Roger Temam

Infinite-Dimensional Dynamical Systems in Mechanics and Physics

Second Edition

力学和物理学中的无限维动力系统

第2版

Applied Mathematical Sciences
68

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Second Edition

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Preface to the Second Edition

Since publication of the first edition of this book in 1988, the study of dynamical systems of infinite dimension has been a very active area in pure and applied mathematics; new results include the study of the existence of attractors for a large number of systems in mathematical physics and mechanics; lower and upper estimates on the dimension of the attractors; approximation of attractors; inertial manifolds and their approximation. The study of multilevel numerical methods stemming from dynamical systems theory has also developed as a subject on its own. Finally, intermediate concepts between attractors and inertial manifolds have also been introduced, in particular the concept of inertial sets.

Whereas we attempted, in the first edition, to cover the subject in an exhaustive way, this goal rapidly appeared to be unthinkable for the second edition. Hence, beside a number of minor alterations and improvements, this second edition includes a limited number of new topics which have been selected in a rather arbitrary way. The additions include a number of sections and subsections concerning the existence of attractors for specific systems for which *the semigroup is not compact* (Sections III.3.2, IV.6, 7, 8, VI.4.2), and two chapters on inertial manifolds and the approximation of attractors and inertial manifolds. Sections III.3.2 and VI.4.2. concern the existence and dimension of attractors for boundary-driven flows, the dimension being reduced from an exponential function of the Reynolds number to a polynomial function. Sections IV.6, 7, 8 address the existence of an attractor in the absence of compactness. Sections IV.6 and 7 are related to reversible wave equations, namely, the weakly damped nonlinear Schrödinger equation and the Korteweg –de Vries equation. In Section IV.8 we show how to prove the existence of the attractor in an unbounded case (the physical domain is unbounded which produces a lack of compactness). The techniques used in these three sections depart in a significant way from the techniques used in the rest of the book.

The new Chapters IX and X are related to inertial manifolds and approximations. In Chapter IX we give, with a different proof, a new result of the existence of inertial manifolds generalizing that of Chapter VIII. This new result applies to the non-self-adjoint case and allows us to relate the concept of inertial manifolds to the concept of slow manifolds encountered in meteorology and oceanography. Finally, Chapter X addresses the approximation of the attractor by smooth finite-dimensional manifolds and at an exponential order (when inertial manifolds are not known to exist); it also produces convergent sequences of simple ("explicit") finite-dimensional manifolds approximating an exact inertial manifold when it exists.

This second edition has benefited from comments from a number of people; unable to mention all of them, I would like to mention, in particular, C. Foias, M. Jolly, O. Manley, S. Wang, X. Wang, and A. Debussche, O. Goubet, A. Miranville, I. Moise, and R. Rosa. The latter have also directly contributed to the writing of the new additions (A.D., Chapters IX and X; O.G., Section IV.6; A.M., Sections IV.3.2 and VI.4.2; I.M., Section IV.7; R.R., Sections IV.7 and 8), and I address my special thanks to them. Finally, I would like to thank Teresa Bunge and Danièle Le Meur who handled the typing kindly and efficiently.

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Preface to the First Edition

The study of nonlinear dynamics is a fascinating question which is at the very heart of the understanding of many important problems of the natural sciences. Two of the oldest and most notable classes of problems in nonlinear dynamics are the problems of celestial mechanics, especially the study of the motion of bodies in the solar system, and the problems of turbulence in fluids. Both phenomena have attracted the interest of scientists for a long time; they are easy to observe, and lead to the formation and development of complicated patterns that we would like to understand. The first class of problems are of finite dimensions, the latter problems have infinite dimensions, the dimensions here being the number of parameters which is necessary to describe the configuration of the system at a given instant of time. Besides these problems, whose observation is accessible to the layman as well as to the scientist, there is now a broad range of nonlinear turbulent phenomena (of either finite or infinite dimensions) which have emerged from recent developments in science and technology, such as chemical dynamics, plasma physics and lasers, nonlinear optics, combustion, mathematical economy, robotics, ...

