and 3, X. S. Wang; Chapters 4, 5, and 6 and Section 7 of Chapter 11, W. L. Hu; Chapters 7 and 8, Z. R. Wang; Chapter 9 and most of Chapter 11, S. J. Yuan; Chapter 10, L. Yuan and Z. R. Wang; Section 7 of Chapter 10, G. Liu; Sections 4, 5, and 6 of Chapter 11, X. Y. Wang, Q. Zhang, and Z. B. He, separately. Also, W. W. Zhang, X. L. Cui, Y. L. Lin, and X. L. Zhang joined in part of the translation work. X. S. Wang and Y. Z. Chen edited the manuscripts of the book according to the request of publication.

I'm obliged to express my special thanks to Ms. Jianbo Liu and Ms. Xueying Zou, editors for the Higher Education Press. It is their precious recommendations that helped bring this book to publication. Sincere thanks also go to Prof. Z. Q. Du, who, as an elder English editor and writer for technical journals, examined almost every sentence of the book in an attempt to ensure the literal suitability. I believe his endeavor will surely be conducive to improving the readability of the book.

Z.R. Wang

2017.5

Contents

Preface *xiii*

1	Fundamentals of Classical Plasticity	1
1.1	Stress	1
1.1.1	The Concept of Stress Components	1
1.1.2	Description of the Stress State	2
1.1.2.1	Stresses on an Arbitrary Inclined Plane	2
1.1.2.2	Stress Components on an Oblique Plane	4
1.1.2.3	Special Stresses	6
1.1.2.4	Common Stress States	7
1.1.3	Stress Tensors and Deviatoric Stress Tensors	7
1.1.4	Mohr Stress Circles	9
1.1.4.1	Mohr Circles for a Two-Dimensional Stress System	9
1.1.4.2	Mohr Circles for a Three-Dimensional Stress System	12
1.1.5	Equations of Force Equilibrium	13
1.2	Strain	15
1.2.1	Nominal Strain and True Strain	15
1.2.2	Strain Components as Functions of Infinitesimal Displacements	17
1.2.3	The Maximum Shear Strains and the Octahedral Strains	20
1.2.4	Strain Rates and Strain Rate Tensors	21
1.2.5	Incompressibility and Chief Deformation Types	23
1.3	Yield Criteria	25
1.3.1	The Concept of Yield Criterion	25
1.3.2	Tresca Yield Criterion	26
1.3.3	Mises Yield Criterion	26
1.3.4	Twin Shear Stress Yield Criterion	27
1.3.5	Yield Locus and Physical Concepts of Tresca, Mises, and Twin Shear Stress Yield Criteria	27
1.3.5.1	Interpretation of Tresca Yield Criterion	29
1.3.5.2	Interpretation of Twin Shear Stress Yield Criterion	30
1.3.5.3	Interpretation of Mises Yield Criterion	31
1.4	A General Yield Criterion	33
1.4.1	Representation of General Yield Criterion	33
1.4.2	Yield Surface and Physical Interpretation	34
1.4.3	Simplified Yield Criterion	34
1.5	Classical Theory about Plastic Stress–Strain Relation	35
1.5.1	Early Perception of Plastic Stress Strain Relations	36

- 1.5.2 Concept of the Gradient-Based Plasticity and Its Relation with Mises Yield Criterion 37
 - 1.5.2.1 Concept of the Plastic Potential 37
 - 1.5.2.2 Physical Interpretation of the Plastic Potential 38
 - 1.5.2.3 Physical Interpretation of Mises Yield Function (Plastic Potential) 39
- 1.6 Effective Stress, Effective Strain, and Stress Type 42
 - 1.6.1 Effective Stress 42
 - 1.6.2 Effective Strain 42
 - 1.6.3 Stress Type 44
- References 44

