普通高等教育“十三五”规划教材
普通高等院校数学精品教材

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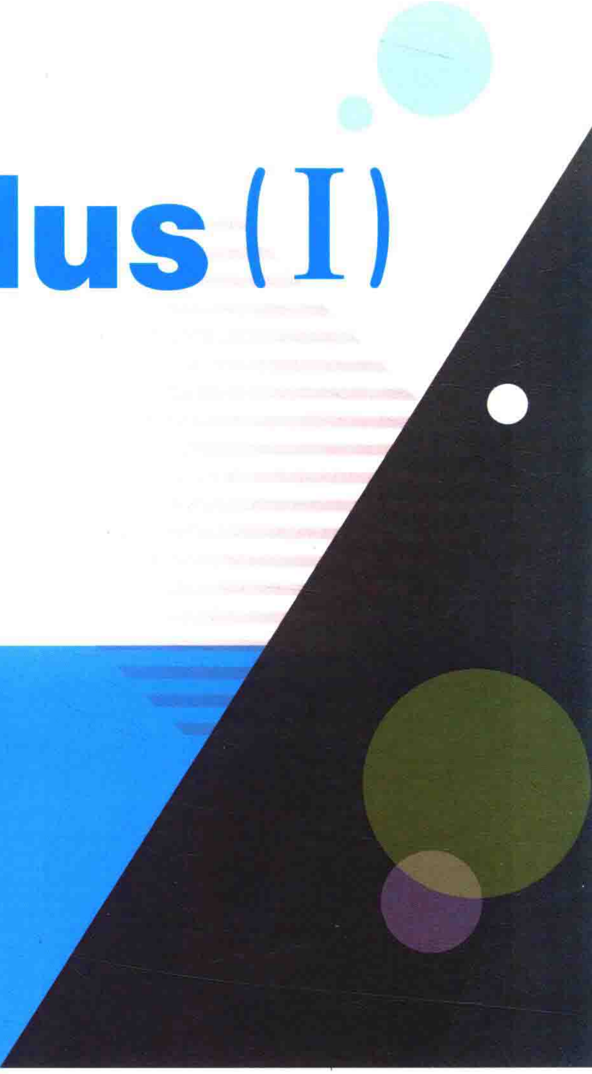
Calculus (Ⅰ)

毛纲源 周海婴／编著

微积分(Ⅰ)

（英文版）



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Abstract

Calculus (Ⅰ) is generally aimed at first year undergraduate students who major in finance or economics. This book strives to provide students and teachers with perspectives and approaches in calculus. The book guides the reader to an appreciation of interrelation among different aspects of the subject. It features examples that illustrate key concept as well as exercises that strengthen understanding.

Chapter 1 provides preliminary knowledge and describes the classical functions. Chapter 2 discusses limits and continuity of functions and presents basic facts about continuous functions. In Chapter 3, the derivative is defined and the basic rules of differentiation are presented. Chapter 4 describes the basic theory of differentiation and Taylor formula. It provides how the derivative used to discuss properties of functions and apply in economics. Chapter 5 defines anti-derivative and indefinite integral. Fundamental integral formulas, change of variable in integral and integration by parts are also discussed. Chapter 6 introduces, through examples of area and distance, the notion of the integral, and the approximate integrals leading to its definition. Fundamental theorem of calculus is proved and applications of integrals are provided.

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Chapter 1 Functions

1.1 Preliminary knowledge

1.1.1 Inequalities and their properties

In this section we work with inequalities that abound in calculus, that is, inequalities involve a variable. To solve an inequality in x is to find the set of numbers x for which the inequality holds.

The way we solve an inequality is very similar to the way we solve an equation, but there is one important difference. We often use the following properties of inequalities.

Proposition 1.1.1 In what follows, a, b, c are numbers.

(1) If $a>b$ and $b>c$, then $a>c$ (transitivity);

(2) If $a<b$ and $c<d$, then $a+c<b+d$ (additivity);

(3) If $a<b$ and $c>0$, then $ac<bc$ (positive multiplicativity);

(4) If $a<b$ and $c<0$, then $ac>bc$ (negative multiplicativity).