In contrast to linear systems, the evolution of nonlinear systems obeys complicated laws that, in general, cannot be arrived at by pure intuition or by elementary calculations. Given a dynamical system starting from a particular initial state, it is not easy to predict if the system will evolve towards rest or towards a simple stationary state, or if it will go through a sequence of bifurcations leading to periodic states or to quasi-periodic states or even to fully chaotic states. The mathematical problem here is the study of the long-term behavior of the system ($t \to \infty$) corresponding to the practical problem of determining which "permanent" state will be observed after a short transient period in, say, a wind tunnel or an electrical circuit. In an attempt to predict the long-term behavior of dynamical systems we encounter several difficulties related to chaos, bifurcation, and sensitivity to initial data:

Chaotic (turbulent) behavior can appear, as well as simple well-ordered states.

We do not know a priori towards which state a given system may evolve, and we do not know when significant changes of the state may occur.

In many cases physical phenomena are stable and small variations in the initial circumstances produce only small variations in the final state. But quoting Maxwell's *Mechanics*, "there are other cases in which small initial variations may produce a very great change in the final state of the system, as when the displacement of the 'points' causes a railway train to run into another instead of keeping its proper course".

Because of its complexity and sensitivity to certain variations, the evolution of a nonlinear system cannot be predicted by mere computations, be it analytical or numerical. They do not offer a satisfactory solution, even if they produce a feasible one; nonlinear phenomena are global and there is a need for a more geometrical view of the phenomena which could provide the proper guidelines for the computations. The limits of the computational methods have been pointed out by Poincaré in his classic work on differential equations; he showed the need to marry analytic and geometric methods, and although he was concerned with asymptotic analytic methods, this also applies to numerical methods since the difficulty is inherent in the problem.

In recent years there has been considerable work on dynamical systems theory. Probably this is due to the favorable convergence of several factors:

The need for the understanding of new phenomena appearing in new areas of science and technology.

The increase in computing power, producing more insight into the behavior of dynamical systems and into the description of chaotic behavior.

New ideas and new mathematical tools such as the work of S. Smale on attractors; the mechanism proposed by D. Ruelle and F. Takens for the explanation of turbulence; the popularization of fractal sets due to B. Mandelbrot and others after him. In related areas we can mention the Kolmogorov–Arnold–Moser theory relating chaos and nonintegrability of Hamiltonian systems; the period-doubling mechanism for mappings of the interval $[0, 1]$; and the associated number discovered by M. Feigenbaum.

Following the ideas of S. Smale, D. Ruelle, and F. Takens, the chaotic behavior of a dissipative dynamical system can be explained by the existence of a complicated attractor to which the trajectories converge as $t \to \infty$; this set can be a fractal, like a Cantor set or the product of a Cantor set and an interval. This attractor is the natural mathematical object describing the observed nonstationary flow, and its complicated structure is the cause (or one of the causes) of the perceived chaos (or the apparent chaos). An understanding of these sets is of course necessary for a better understanding of the flow that they describe, and for the discovery of the laws and structures of the flow underlying the small-scale chaos. Already finite-dimensional systems (i.e., those whose state is described by a finite number of parameters) lead to

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complicated attractors as shown by the classic example of the Lorenz attractor in space dimension 3.

The study of turbulence in finite-dimensional systems suggests that the level of complexity of the phenomena increases with the level of complexity of the system. Thus we may wonder what is the level of complexity of infinite-dimensional dynamical systems such as those arising in continuum mechanics or continuum physics; this is one of the questions addressed in this book: we will see that, fortunately, the number of degrees of freedom of such systems is finite although it may be high. Thus in infinite dimensions the complexity of motion is due at the same time to the large (but finite) number of degrees of freedom and to the possible chaotic behavior of some of them.