- 2 Experimental Research on Material Mechanical Properties under Uniaxial Tension 47**
 - 2.1 Stress–Strain Relationship of Strain-Strengthened Materials under Uniaxial Tensile Stress State 47
 - 2.2 The Stress–Strain Relationship of the Strain-Rate-Hardened Materials in Uniaxial Tensile Tests 48
 - 2.3 Stress–Strain Relationship in Uniaxial Tension during Coexistence of Strain Strengthening and Strain Rate Hardening 50
 - 2.4 Bauschinger Effect 56
 - 2.5 Tensile Tests for Automotive Deep-Drawing Steels and High-Strength Steels 57
 - 2.5.1 Test Material and Experiment Scheme 57
 - 2.5.2 True Stress–Strain Curves in Uniaxial Tension 58
 - 2.5.3 Mechanical Property Parameters of Sheets 58
 - 2.5.3.1 Strain-Hardening Exponent n 59
 - 2.5.3.2 Lankford Parameter R 62
 - 2.5.3.3 Plane Anisotropic Exponent ΔR 62
 - 2.5.3.4 Yield-to-Tensile Ratio σ_s/σ_b 62
 - 2.5.3.5 Uniform Elongation δ_m 62
 - 2.6 Tensile Tests on Mg-Alloys 63
 - 2.7 Tension Tests on Ti-Alloys 63
 - 2.7.1 Mechanical Properties of Ti-3Al-2.5V Ti-Alloy Tubes at High Temperatures 65
 - 2.7.2 Strain Hardening of Ti-3Al-2.5V Ti-Alloy in Deformation at High Temperatures 69
 - References 71

- 3 Experimental Research on Mechanical Properties of Materials under Non-Uniaxial Loading Condition 73**
 - 3.1 P - p Experimental Results of Thin-Walled Tubes 73
 - 3.1.1 Lode Experiment 73
 - 3.1.2 P - p Experiments on Thin-Walled Tubes Made of Superplastic Materials 78
 - 3.1.2.1 Experiment Materials and Specimens 78
 - 3.1.2.2 Loading Methods 80
 - 3.1.2.3 Experimental Results and Analysis 80
 - 3.1.3 Experiments on Tubes Subjected to Internal Pressure and Axial Compressive Forces 86
 - 3.1.3.1 Experimental Device 86
 - 3.1.3.2 Material Properties 88
 - 3.1.3.3 Experimental Results 89

3.2	Results from <i>P-M</i> Experiments on Thin-Walled Tubes	91
3.2.1	Taylor-Quinney Experiments	91
3.2.2	<i>P-M</i> Experiments on Superplastic Material	94
3.3	Biaxial Tension Experiments on Sheets	95
3.3.1	Equipment for Biaxial Tension of Cruciform Specimens	96
3.3.2	Design of Cruciform Tensile Specimens	96
3.3.3	Application of Cruciform Biaxial Tensile Test	97
3.3.3.1	Forming Limit	97
3.3.3.2	Prediction of Yielding Locus	97
3.3.3.3	Analysis of Composite Materials	99
3.4	Influences of Hydrostatic Stress on Mechanical Properties of Materials	100
3.4.1	Testing Technique in High-Pressure Experiments	101
3.4.2	Influences of Hydrostatic Stresses on Flow Behavior of Materials	103
3.4.3	Influences of Hydrostatic Pressure on Fracture Behavior of Materials	106
3.5	Experimental Researches Other Than Non-Uniaxial Tension	114
3.5.1	Plane Compression Experiments	114
3.5.2	Loading Experiments along Normal and Tangential Directions	118
3.5.3	Other Combined Loading Methods	119
	References	119
4	Yield Criteria of Different Materials	123
4.1	Predicting Capability of a Yield Criterion Affected by Multiple Factors	123
4.2	Construction of a Proper Yield Criterion in Consideration of Multifactor-Caused Effects	129
4.2.1	A Proper Frame of Yield Criterion	130
4.2.2	Practical Yield Criterion with Multifactor-Caused Effects	133
4.2.3	Material Yielding Behavior Affected by Different Factors	136
4.2.3.1	Convexity of Yield Locus at Plane Stress State	137
4.2.3.2	Stress-Type-Caused Effects	143
4.2.3.3	Hydrostatic-Stress-Caused Effects	145
4.2.4	Simplified Forms of the Yield Criterion	148
4.2.5	Verification of the Yield Criterion Through Experiments	151
4.3	Anisotropic Materials	156
4.3.1	Experimental Description of Anisotropic Behavior of Rolled Sheet Metals	156
4.3.1.1	Uniaxial Tension	157
4.3.1.2	Biaxial Tension	159
4.3.2	Brief Review of the Anisotropic Yield and Plastic Potential Functions	160
4.3.3	Nonassociated-Flow-Rule-Based Yield Function and Plastic Potential	165
4.3.3.1	Anisotropic Yield Criterion	165
4.3.3.2	Anisotropic Plastic Potential	172
4.3.4	Associated-Flow-Rule-Based Anisotropic Yield Criterion	174
4.3.5	Experimental Verification of Two Kinds of Anisotropic Yield Criteria	178
	References	184
5	Plastic Constitutive Relations of Materials	187
5.1	Basic Concepts about Plastic Deformation of Materials and Relevant Plastic Constitutive Relations	187
5.1.1	Effects of Material Strength Property Transformation on Material Plastic Deformation	187

