



Intermediate Mechanics of Materials

中级材料力学

J. R. Barber

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清华大学出版社

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Intermediate Mechanics of Materials

影 印 版 序

材料力学是高等理工院校的力学基础课程，国内外已经编写了不少著名的经典教材。随着近代工业和现代高科技的发展，工程师们越来越觉得需要补充比初等材料力学更为深入和广泛的固体力学基础知识，例如弹性力学、塑性力学、有限元法、板壳理论等。另一方面，高等院校中课程教学内容的改革也产生了同样的趋势。于是，国外近年来出版了一些新编的“高等材料力学”教材，或称为“材料力学二级教程”。由于固体力学的范围甚广，新编教材各有自己的特色和侧重点，但基本内容都是一致的。从讲授内容来看，本书属于这类教材之一。

本书的特点是更重视力学概念的物理直观阐述和在工程设计中的灵活应用，通过日常生活和工程应用实例以及试验观测和数值计算图表帮助读者从感性上深入理解基本概念和基本原理。讲述方法更接近于初等材料力学的习惯，数学上力求简明易懂，不追求理论的系统性和完整性。正是基于这些特点，作者把本书称为“中级材料力学”。

作者是美国密歇根大学机械工程与应用力学系的教授，曾写过弹性力学教材，担任过大量工程设计中应力分析问题的咨询工作。本书积累了作者的教学和工程咨询经验，对一些通常认为枯燥而困难的问题给出了新颖的处理方法。

本书共分 12 章。第 1 章“引论”，介绍工程强度设计和材料力学基本概念。第 2 章“材料特性与破坏”，讲述材料的力学性质及其在恒定与循环载荷作用下的破坏形式与设计准则。第 3 章“能量法”，包括最小势能原理、瑞利—里兹法和卡氏第一、第二定理。第 4 章“非对称弯曲”和第 5 章“非线性和弹塑性弯曲”都研究各种截面形状的梁。第 6 章“薄壁梁的剪切与扭转”，包括开口、闭口和多闭室薄壁杆件。第 7 章“弹性基础上的梁”在土木、水利工程中具有重要的应用背景。第 8 章“轴对称壳的薄膜应力”、第 9 章“圆柱壳的轴对称弯曲”、第 10 章“厚壁筒和圆盘”和第 11 章“曲梁”都讲述的是在机械、石油化工、航空航天、核能、土木、水利等工程领域中有广泛应用的典型部件。第 12 章“弹性稳定性”，讨论在受压缩的细长杆和薄壁壳以及高速回转机械中出现的丧失稳定性的破坏现象。最后在附录 A 中概述了当代工程计算中越来越有用的有限单元法。

在每章后面附有许多与工程紧密结合的习题，包括基本训练题和具有挑战性的高难度习题（用 * 号标明）。在附录 D 中给出了双号习题的答案。

本书是一本面向工程设计的力学教材，对从事应力分析和强度设计的科技工作者、讲授高等材料力学和应力分析课程的教师以及理工院校的高年级学生和研究生是一本很好的参考书。

陆明万

清华大学工程力学系

ABOUT THE AUTHOR

J. R. BARBER graduated in Mechanical Sciences from Cambridge University in 1963 and joined British Rail, who later sponsored his research at Cambridge between 1965 and 1968 on the subject of thermal effects in braking systems. In 1969 he became Lecturer and later Reader in Solid Mechanics at the University of Newcastle upon Tyne. In 1981 he moved to the University of Michigan, where he is presently Professor of Mechanical Engineering and Applied Mechanics. His current research interests are in solid mechanics with particular reference to thermoelasticity, contact mechanics and tribology. He is a chartered engineer in the U.K., Fellow of the Institution of Mechanical Engineers and has engaged extensively in consulting work in the field of stress analysis for engineering design. He is author of a book on elasticity and over 150 articles in the fields of elasticity, thermoelasticity, contact mechanics, tribology, heat conduction and elastodynamics.

PREFACE

Most engineering students first encounter the subject of mechanics of materials in a course covering the concepts of stress and strain and the elementary theories of axial loading, torsion, bending and shear. There is broad agreement as to the content of such courses, there are many excellent textbooks and it is easy to motivate the students by using simple examples with obvious engineering relevance.

The second course in the subject presents considerably more challenge to the instructor. There is a very wide range of possible topics and different selections will be made (for example) by civil engineers and mechanical engineers. The concepts tend to be more subtle and the examples more complex making it harder to motivate the students, to whom the subject may appear merely as an intellectual exercise. Existing second level texts are frequently pitched at too high an intellectual level for students, many of whom will still have a rather imperfect grasp of the fundamental concepts.

Most undergraduate students are looking ahead to a career in industry, where they will use the methods of mechanics of materials in design. Many will get a foretaste of this process in a capstone design project and this provides an excellent vehicle for motivating the subject. In mechanical or aerospace engineering, the second course in mechanics of materials will often be an elective, taken predominantly by students with a design concentration. It is therefore essential to place emphasis on the way the material is used in design.

