



高等教育“十三五”工科类全英教材

Fluid Mechanics in Civil Engineering

工程流体力学（土木类）

尹小玲 (YIN XIAOLING) 唐小南 (TANG XIAONAN) 主编



华南理工大学出版社
SOUTH CHINA UNIVERSITY OF TECHNOLOGY PRESS



高等教育“十三五”工科类全英教材

Fluid Mechanics in Civil Engineering

工程流体力学（土木类）

主编 尹小玲 (YIN XIAOLING) 唐小南 (TANG XIAONAN)

主审 【英】DONALD W KNIGHT



华南理工大学出版社
SOUTH CHINA UNIVERSITY OF TECHNOLOGY PRESS

· 广州 ·

图书在版编目(CIP)数据

工程流体力学:土木类 = Fluid Mechanics in Civil Engineering:英文/尹小玲,唐小南主编. —广州:华南理工大学出版社,2017.12

高等教育“十三五”工科类全英教材

ISBN 978-7-5623-5474-1

I. ①工… II. ①尹… ②唐… III. ①工程力学-流体力学-高等学校-教材-英文 ②土木工程-流体力学-高等学校-教材-英文 IV. ①TB126 ②TU

中国版本图书馆 CIP 数据核字(2017)第 285685 号

Fluid Mechanics in Civil Engineering

工程流体力学(土木类)

尹小玲 唐小南 主编

出版人:卢家明

出版发行:华南理工大学出版社

(广州五山华南理工大学17号楼,邮编510640)

<http://www.scutpress.com.cn> E-mail:scutc13@scut.edu.cn

营销部电话:020-87113487 87111048(传真)

策划编辑:赖淑华

责任编辑:骆婷 庄彦

印刷者:虎彩印艺股份有限公司

开本:787mm×1092mm 1/16 印张:20.5 字数:682千

版次:2017年12月第1版 2017年12月第1次印刷

定价:74.00元

版权所有 盗版必究 印装差错 负责调换

Preface

This book is written for civil engineering students who are interested in learning some basic principle and application of fluid mechanics. We make effort to present the basic principles thoroughly to help understand physical circumstances, emphasizing physical concepts rather than mathematics. For the sake of the majority of students who work as civil engineers in future, emphasis of this book is more on engineering application of fundamental fluid mechanics. However, students should have background knowledge in statics and calculus.

This book is organized in two parts: basic theory and engineering application. The part of basic theory includes introduction, hydrostatics, hydrodynamics, flow friction and head loss. The part of engineering application includes orifice flow, nozzle flow, hydraulic calculation of long and short pipelines, pipe networks, steady open – channel flow, and seepage flow. The contents are feasibly arranged and suit for flexibly teaching hours.

This book is also organized to support the development of skills for problem solving. A brief summary of every chapter is given at the end of each chapter, followed by exercises, which include multiple-choice questions and problems. The answers to the selected problems are given at the end of the book.

We are most grateful to all who gave us support in the preparation of this book. We particularly thank Professor Donald W Knight (University of Birmingham, UK) to read the whole manuscript and give valuable comments. Also we would like to express our gratitude to Ms Bu Yu (former professor at South China University of Technology, China) who provided much of the book's framework and contents that was based heavily upon her earlier book titled Hydraulics.

Xiaonan Tang

(Xi'an Jiaotong-Liverpool University)

Xiaoling Yin

(South China University of Technology)

July 2017

Contents

Chapter 1 Introduction	1
1.1 What is about hydraulics?	1
1.2 Fluids and their properties	2
1.2.1 The basic characteristic of fluid	2
1.2.2 International system of units (SI) and engineering units	2
1.2.3 The main physical properties of liquids	4
1.3 Forces acting on the fluid	14
1.3.1 Surface force	14
1.3.2 Body force	15
Multiple-choice questions (one option)	15
Problems	16
Chapter 2 Hydrostatics	18
2.1 Concept of hydrostatic pressure	18
2.1.1 Definition of hydrostatic pressure	18
2.1.2 Features of hydrostatic pressure	19
2.2 Hydrostatic differential equation and isobaric surface	21
2.2.1 Differential equation of fluid in equilibrium	21
2.2.2 Isobaric surface	23
2.3 Distribution of hydrostatic pressure under gravity	24
2.3.1 Basic formula of hydrostatic pressure under gravity	24
2.3.2 Absolute, relative and vacuum pressures	26
2.3.3 Energy significance and geometric meaning of the basic hydrostatic pressure equation	30
2.4 The application of hydrostatics in measurement	32
2.4.1 Piezometer	32
2.4.2 Differential gauge	33
2.5 Total hydrostatic force acting on a plane surface	35
2.5.1 Graphic method	36
2.5.2 Analytical method	40
2.6 Total hydrostatic forces acting on curved surfaces	46
2.6.1 Magnitude of total hydrostatic force on a curved surface	47
2.6.2 Direction of total hydrostatic force	50