- 5.1.2 General Description of Subsequent Hardening Increments and Convexity of Yield Function 189
- 5.1.3 Effects of Flow Rules on Judgment of Condition of Stable Plastic Deformation of Materials 196
- 5.2 Equivalent Hardening Condition in Material Plastic Deformation 197
- 5.2.1 Universal Forms of Plastic Potential and Yield Criterion in Constructing Plastic Constitutive Relations 198
- 5.2.2 Relationship between Yield Function and Plastic Potential in Describing Equivalent Hardening Increments 199
- 5.2.3 Equivalent Hardening Condition Corresponding to Associated Flow Rule 201
- 5.2.4 Equivalent Hardening Condition Related to Nonassociated Flow Rule 206
- 5.3 “Softening” and Strength Property Changes in Plastic Deformation of Materials 209
- 5.3.1 Mechanical Models Mimicking Plastic Deformation of Sensitive-to-Pressure Materials 210
- 5.3.2 Dynamic Models to Imitate the Stress–Strain Relation of Anisotropic Material 215
- 5.3.3 Softening and Material Strength Property Changes in a Stable Plastic Deformation 219
- 5.4 Influences of Loading Path on Computational Accuracy of Incremental Theory 227
- 5.4.1 Discontinuous Stress Path 227
- 5.4.2 Unrealistic Strain Path 229
- References 231

- 6 Description of Material Hardenability with Different Models 233**
- 6.1 Plastic Constitutive Relations of Sensitive-to-Pressure Materials 233
- 6.1.1 Experimental Characterizations of Yield Function and Corresponding Plastic Potential 234
- 6.1.2 Theoretical Predictions in Comparison with Experimental Results 237
- 6.1.2.1 Influences of Hardening Models upon Description of Plastic Deformation of Materials 238
- 6.1.2.2 Yieldability and Plastic Flowability of Sensitive-to-Pressure Materials 239
- 6.1.2.3 Prediction of the Volumetric Plastic Strain 240
- 6.1.2.4 Predictions of Stress–Strain Relations in Uniaxial Tension and Compression 243
- 6.1.2.5 Stress–Strain Relations in Compression Affected by Superimposed Pressures 247
- 6.2 Anisotropic Hardening Model of Rolled Sheet Metals Characterized by Multiple Experimental Stress–Strain Relations and Changeable Anisotropic Parameters 248
- 6.2.1 A Constitutive Model to Describe Anisotropic Hardening and Anisotropic Plastic Flow of Rolled Sheet Metals 249
- 6.2.2 Transformation from Special 3D Stress State into 2D Stress States 252
- 6.2.3 Predictions of Anisotropic Hardening and Plastic Flow Behavior 254
- 6.2.3.1 Subsequent Yield Locus of Anisotropic Materials 254
- 6.2.3.2 Predictions of All Experimental Stress–Strain Relations in Yield Function 260
- 6.2.4 Experimental Verification 262
- 6.2.4.1 Predictions of Stress–Strain Relations in Uniaxial Tensions in Different Directions 262
- 6.2.4.2 Predictions of Changeable Anisotropic Parameters 267
- 6.3 Plastic Constitutive Relation with the Bauschinger Effects 271