The material treated in this book is limited to infinite-dimensional dissipative dynamical systems; very few finite-dimensional systems are presented, and then only to serve as reference examples or model problems; most of the systems considered are derived from evolutionary partial differential equations associated with boundary-value problems. Of course, in the study of such systems we are not only faced with the difficulties of nonlinear dynamics, but also with the difficulties related to evolutionary partial differential equations. Unlike the case of ordinary differential equations, no general theorem of existence and uniqueness of solutions exists for such problems and each partial differential equation necessitates a particular study. As a rule, a proper treatment of such problems necessitates the use of several different function spaces. It is one of our aims in this book to combine the problems and methods of both theories, dynamical systems and evolutionary partial differential equations, in an attempt to fill the gap between them and to make all the aspects of the theory accessible to the nonspecialist. We did not try to develop an abstract setting, but rather we deliberately chose to study specific equations and to remain close to their physical context.

For each equation, the following questions are addressed:

Existence and uniqueness of the solution and continuous dependence on the initial data. These preliminary results are not new, but they are part of the definition of the dynamical system and most of the necessary tools are also needed in other parts of the study. We have thus included these results for the sake of completeness.

Existence of absorbing sets. These are sets which all the orbits corresponding to the different initial data eventually enter. The existence of such sets is a step in the proof of the existence of an attractor. It is also an evidence (or a consequence) of the dissipative nature of the equation. In finite dimensions J.E. Billoti and J.P. La Salle [1] propose it as a definition of dissipativity; unfortunately, some difficulties specific to infinite dimensions make it, in that case, a less natural definition of dissipativity.

Existence of a compact attractor. It is shown that the equation possesses an attractor towards which all the orbits converge. Several adjectives are attributed to this attractor: we call it the *global* or *universal* attractor since it describes all the possible dynamics that a given system can produce; we also call it the *maximal* attractor since it is maximal (for the inclusion relation) among all bounded attractors. This set $\mathscr{A}$ attracts all bounded sets and it is an invariant set for the flow: by this we mean that any point of $\mathscr{A}$ belongs to a complete orbit lying in $\mathscr{A}$; this is particularly unusual in infinite dimensions where the backward initial-value problem is usually not well posed. The three names, global, universal, and maximal attractor, have the same meaning; we did not want to favor one of them and, at this point, leave the choice to the reader.

Finite dimensionality and estimate of the dimension of the attractor. Another aspect of dissipativity is that the attractor has a finite dimension, so that the observed permanent regime depends on a finite number of degrees of freedom. This was first proved for a delay evolution equation by J. Mallet-Paret [1] and in the case of the Navier–Stokes equations by C. Foias and R. Temam [1]. Here, besides showing the finite dimensionality of the attractor, we actually estimate an upper bound of its dimension in terms of the physical data and, in some specific cases, there are indications that these bounds are physically relevant. For instance, in the three-dimensional turbulent fluid flows we recover exactly the estimate of the number of degrees of freedom predicted by the Kolmogorov theory of turbulence (see P. Constantin, C. Foias, O. Manley, and R. Temam [1]).

* * *

Let us now describe the content of this book in its chronological order. The first chapter is a general introduction which further develops the present Preface, the motivations for this work, and a description of the main results. It also contains a "User's Guide" which is intended for more physics-oriented readers who are interested in the questions discussed in this book, but not in all of the mathematical aspects. In Chapter I we introduce the general results and concepts on invariant sets and attractors. This chapter contains all the basic definitions and properties and, in particular, a general criterion of existence of a global attractor which we use repeatedly in the sequel. This chapter contains, as an illustration, simple examples drawn from ordinary differential equations, in particular, the well-known Lorenz model. Another example treated there is that of fractal interpolation producing an interesting application in infinite dimensions which does not require a complicated functional framework. Chapter II is a technical chapter containing some elements of functional analysis: function spaces, linear operators, linear evolution equations of first and second order in time. This chapter is conceived as a reference chapter; it is not suggested that it be read in its entirety before reading the subsequent chapters, but rather that it be read "locally" as needed.