Note The multiplication laws (3) and (4) must be carefully observed. We can maintain an inequality only by multiplying both sides by the same positive number. But if we multiply by a negative number, then the inequality is reversed.

To illustrate their use we will solve several examples. In each, the problem is to "solve for x", which means to find all real numbers that satisfy a given inequality.

1.1.1.1 The usual way to solve a linear inequality of x

The usual way to solve the linear inequality is using the properties of inequality, that is the proposition 1.1.1. There are generally several ways to solve a given inequality.

Example 1.1.1 Solve the inequality $-3(4-x)<12$.

Solution 1 Multiplying both sides of the inequality by $-\frac{1}{3}$, we have $4-x>-4$, where the inequality has been reversed.

Subtracting 4 at both sides of the inequality, we get $-x>-8$.

To isolate x, we multiply both sides by -1. This gives $x<8$, here the inequality has been reversed again.

The solution is the interval $(-\infty,8)$.

Solution 2　From $-3(4-x)<12$, we get $-12+3x<12$.

Add both sides by 12, we have $3x<24$. Now we have $x<8$ by dividing both sides by 3.

The solution, of course, is $(-\infty,8)$.

1.1.1.2　The usual way to solve a quadratic inequality

The usual way to solve a quadratic inequality is to factor the quadratic.

Example 1.1.2　Solve the inequality $\frac{1}{5}(x^2-4x+3)<0$.

Solution　First we eliminate the outside factor $\frac{1}{5}$ by multiplying by 5. This gives $x^2-4x+3<0$.

Factoring the quadratic, we have

$$(x-1)(x-3)<0$$

The product $(x-1)(x-3)$ is zero at 1 or 3. We mark these points on a number line, as shown in Figure 1.1.1.

Figure 1.1.1

The points 1 and 3 divide the number line into three intervals:

$$(-\infty,1),\quad (1,3),\quad (3,+\infty)$$

On each of these intervals the product $(x-1)(x-3)$ keeps a constant sign:

on $(-\infty,1)$ (to the left of 1), $\operatorname{sgn}[(x-1)(x-3)]=(-)(-)=+$;

on $(1,3)$ (between 1 and 3), $\operatorname{sgn}[(x-1)(x-3)]=(+)(-)=-$;

on $(3,+\infty)$ (to the right of 3), $\operatorname{sgn}[(x-1)(x-3)]=(+)(+)=+$.

The product $(x-1)(x-3)$ is negative only on $(1,3)$. The solution is the open interval $(1,3)$.

Example 1.1.3　Solve the inequality $2x^2-2x>4$.

Solution　From the given inequality, we have $2x^2-2x-4>0$ (subtracted 4 on both sides), then we have $x^2-x-2>0$ (divided by 2), and $(x+1)(x-2)>0$ (factored).

On the number line we mark the points -1 and 2 where the product

$(x+1)(x-2)$ is zero, as illustrated in Figure 1. 1. 2. These points separate the number line into three intervals $(-\infty,-1),(-1,2),(2,+\infty)$.

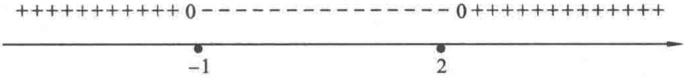

Figure 1. 1. 2

On each of these intervals the product $(x+1)(x-2)$ keeps a constant sign:

on $(-\infty,-1)$ (to the left of -1), $\operatorname{sgn}[(x+1)(x-2)]=(-)(-)=+$;

on $(-1,2)$ (between -1 and 2), $\operatorname{sgn}[(x+1)(x-2)]=(+)(-)=-$;

on $(2,+\infty)$ (to the right of 2), $\operatorname{sgn}[(x+1)(x-2)]=(+)(+)=+$.

The product $(x+1)(x-2)$ is positive on $(-\infty,-1)$ and $(2,+\infty)$. The solution is the union of these two intervals $(-\infty,-1)\cup(2,+\infty)$.