- 6.3.1 Basic Concepts of the Bauschinger Effects 271
- 6.3.2 Consideration of the Bauschinger Effect in Constructing a Constitutive Relation 274
- 6.3.3 Exotic Anisotropic Behavior of Material Element Induced by Kinematic Hardening Model Based on Associated Flow Rule 276
 - 6.3.3.1 Anisotropic Flowability Borne of Kinematic Yield Model 276
 - 6.3.3.2 Calculations of the Exotic Anisotropy by Means of Yoshida's Modified Kinematic Model 281
- 6.3.4 A Method to Generate a Kinematic Plastic Potential Function 286
- References 293

- 7 Sequential Correspondence Law between Stress and Strain Components and Its Application in Plastic Deformation Process 295**
 - 7.1 Sequential Correspondence Law between Stress and Strain Components and Its Experimental Verification 295
 - 7.1.1 Sequential Correspondence Law between Stress and Strain Components 295
 - 7.1.2 Experimental Verification of the Sequential Correspondence Law between Stress and Strain Components 298
 - 7.1.3 Application of the Sequential Correspondence Law between Stress and Strain Components 300
 - 7.2 Zoning of Mises Yield Ellipse and Typical Plane Stress Forming Processes on It 302
 - 7.3 Stress and Strain Analysis of Plane-Stress Metal-Forming Processes 306
 - 7.3.1 Tube Drawing 306
 - 7.3.2 Deep Drawing 307
 - 7.3.3 Tube Hydroforming 308
 - 7.4 Spreading of Mises Yield Cylinder and Characterization of Three-Dimensional Stresses Therein 309
 - 7.5 Zoning in Three-Dimensional Stress Yield Locus and Positioning Typical Forming Processes Thereon 311
 - References 316

- 8 Stress and Strain Analysis and Experimental Research on Typical Axisymmetric Plane Stress-Forming Process 317**
 - 8.1 Incremental-Theory-Based Solution to Stress and Strain Distribution of Steady Axisymmetric Plane Stress-Forming Processes 317
 - 8.1.1 Two Expressions of Stress and Strain Distribution 317
 - 8.1.2 Division of Steady Thin-Walled Tube-Forming Processes 319
 - 8.1.3 Basic Formulas and Assumption 320
 - 8.1.4 Stress and Strain Distribution in Steady Frictionless Forming Process 321
 - 8.1.4.1 General Equilibrium Equation 321
 - 8.1.4.2 Stress Distribution $\sigma(r)$ 322
 - 8.1.4.3 Strain Rate $d\varepsilon/d\varphi$ 324
 - 8.1.4.4 Strain Distribution $\varepsilon(\varphi)$ 325
 - 8.1.5 Stress and Strain Distribution in Steady Forming Processes in the Presence of Friction 328
 - 8.1.5.1 General Equilibrium Equation 329
 - 8.1.5.2 Stress and Strain Distribution 331