Mechanical design typically involves an initial conceptual stage during which many options are considered. During this phase, quick approximate analytical methods are crucial in determining which of the initial proposals are feasible. The ideal would be to get within ± 30 percent with a few lines of calculation. The designer also needs to develop experience as to the kinds of features in the geometry or the loading that are most likely to lead to critical conditions. With this in mind, I try wherever possible to give a physical and even an intuitive interpretation to the problems under investigation. For example, students are encouraged to estimate the location of flexible and stiff bending axes and the resulting neutral axis of bending by eye, and methods are discussed for getting good accuracy with a simple one degree of freedom Rayleigh-Ritz approximation. Students are also encouraged to develop a feeling for the mode of deformation of engineering components by performing simple experiments in their outside environment, for example, estimating the radius to which an initially straight bar can be bent without producing permanent deformation, or convincing themselves of the dramatic difference between torsional and bending stiffness for a thin-walled open beam section by trying to bend and then twist a structural steel beam by hand-applied loads at one end.

In choosing dimensions for mechanical components, designers will expect to be guided by criteria of minimum weight, which with elementary calculations, often

lead to a thin-walled structure as the optimal solution. This demands that students be introduced to the limits imposed by elastic instability. Some emphasis is also placed on the effect of manufacturing errors on such highly designed structures—for example, the effect of load misalignment on a beam with a large ratio between principal stiffnesses and the large magnification of initial alignment or loading errors in a column below, but not too far below, the buckling load.

No modern text of mechanics of materials would be complete without a discussion of the finite element method. However, students and even some instructors are often confused as to the respective roles played by analytical and numerical methods in engineering practice. Numerical methods provide accurate solutions for complex practical problems, but the results are specific to the geometry and loading modelled and the solution involves a significant amount of programming effort. By contrast, analytical methods may be very idealized and hence approximate, but they are often quick to apply and they provide generality, permitting a whole family of designs to be compared or even optimized.

The traditional approach to mechanics is to define the basic concepts, derive a general theory and then illustrate its application in a variety of examples. As a student and later as a practicing engineer, I have never felt comfortable with this approach, because it is impossible to understand the nuances of the definitions or the general treatment until after they are seen in examples which are simple enough for the mathematics and physics to be transparent. Over the years, I have therefore developed rather untraditional ways of proving and explaining things, relying heavily on simple examples during the derivation process and using only the bare minimum of specialist terminology. I try to avoid presenting to the student anything which he or she cannot reasonably be expected to understand fully *now*.

The problems provided at the end of each chapter range from routine applications of standard methods to more challenging problems. Particularly lengthy or difficult problems are identified by an asterisk. The Solutions Manual to accompany this book is prepared to the same level of detail as the example problems in the text and in many cases introduces additional discussion. Answers to even-numbered problems are provided in Appendix D.

This book evolved out of a set of notes which I wrote for a second level course at the University of Michigan and the resulting interaction with my students and colleagues has played a crucial role in the development of my thinking about the subject. Special thanks go to Przemislaw Zagrodzki of Warsaw University of Technology and Raytech Composites Inc. for his invaluable help with the Appendix on finite element methods. I am very grateful to the staff at McGraw-Hill, whose insight into the needs of the readership have proved invaluable in suggesting changes in both style and content. I also wish to thank those reviewers who provided detailed comments on earlier versions of this manuscript, including Gordon Pennock, Purdue University; Lester Schmerr, Iowa State University; Larry Lessard, McGill University; Paul Heyliger, Colorado State University; Victor Giurgiutiu, University of South Carolina;

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chapter

1

Introduction

Mechanics of materials is principally concerned with the determination of the stresses and deformation of engineering devices and structures. Stresses are important because, in combination with fundamental tests on engineering materials, they enable us to determine the loading conditions under which material failure will occur. Deformations may also be important insofar as they affect the kinematic function of a device.

As engineers, the principal use we make of such calculations is to provide guidance in the design process. Typically, we may perform a mechanics of materials calculation to determine the minimum dimension for a component if it is not to fail under the expected loading or, more generally, to choose between several competing designs on the basis of strength or weight. To ask the right questions in mechanics of materials, it is therefore important to have some understanding of the process of *engineering design*.

1.1 THE ENGINEERING DESIGN PROCESS

The engineering designer typically starts with a well-defined specification that the proposed device is required to meet but no very clear idea of the form it is to take. The first step is therefore to generate a large number of potential solutions to the problem in an initial conceptual phase. This is a creative process and it is advantageous to give the imagination free rein by refraining deliberately from eliminating possible solutions based on any criteria other than the fundamental function of the device.