Summing up the argument of the above two examples, we conclude the following results:

Proposition 1. 1. 2 Let a and b be two numbers and $a<b$, then the solution of inequality $(x-a)(x-b)<0$ is the interval (a,b); the solution of $(x-a)(x-b)>0$ is the union of two intervals $(-\infty,a)\cup(b,+\infty)$.

Example 1. 1. 4 Solve for x in $x^2+4x-2\leqslant 0$.

Solution To factor the quadratic we first transpose the -2 and then complete the square on the left:

$$x^2+4x\leqslant 2,\quad x^2+4x+4\leqslant 2+4=6,\quad (x+2)^2\leqslant 6.$$

Now we subtract 6 from both sides: $(x+2)^2-6\leqslant 0$ and factor the expression as the difference of two squares:

$$(x+2+\sqrt{6})(x+2-\sqrt{6})\leqslant 0.$$

The product is 0 at $-2-\sqrt{6}$ and $-2+\sqrt{6}$. It is negative between these two numbers. By proposition 1. 1. 2 we get that the solution is the closed interval $[-2-\sqrt{6},-2+\sqrt{6}]$. i. e. , $-2-\sqrt{6}\leqslant x\leqslant -2+\sqrt{6}$.

1.1.1.3 The usual way to solve an inequality that involves quotients

We usually use the following proposition to solve it.

Proposition 1. 1. 3 Suppose that $P_m(x)$ and $Q_n(x)$ are the mth degree, the nth degree polynomials respectively.

(1) $\dfrac{P_m(x)}{Q_n(x)}>0$ if $P_m(x)\cdot Q_n(x)>0$;

(2) $\dfrac{P_m(x)}{Q_n(x)}<0$ if $P_m(x)\cdot Q_n(x)<0$.

Example 1.1.5 Solve for x in $\frac{x}{x-3}<2$.

Solution 1 By reduction the fraction $\frac{x}{x-3}-2<0$ to a common denominator, we get $\frac{x-2(x-3)}{x-3}=\frac{6-x}{x-3}<0$. By proposition 1.1.3 we have $(6-x)(x-3)<0$. Multiplying both sides by -1, we get $(x-6)(x-3)>0$. By proposition 1.1.2, we obtain the union of two intervals $(-\infty,3)\cup(6,+\infty)$ is the solution of the inequality.

Solution 2 We multiply the inequality by $x-3$ for the cases $x-3>0$ and $x-3<0$. The case $x-3=0$ can not occur.

Case 1: $x-3>0$ or $x>3$. By proposition 1.1.1(3), we get $x<2(x-3)$ or $x<2x-6$.

This implies $x>6$. Hence a partial solution is $(6,+\infty)$.

Case 2: $x-3<0$ or $x<3$. This time when we multiply both sides by $x-3$, by proposition 1.1.1(4), we reverse the inequality and obtain $x<6$. The solution for this case are those values of x for which $x-3<0$ and $x<6$. Thus the numbers in $(-\infty,3)$ are the solutions for case 2.

Combining the results of the two cases, we conclude that any number in $(-\infty,3)$ and $(6,+\infty)$ satisfies the given inequality.

Example 1.1.6 Solve the inequality $\frac{(x+4)(x-5)^2}{x^3(x-2)}>0$.

Solution By proposition 1.1.3, this inequality has the same solution as the inequality

$$(x+4)(x-5)^2x^3(x-2)>0.$$

We will work with the second inequality. The product on the left is 0 at the points $x=-4, x=5, x=0, x=2$.

We rewrite these numbers in increasing order:

$$x=-4,\quad x=0,\quad x=2,\quad x=5.$$

Then we rewrite the product as $(x+4)x^3(x-2)(x-5)^2$. The points $x=-4, x=0, x=5, x=2$ separate the x axis into five subintervals: $(-\infty,-4)$, $(-4,0)$, $(0,2)$, $(2,5)$, $(5,+\infty)$.

On each of these intervals the product $(x+4)x^3(x-2)(x-5)^2$ keeps constant sign, which is

negative on $(-\infty,-4)$ (7 negative factors);

positive on $(-4,0)$ (6 negative factors);

negative on $(0,2)$ (3 negative factors);

positive on $(2,5)$ (2 negative factors);

positive on $(5,+\infty)$ (0 negative factors).