8.2	Experimental Study on Thickness Distribution in Tube Necking and Tube Drawing	331
8.2.1	Thickness Distribution in Tube-Necking Processes	331
8.2.2	Experimental Research on Thickness Distribution during Tube Drawing	333
8.3	Experiments on Thin-Walled Tube under Action of Biaxial Compressive Stresses	336
8.3.1	Introduction of Experimental Setup	337
8.3.2	Results and Discussion	339
	References	341
9	Shell and Tube Hydroforming	343
9.1	Mechanics of Dieless Closed Shell Hydro-Bulging	343
9.1.1	Equilibrium Equation for an Internally Pressurized Closed Shell	343
9.1.2	Yield Equation of an Internally Pressurized Closed Shell	345
9.1.3	Principle of Spheroidization of Plastic Deformation in Shell Hydro-Bulging	345
9.2	Dieless Hydro-Bulging of Spherical Shells	347
9.2.1	Stress Analysis of Dieless Hydro-Bulging of Spherical Shells	347
9.2.2	Manufacture of Spherical Shells	347
9.2.3	Shell Structure before Hydro-Bulging	348
9.2.4	Dieless Hydro-Bulging of Single-Curvature Polyhedral Shells	349
9.3	Dieless Hydro-Bulging of Ellipsoidal Shells	350
9.3.1	Stress Analysis of Internally Pressurized Ellipsoidal Shells	351
9.3.2	Wrinkling of Internally Pressurized Ellipsoidal Shell and Anti-Wrinkling Measures	352
9.4	Dieless Hydro-Bulging of Elbow Shell	355
9.5	Tube Hydroforming	356
9.5.1	Principle of Tube Hydroforming and Its Stress States	356
9.5.2	Yield Criterion for Tube Hydroforming	357
9.5.3	Position of Tube Hydroforming on Yield Ellipse	358
9.5.4	Typical Stress States and Their Distribution on Yield Ellipse	358
9.5.5	Effect of Stress State on the Tube Deformation Characteristics	359
9.5.6	Formation Mechanism of Wrinkles in Thin-Walled Tube Hydroforming	360
	References	362
10	Bulk Forming	365
10.1	Load Calculation in Tool Movement Direction	365
10.2	Upsetting of Cylinders and Rings	368
10.2.1	Load Calculation for Cylinder Upsetting	369
10.2.2	Inhomogeneous Deformation in Cylinder Upsetting	373
10.2.3	Metal Flow and Pressure Distribution during Ring Compression	376
10.3	Characteristics of Die Forgings and Calculation of Required Loads	378
10.4	Isothermal Forging	381
10.4.1	Stress Analysis in Isothermal Forging	381
10.4.2	Stress Analysis of a Single Rib Piece in Isothermal Forging	382
10.4.3	Isothermal Forming of Cross-Rib-Born Pieces	384
10.4.3.1	Analysis of Forming Processes	384
10.4.3.2	Stress Analysis	384
10.4.4	Control and Analysis of Flow Defects during Isothermal Forging	386
10.4.4.1	Folds	386

- 10.4.4.2 Formation and Control of Flow Lines 388
- 10.5 Calculation of Required Load in Rolling 389
- 10.5.1 Derivation of Formula for Calculating Unit Pressure Distribution on Rollers' Contact Arc Surface 391
- 10.5.2 Total Rolling Force and Average Pressure 395
- 10.5.3 Rolling Torque 396
- 10.5.4 Energy Consumption in Rolling 397
- 10.6 Extrusion and Drawing 397
- 10.6.1 Extrusion 397
- 10.6.2 Drawing 400
- 10.7 Rotary Forging 403
- 10.7.1 Introduction 403
- 10.7.2 Stress and Strain Analysis in Rotary Forging of Cylinders 403
- 10.7.3 Stress–Strain Analysis in Rotary Forging of Discs 409
- 10.8 Strain Distribution Measurement in Bulk Forming 411
- 10.8.1 Introduction 411
- 10.8.2 Screw Method 412
- 10.8.3 Applications of Screw Method in Determining Strain Distribution 414
- References 419

- 11 Sheet Forming 421**
- 11.1 Deep Drawing 421
- 11.1.1 Basic Principles 421
- 11.1.2 Strain Analysis in Flange Area 421
- 11.1.3 Stress Analysis of the Flange Area 424
- 11.1.3.1 Equilibrium Equation 424
- 11.1.3.2 Yield Criteria 425
- 11.2 Sheet Hydroforming Process 426
- 11.2.1 Basic Principles 426
- 11.2.2 Characteristics and Application Scope 427
- 11.2.3 Assessment of Experimental Parameters 428
- 11.2.3.1 Critical Liquid Pressure p_{cr} 428
- 11.2.3.2 Drawing Force 429
- 11.2.3.3 Blank Holder Force (BHF) 429
- 11.2.4 Influences of Normal Stress on SHP 430
- 11.2.5 Influences of Pre-Bulging on the Deformation Uniformity in SHP 430
- 11.3 Hole-Flanging 434
- 11.3.1 Basic Principles 434
- 11.3.2 Analysis of Stress and Strain 434
- 11.3.3 Limiting Flanging Coefficient 436
- 11.4 Viscous Pressure Forming 438
- 11.4.1 Mechanism and Features 438
- 11.4.1.1 Forming Sequence 438
- 11.4.1.2 Properties of Pressure Medium 439
- 11.4.1.3 Reverse Pressure 439
- 11.4.1.4 Surface Quality 439
- 11.4.2 Constitutive Equations of Viscous Medium 439
- 11.4.3 Influences of BHP on Forming Process 441
- 11.5 Multipoint Sandwich Forming 445