Chapters III and IV contain a systematic study of several infinite-dimensional dynamical systems arising in chemistry, mechanics, and physics. In Chapter III we consider the reaction–diffusion equations; pattern formation equations; fluid mechanics equations which include the Navier–Stokes equations in space dimension 2; and some other equations: magneto-hydrodynamics, thermohydraulics, fluid driven by its boundary, and geophysical flows (flows on a manifold). Chapter IV deals with damped nonlinear wave equations. For each equation we provide the appropriate functional setting and the results of existence and uniqueness of solutions; we prove the existence of absorbing sets for various norms, as needed, and we show the existence of a maximal attractor. This is the mathematical object describing all the possible dynamics (behaviors) of the system. Chapters III and IV also contain some technical results (like the injectivity of the semigroup which is equivalent to backward uniqueness), or some aspects that we mention without developing them thoroughly: e.g., regularity results and stability of attractors with respect to perturbations.

In Chapter V we introduce some new tools and results which we then apply in Chapter VI to all the equations considered earlier. The central theme of Chapter V is that of Lyapunov exponents and Lyapunov numbers. The chapter starts with a technical section on linear and multilinear algebras which recapitulates some known results and gives some extensions. We then introduce the Lyapunov exponents and show their relation to the distortion of volumes generated by the semigroup on the attractor. The chapter also contains general results concerning the Hausdorff and fractal dimensions of attractors which were proved in P. Constantin, C. Foias, and R. Temam [1]. These results, when properly applied, show the finite dimensionality of the attractor, and they allow a sharp explicit estimate of the dimension in terms of some quantities directly related to the physical problems. For attractors which are expected to be complicated (fractal) sets, the dimension is one of the few mathematical pieces of information on the geometry of such sets. On the physical and numerical sides, this dimension gives one an idea of the number of degrees of freedom of the system, and therefore of the number of parameters and the size of the computations needed in numerical simulations.

As indicated above, Chapter VI contains a systematic application of these results to the equations of Chapters III and IV and to the Lorenz equation which serves as a simple model. We derive various estimates for the Lyapunov exponents, the evolution of the volume element in the phase space, and, most important for the dimension of the attractor, the bound on the dimension being expressed as explicitly as possible in terms of physical entities.

Chapter VII contains some miscellaneous topics: the extension of the results to non-well-posed problems, and some extensions regarding unstable manifolds leading to a detailed description of the global attractor of a semigroup possessing a Lyapunov function; and on the other hand, to a briefly sketched method for deriving lower bounds on dimensions of attractors.

The last chapter, Chapter VIII, is devoted to inertial manifolds, new mathematical objects which have been recently introduced in relation with the study of the long-time behavior of dynamical systems. These are finite-dimensional Lipschitz manifolds, which attract exponentially all the orbits; they are positively invariant for the flow and contain of course the global attractor. This question is the object of much current investigation, and we thought it was desirable to include here some typical recently proved results.

The book ends with an Appendix providing some collective Sobolev inequalities which are used in Chapter VI for refined estimates of the traces of certain linear operators (the linearized operator corresponding to the first variation equation).

Most of the topics developed here are new or have appeared recently. Relevant questions which are not developed here include those of partly dissipative systems, and of nonautonomous systems. These aspects are currently being investigated and will appear in articles elsewhere (see M. Marion [2], J.M. Ghidaglia and R. Temam [6]). Also the theory of attractors for stochastic differential equations (see A. Bensoussan and R. Temam [1], [2], P. Malliavin [1]) has not yet been approached.

In conclusion, I would like to thank all those who helped in the realization of this book through encouragement, advice, or scientific exchanges: Peter Constantin, Jean-Michel Ghidaglia, Jack Hale, Martine Marion, Basil Nicolaenko, David Ruelle, Jean-Claude Saut, Bruno Scheurer, George Sell, and I.M. Vishik. More particularly, I would like to thank Ciprian Foias for a continued and friendly collaboration on which part of this book is based, and Oscar Manley who undertook the considerable task of improving the English language.

Madame Le Meur typed the manuscript struggling through hundreds of pages while being introduced to the word processor. I would like to thank her for her kind and patient cooperation.

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