The solution is the union $(-4,0)\cup(2,5)\cup(5,+\infty)$.

Note For an expression of the form

$$(x-a_1)(x-a_2)\cdots(x-a_n)$$

with $a_1<a_2<\cdots<a_n$, it is positive on those intervals where an even number of terms are negative; and it is negative on those intervals where an odd number of terms are negative.

1.1.2 Absolute value and its properties

1.1.2.1 Definition of absolute value

Definition 1.1.1 Let x be a real number, if

$$|x|=\begin{cases} x, & x\geqslant 0, \\ -x, & x<0, \end{cases}$$

then the nonnegative number $|x|$ is called the absolute value of x.

By the above definition we easily know that the absolute value of a number is always equal or greater than 0. The absolute value of a positive number is always positive.

Geometrically we may interpret the absolute value $|x|$ as the distance from the original point 0 to the point x. Whereas $|x-y|$ means the distance between x and y.

With an eye toward chapter 3 we introduce two Greek letters: δ (delta) and ε (epsilon).

We begin with the inequality $|x|<\delta$, where δ is some positive number. To say that $|x|<\delta$ means that x lies within δ units of 0, equivalently, that x lies between $-\delta$ and δ. Thus

$$|x|<\delta \quad \text{if} \quad -\delta<x<\delta,$$

as shown in Figure 1.1.3.

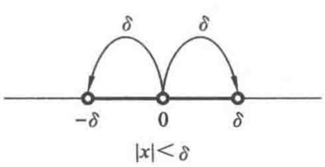

Figure 1.1.3

To say that $|x-x_0|<\delta$ means that x lies within δ units of x_0, or, equivalently, that x lies between $x_0-\delta$ and $x_0+\delta$. Thus

$$|x-x_0|<\delta \quad \text{if} \quad x_0-\delta<x<x_0+\delta,$$

as shown in Figure 1.1.4.

Somewhat more delicate is the inequality

$$0<|x-x_0|<\delta.$$

Here we have $|x-x_0|<\delta$ with the additional requirement that x cannot be x_0. Consequently

$$0<|x-x_0|<\delta \quad \text{if} \quad x_0-\delta<x<x_0 \quad \text{or} \quad x_0<x<x_0+\delta,$$

as illustrated in Figure 1.1.5.

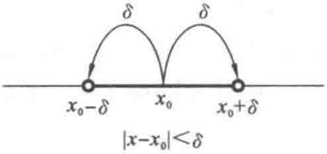

Figure 1.1.4

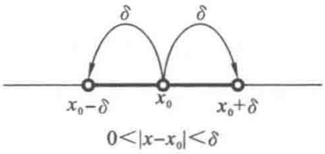

Figure 1.1.5

Example 1.1.7 Solve the inequality $|3x-4|<2$.

Solution Since

$$|3x-4|=|3(x-4/3)|=|3|\,|x-4/3|=3|x-4/3|,$$

the inequality can be rewritten as $3|x-4/3|<2$. This gives $|x-4/3|<2/3$, and $4/3-2/3<x<4/3+2/3$, then $2/3<x<2$. The solution is the open interval $(2/3,2)$.

Let $\varepsilon>0$. If we consider $|x|$ as the distance between x and 0, then obviously

$$|x|>\varepsilon \quad \text{if} \quad x>\varepsilon \quad \text{or} \quad x<-\varepsilon,$$

as shown in Figure 1.1.6.

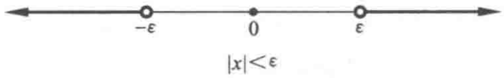

Figure 1.1.6

Example 1.1.8 Solve the inequality $|2x+3|>5$.

Solution As shown above, in general

$$|x|>\varepsilon \quad \text{if} \quad x>\varepsilon \quad \text{or} \quad x<-\varepsilon.$$

So here $2x+3>5$ or $2x+3<-5$.

The first inequality gives $2x>2$ and thus $x>1$.

The second inequality gives $2x<-8$ and thus $x<-4$.