11.5.1	Introduction	445
11.5.2	Working Principles of MPSF	446
11.5.3	Advantages of MPSF and Applications	447
11.5.4	FE Model of MPSF	448
11.5.5	Forming of Ellipsoidal Workpiece	451
11.5.6	Saddle-Type Pieces Forming	455
11.6	Formability of Sheet Metals	462
11.6.1	Introduction	462
11.6.2	Forming Limit Diagram	462
11.6.3	Experimental Determination of FLC	464
11.6.3.1	Uniaxial Tensile Test	465
11.6.3.2	Hydro-Bulging Test	465
11.6.3.3	Nakazima Test	465
11.6.4	Advanced Experimental Methods	466
11.6.5	Theoretical Prediction of FLC	469
11.6.6	New Developments in FLCs	475
11.7	Improvements of Panel Stamping Process	478
11.7.1	Designs of Draw-Bars Corresponding to the Wrinkling Types	479
11.7.2	Replacement of Stretching Wall with Local Nondeformable Design	482
	References	484
	Index	489

Fundamentals of Classical Plasticity

1.1 Stress

1.1.1 The Concept of Stress Components

When a set of directional forces P_1, P_2, P_3, \dots acts on a deformable material element (see Figure 1.1) and remains balanced without causing a displacement and/or rotation, a set of balanced internal stresses must be generated because of the deformation taking place in the material element. Generally, if stress components distribute uniformly on a plane, the stress unit is equal to the force per unit area. Despite an inherent relation that exists between the stress and the acting force, the stress and the force are entirely different in their physical concepts that we could not confuse.

In analyzing displacement and rotation of a rigid body, all acting forces are vectors and can be converted into a single one. For example, the forces P_1, P_2, P_3, \dots shown in Figure 1.1 can be turned into a single vector P :

$$P = P_1 + P_2 + P_3 + \dots \quad (1.1)$$

Equation (1.1) means that if $P \neq 0$, this loaded body must move and if P does not pass the body's center, the body must rotate simultaneously.

However, it is incorrect to use the force P resulted from vector addition to analyze the elastic or plastic changes in shape of the material element. Different sets of directional forces will respond to different stress distribution on a plane cut out of this material element even if they have the same vector composition. Figure 1.2 illustrates the case of a simple uniaxial tension.

Stress components on different planes of this loaded material element are different. For example, the stress component on the plane vertical to the axis in Figure 1.2 can be expressed by

$$\sigma_0 = P/S_0 \quad (1.2)$$

where P is the axial force, and S_0 is the cross section.

If this material element is a unit body with each edge equal to 1 unit, Equation (1.2) becomes

$$\sigma_0 = P \quad (1.3)$$

Equation (1.3) builds up a relationship between the force and the stress, however, it holds true only in the analysis of equilibrium system. Their physical concepts are essentially different. The force is mainly to make the forced material element to move or move with rotation, and the stress deals with the "shape changing" of the stressed material element.

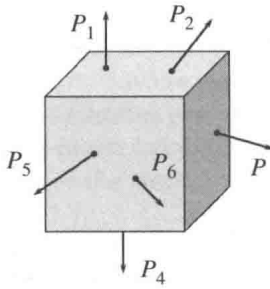


Figure 1.1 Directional forces acting on a unit element.

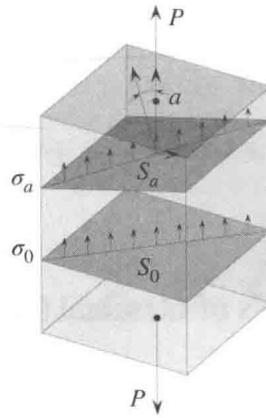


Figure 1.2 Relationship between forces and stresses on a plane cut out of a loaded body under uniaxial tension.