Once a collection of potential concepts has been established, the choice can be narrowed down by evaluative arguments involving estimates of cost, weight, maximum stress, manufacturability, etc. For a complex device, these arguments will initially only concern major characteristics of the design, details being left for

determination at a later stage. Towards the end of the process, the choices will have been narrowed down to one or two only and calculations will be performed to determine appropriate dimensions for the components.

1.1.1 Economics of Design Calculations

An important point to recognize is that, even when a design is more or less finalized, it is seldom possible to perform detailed stress calculations for all the components because of time and cost constraints. For example, a finite element calculation of the stresses in a fairly simple three-dimensional component might involve two-days work for an experienced analyst and cost \$2000, including salary, computing services and overhead (indirect) costs. At this rate, the stress analysis alone for a fairly modest device might cost around \$50,000. In addition, we shall probably need to perform other design calculations associated with dynamics, hydraulic or electrical control, lubrication, selection of bearings, thermal effects, materials selection, etc. If the device is to market for \$100, these costs would be prohibitive unless we expect to sell at least 100,000 units. Even when the costs are acceptable, the time required to perform the analysis may place an unreasonable delay between the initial concept stage and the device coming into production.

Clearly, the size of the production run is a critical parameter in deciding how much time and money can be invested in design calculations. For this reason, some of the simplest and most common artifacts are the subject of intensive and sophisticated design and manufacturing studies. An example is the formed aluminum beverage can, for which modest cost reductions (typically by a reduction in the amount of aluminum required) have led to significant savings because of the millions of items purchased each year.

1.2 DESIGN OPTIMIZATION

Design can be seen as a complex optimization process. In the broadest terms, we wish to find the design which meets a specified function at minimum cost.¹ The functional requirements will often be defined as *constraints* which take the form of inequalities.² For example, we want a design that will not fail in service, so the stresses must be below some failure threshold, but we don't mind it being too strong, provided the cost is not thereby increased. However, reducing the size and hence the weight of a component will typically reduce the cost and increase the maximum stress, so the optimum design will often be one that is limited by a maximum stress criterion.

It is tempting to generalize this argument and to seek a design in which every point in the body is at its maximum permissible stress under the most severe service

¹Notice that, as remarked above, the costs of developing the design must themselves be figured into this calculation, which threatens to make the argument circular!

²Constraints involving inequalities are sometimes referred to as *unilateral constraints*.

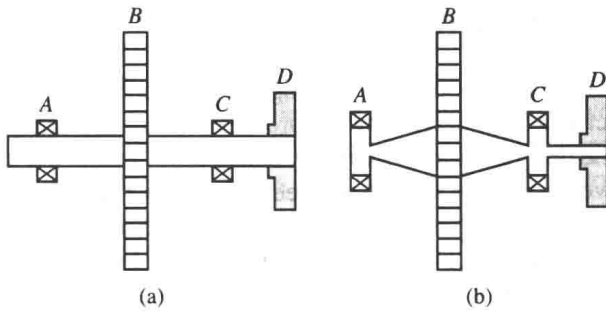


Figure 1.1 (a) Geometry of gearbox shaft; (b) “optimal” design based on elementary bending and torsion calculations

conditions.³ However, the diversity of function in mechanical engineering components generally makes it impossible to do this. Consider, for example, the simple gearbox shaft shown in Figure 1.1(a). The shaft is supported in bearings at *A*, *C* and it carries a gear at *B* and a coupling to transmit torque to another component at *D*. The gear force will be equally shared between the two bearings due to symmetry, and the bending moment will be a maximum at *B*, falling to zero at *A*, *C*. The transmitted torque is constant between *B* and *D*, but zero in *AB*. Because the shaft transmits only a shear force at *A*, a strictly optimal design would require only a very small local diameter there, but bearing design considerations will probably require a larger diameter. It is superficially tempting to save material by choosing a design like that of Figure 1.1(b), but the changes in section will themselves introduce locally high stresses due to stress concentration effects and the forming or machining costs will probably more than outweigh any saving in material costs due to the reduced weight. Also, the reduced stiffness of the design of Figure 1.1(b) will lower the natural frequency of vibration of the shaft and if this falls in the operating speed range, there is a danger of excessive noise and accelerated fatigue failure.

More generally, the maximum stresses reached are only one of many criteria to be satisfied by a successful design and this fact plus considerations of simplicity in manufacture and assembly will usually result in a design for which the maximum acceptable stress level is reached only at certain critical locations. If we can identify these locations, we can then also save time and money in stress analysis by concentrating attention on the critical points. Details of the geometry and loading distant from these points can be safely approximated by idealized shapes and distributions. Thus, the two crucial questions to ask at the beginning of a mechanics of materials problem are:

- (i) How and where is the component or device most likely to fail?
- (ii) Can we estimate the stresses and/or displacements in critical regions by a simple calculation (in particular, without calculating the stresses and displacements everywhere in the component)?

³ Some structural optimization methods use an algorithm not too different from this.