The solution is therefore the union $(-\infty,-4)\cup(1,+\infty)$, as illustrated in Figure 1.1.7.

1.1.2.2 Properties of absolute values

Absolute values behave nicely for multiplication and division, but not so well for addition and subtraction.

Proposition 1.1.4 For all real numbers a and b, absolute values have the

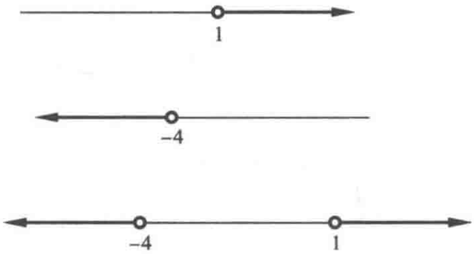

Figure 1.1.7

following basic properties:

(1) $|-x|=|x|=\sqrt{x^2}$; (2) $-|x|\leqslant x\leqslant |x|$;

(3) $|x\pm y|\leqslant |x|+|y|$ (triangle inequality);

(4) $||x|-|y||\leqslant |x-y|$; (5) $|x\cdot y|=|x||y|$;

(6) $\left|\frac{x}{y}\right|=\frac{|x|}{|y|}(y\neq 0)$; (7) $|x|<|y|$ if $x^2<y^2$.

In the following part we give the proof of proposition 1.1.4 (3)、(4).

By the property (2), we obtain $-|x|\leqslant x\leqslant |x|$ and $-|y|\leqslant y\leqslant |y|$. Therefore $-(|x|+|y|)\leqslant x+y\leqslant |x|+|y|$, i. e., $|x+y|\leqslant |x|+|y|$.

Similarly, we can prove that

$$|x-y|=|x+(-y)|\leqslant |x|+|-y|=|x|+|y|.$$

By the property (3) we obtain

$$|x|=|x-y+y|\leqslant |x-y|+|y|, \text{i. e., } |x|-|y|\leqslant |x-y|.$$

Similarly we obtain $|y|-|x|\leqslant |y-x|$, multiplying both sides by (-1), we obtain

$$|x|-|y|\geqslant -|y-x|=-|x-y|.$$

Therefore $$-|x-y|\leqslant |x|-|y|\leqslant |x-y|,$$

i. e., $$||x|-|y||\leqslant |x-y|.$$

Example 1.1.8 Solve for x in $|x+2|<|x-1|$ and express the set of solution to the inequality as an interval.

Solution 1 This inequality is more difficult to solve than earlier examples, because there are two absolute value signs. We can remove both of them by using proposition 1.1.4 (7).

$$|x+2|<|x-1|\Leftrightarrow (x+2)^2<(x-1)^2$$
$$\Leftrightarrow x^2+4x+4<x^2-2x+1\Leftrightarrow 6x+3<0,$$

i. e., $x<-\frac{1}{2}$. It is expressed as interval $\left(-\infty,-\frac{1}{2}\right)$.

Solution 2 By the geometrical meaning of the absolute value, we know that we

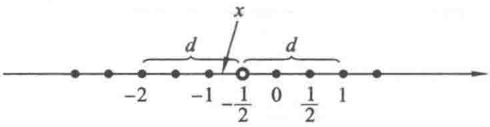

Figure 1. 1. 8

need find the set of points x such that the distance from x to point -2 is less than the distance from x to point 1. By Figure 1. 1. 8, we can easily see that the point $-\frac{1}{2}$ is the median point from -2 to the point 1. Thus the distance from $-\frac{1}{2}$ to -2 is equal to the distance from $-\frac{1}{2}$ to 1. Therefore when $x<-\frac{1}{2}$, the distance from x to -2 is less than the distance from x to 1, as shown in Figure 1. 1. 8. So the solution set of the given inequality is $\left\{x \middle| x<-\frac{1}{2}\right\}$. It is expressed as the interval $\left(-\infty,-\frac{1}{2}\right)$.

1. 1. 3 The range of variable

A variable takes on a series of numerical values. The set of all numerical values of a variable quantity is called the range of the variable.

In the next part we shall define the following ranges of a variable that will be frequently used later.