On a cut plane tilting at an angle α against the axis, when the area of the plane increases from S_0 to $S_0/\cos \alpha$, the stress on it becomes

$$\sigma_\alpha = P \cos \alpha \quad (1.4)$$

With the angle α increasing, the cut plane should get more inclined to decrease the stress σ_α so as to maintain the force in equilibrium. Equation (1.4) indicates that the value of stress σ_α is completely predicated on the orientation of the cut plane. But variation of the stress value σ_α does not bring any influences upon the deformation type of the uniaxial tension, which means that the strain state of the element remains unchanged.

1.1.2 Description of the Stress State

It should thus be clear that the stress state is very important—we must understand how the material element responds to the deformation caused by the stress components. On the other hand, in the case of inhomogeneous stress distribution on cut planes, which is most common in reality, it is required to analyze the stress state of a deforming body from one point inside the body to the other. Generally, the stress condition of a point inside a deforming body is often defined by a cubic element. Further understanding stress components in relation to any complex stress state would be essential to fully grasp the stress and stress tensor concepts.

1.1.2.1 Stresses on an Arbitrary Inclined Plane

Let's investigate the necessary condition by analyzing the stress state at a point inside the deformable body. Suppose that the point to be analyzed is O . Usually, three mutually perpendicular planes XOY , YOZ , and ZOX (see Figure 1.3) are set up to analyze its stress state. Stress components on each plane are divided into one normal stress, symbolized by σ , vertical to the plane and two shear stresses by τ , parallel to the coordinate axes. In order to identify what plane the stress components act on, one subscript is used for the normal stress and two subscripts for each of the shear stresses. The subscript for the normal stress denotes its acting direction.

The first of the two subscripts of the shear stress denotes the normal direction of the acting plane of the shear stress, and the second the acting direction of the shear stress (see Figure 1.3). The value of all stress components is not arbitrary but is determined by the equilibrium

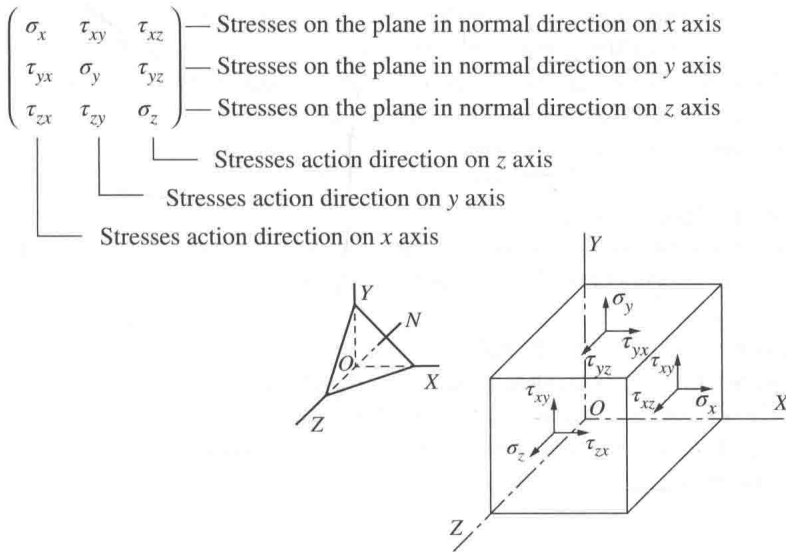


Figure 1.3 Stress components on three mutually perpendicular planes.

between stress components on the cut plane and the associated external force (see the example in Figure 1.2). Customarily, positive normal stresses are supposed to be tensile ones and negative normal stresses are compressive. Normally, it doesn't matter whether a shear stress is positive or negative, because shear stresses always exist in pairs. Nevertheless, because some materials show anisotropic yield behavior and/or strength differential in tension and compression, the change in loading direction or stress state might change the yielding behavior in value of the shear stresses. Therefore, for some materials, when the stress direction changes (e.g., from tension to compression or vice versa), we must still define whether the shear stresses are positive or negative, based on the action direction. Namely, when the second subscript of shear stress implies the positive direction of the axis, this shear stress is positive, and vice versa. Thus, stress components on three planes in a xyz coordinate system can be expressed in the matrix form (see Figure 1.3).