An interval is the set of all numbers x lying between two given points a and b, where a and b are the end points. The word "number" means a real number throughout this book.

If $a<b$, we denote by (a,b) the set of all numbers x satisfying the inequalities $a<x<b$ and refer to this set as the open interval from a to b. We write it as

$$(a,b)=\{x \mid a<x<b\},$$

and read as: x is greater than a and less than b.

A closed interval is the set of all numbers x lying between and including two given numbers a and b. It is denoted by $[a,b]$, or by the inequalities $a\leqslant x\leqslant b$. That is, $[a,b]=\{x \mid a\leqslant x\leqslant b\}$, and read as: x is greater than or equal to a, and less than or equal to b.

If one of the two numbers a and b, say a, belongs to the interval, while the other one does not, we have a partial closed (half-closed) interval, which may be given by the inequalities $a\leqslant x<b$ and is denoted by $[a,b)$. If the number b belongs to the set while a does not, we have the half-closed interval $(a,b]$, which can be

given by the inequalities $a<x\leqslant b$.

The above intervals are called finite intervals. The number $b-a$ is called the length of the interval.

In addition, there exist infinite intervals. We introduce the symbols $+\infty$ (read as plus infinity) and $-\infty$ (read as minus infinity) to express the infinite interval.

If the variable x assumes all possible values greater than a, such an interval is denoted as $(a,+\infty)$ and represented by the conditional inequalities $a<x<+\infty$. In the same way we regard the infinite intervals and half-closed infinite intervals represented by the following conditional inequalities:

$$a\leqslant x<+\infty,\quad -\infty<x<b,\quad -\infty<x\leqslant b,\quad -\infty<x<+\infty;$$

Or we can denote them by the following intervals respectively:

$$[a,+\infty)=\{x\mid x\geqslant a\},\quad (-\infty,b)=\{x\mid x<b\},$$

$$(-\infty,b]=\{x\mid x\leqslant b\},\quad (-\infty,+\infty)=\{x\mid -\infty<x<+\infty\}.$$

The graphs of the two kinds of intervals are shown in Figure 1. 1. 9.

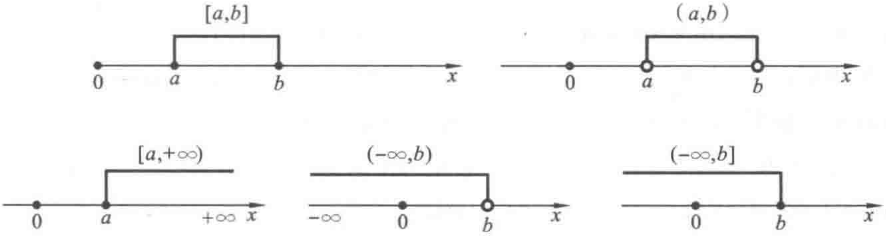

Figure 1. 1. 9

Any open interval containing a point x_0 as its midpoint is called a neighborhood of x_0. We denote it by $U(x_0,\delta)=\{x\mid x_0-\delta<x<x_0+\delta\}$, as shown in Figure 1. 1. 10.

The positive number δ is called the radius of the neighborhood, and the point x_0 is called the center of the neighborhood.

The inequalities $x_0-\delta<x<x_0+\delta$ are equivalent to $-\delta<x-x_0<\delta$, and to $|x-x_0|<\delta$. Thus, $U(x_0,\delta)$ consists of all points whose distance from x_0 is less than δ. Therefore a neighborhood of radius δ of a point x_0 means the set of all points x satisfying $|x-x_0|<\delta$. In one-dimensional space (a line), a neighborhood is an open interval; in two-dimensional space, a neighborhood is the inside of a circle; in three-dimensional space, it is the inside of a sphere, as shown in Figure 1. 1. 10.

If the point x_0 is deleted from the neighborhood $(x_0-\delta,x_0+\delta)$, then the set $\{x\mid 0<|x-x_0|<\delta\}$ is called the deleted neighborhood of x_0, which is denoted by $\overset{\circ}{U}(x_0,\delta)$.

In general, any open interval with x_0 as a center is called the neighborhood of