From Figure 1.3, we know that the nine stress components on the three mutually perpendicular planes share a common feature. That is, shear stresses exist in pairs with the same value and the subscripts composed of two identical English letters in opposite order. Namely,

$$\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz} \quad (1.5)$$

Equation (1.5) means that there are only six independent stress components in a symmetric form (Figure 1.3), which can be represented by a matrix as follows:

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ & \sigma_y & \tau_{yz} \\ & & \sigma_z \end{pmatrix} \quad (1.6)$$

In the case that, on the three mutually perpendicular planes, there are only three normal stresses, called the principal stresses: σ_1 , σ_2 , and σ_3 , without any shear stress, Equation (1.6) becomes

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \quad (1.7)$$

which represents the principal coordinate system.

Obviously, once the six stress components at the point O are given with respect to the $x, y,$ and z coordinate axis, the stress state at the point O is fixed. Any change in the value of one of the stress components, as long as it is not due to the conversion of coordinate system, would mean a change in the stress state at the point O . In other words, the stress state at a point must be described with six stress components or three principal stresses.

1.1.2.2 Stress Components on an Oblique Plane

It has been proved that given six stress components or three principal stresses at a point, the normal and shear stresses on any oblique plane relative to the $x, y,$ and z coordinate axes can be determined.

On the oblique plane represented by a triangle ABC (see Figure 1.4), the normal N (ON) is denoted by directional cosines (l, m, n) with three angles α_x, α_y and α_z formed between ON and separately $OX, OY,$ and OZ . Let ΔA denote the area of the triangle $ABC, \Delta A_x, \Delta A_y,$ and ΔA_z the areas formed by projecting $\Delta A,$ respectively, on the three coordinate surfaces:

$$\begin{aligned} \Delta A_x &= \Delta A \cdot l \\ \Delta A_y &= \Delta A \cdot m \\ \Delta A_z &= \Delta A \cdot n \end{aligned} \tag{1.8}$$

Let the resultant stress on the triangle ABC be denoted by $S,$ which has a direct stress component σ_N normal to the plane ABC and a shear stress component τ on it. Thus, the equilibrium of the forces on the tetrahedron $OABC$ in the direction $OX, OY,$ and $OZ,$ respectively, can be described by

$$\left. \begin{aligned} S_x \cdot \Delta A &= \sigma_x \cdot \Delta A \cdot l + \tau_{yx} \cdot \Delta A \cdot m + \tau_{zx} \cdot \Delta A \cdot n \\ S_y \cdot \Delta A &= \tau_{xy} \cdot \Delta A \cdot l + \sigma_y \cdot \Delta A \cdot m + \tau_{zy} \cdot \Delta A \cdot n \\ S_z \cdot \Delta A &= \tau_{xz} \cdot \Delta A \cdot l + \tau_{yz} \cdot \Delta A \cdot m + \sigma_z \cdot \Delta A \cdot n \end{aligned} \right\} \tag{1.9}$$

where $S_x, S_y,$ and S_z are the components of the resultant stress S in parallel with $OX, OY,$ and $OZ,$ respectively.

Simplification of Equation (1.9) gives

$$\left. \begin{aligned} S_x &= \sigma_x \cdot l + \tau_{yx} \cdot m + \tau_{zx} \cdot n \\ S_y &= \tau_{xy} \cdot l + \sigma_y \cdot m + \tau_{zy} \cdot n \\ S_z &= \tau_{xz} \cdot l + \tau_{yz} \cdot m + \sigma_z \cdot n \end{aligned} \right\} \tag{1.10}$$

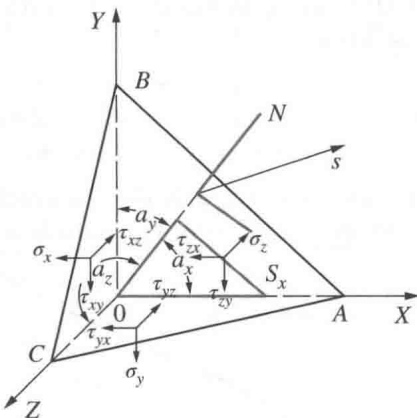


Figure 1.4 Stress components on an oblique plane.