2. 6. 3	Acting point of total hydrostatic force	50
2. 7	Total hydrostatic force on a body, buoyancy, stability of a floating body	54
2. 7. 1	Total hydrostatic force acting on a body —Archimedes principle	54
2. 7. 2	Equilibrium of a sinking body, submerged body and floating body	55
Chapter summary	56
Multiple-choice questions (one option)	57
Problems	58
Chapter 3	Basic equations of steady total flow	67
3. 1	Two methods for describing motion of fluid	67
3. 1. 1	Lagrangian method and Eulerian method	67
3. 1. 2	Acceleration of particle; local, convective and total acceleration	70
3. 1. 3	Some basic concepts of fluid movement	73
3. 2	Continuity equation of steady total flow	79
3. 3	Energy equation of steady total flow	82
3. 3. 1	Energy equation of steady streamtube flow of ideal fluid	82
3. 3. 2	Energy equation of steady streamtube flow of real fluid	85
3. 3. 3	Energy equation of steady total flow of real fluid	86
3. 4	Momentum equation of steady total flow	104
3. 4. 1	Derivation of the momentum equation	104
3. 4. 2	Conditions and tips in the application of the momentum equation	109
3. 4. 3	Application examples of the momentum equation	111
3. 4. 4	Similarities and differences between the momentum equation and energy equation	116
Chapter summary	116
Multiple-choice questions (one option)	117
Problems	119
Chapter 4	Types of flow and head loss	128
4. 1	The classification of flow resistance and head loss	128
4. 1. 1	The classification of flow resistance	128
4. 1. 2	The classification of head losses	131
4. 1. 3	The superposition principle of head losses	131
4. 2	Two regimes of real fluid flow	132
4. 2. 1	Reynolds' experiment	132
4. 2. 2	The identification of laminar and turbulent flows	135
4. 2. 3	The physical meaning of Reynolds number	136
4. 3	The relationship between frictional head loss and shear stress of uniform flow	138
4. 3. 1	The relationship between frictional head loss and wall shear stress	138

4.3.2	The relationship between frictional head loss and shear stress	139
4.3.3	The general calculation formula for frictional head loss	141
4.4	Laminar flow in circular pipes	143
4.4.1	The velocity distribution of laminar flow	143
4.4.2	The mean flow velocity of laminar flow	144
4.4.3	The flow rate of laminar flow	144
4.4.4	The frictional head loss of laminar flow	144
4.4.5	The kinetic correction coefficient of laminar flow	145
4.5	The basic concepts of turbulent flow	146
4.5.1	Developing process of turbulent flow	146
4.5.2	Fluctuation and time averaged motion of turbulent flow	148
4.5.3	The shear stress and Prandtl's theory of turbulent flow	152
4.5.4	The viscous sublayer and flow zone of turbulent flow	156
4.5.5	The velocity distribution of turbulent flow	159
4.6	Frictional head losses of turbulent flow	164
4.6.1	Experiment of frictional resistance coefficient	164
4.6.2	Frictional resistance coefficient of commercial pipes	169
4.6.3	Empirical formulae for frictional head loss	173
4.7	Local head loss	177
4.7.1	Local head loss of sudden expansion of pipe	177
4.7.2	Local head loss coefficient	179
4.8	Basic concepts of boundary layer and flow resistance around an object	185
4.8.1	Basic concept of boundary layer	185
4.8.2	Separation of boundary layer and flow resistance	187
	Chapter summary	191
	Multiple-choice questions (one option)	192
	Problems	194
Chapter 5	Steady orifice, nozzle and pipe flow	197
5.1	Introduction	197
5.2	Basic formulae for steady flow through orifice and nozzle	198
5.2.1	Steady flow through thin-wall orifice	198
5.2.2	Steady flow through nozzle	201
5.3	Steady flow in pressurized pipes	205
5.3.1	Hydraulic calculation of hydraulically short pipes	205
5.3.2	Hydraulic calculation of hydraulically long pipes	212
5.3.3	Hydraulic calculation for pipeline networks	221
	Chapter summary	226
	Review questions	227

Multiple-choice questions (one option)	227
Problems	229
Chapter 6 Steady flow in an open channel	233
6.1 Geometry of open channel	233
6.1.1 Longitudinal bed slope of open channel	233
6.1.2 Cross-section of open channel	234
6.1.3 Geometrical parameters of flow cross-section	234
6.1.4 Prismatic and non-prismatic channel	235
6.2 Uniform flow in open channel	236
6.2.1 Characteristics and conditions of uniform open-channel flow	236
6.2.2 Basic equations for uniform open-channel flow	238
6.2.3 Hydraulic calculation of uniform open-channel flow	240
6.2.4 The optimum hydraulic cross-section	243
6.3 Steady non-uniform open-channel flow	245
6.3.1 Flow regime of open-channel flow	245
6.3.2 Specific energy	248
6.3.3 Critical depth	249
6.3.4 Critical bed slope	250
6.3.5 Hydraulic jump and hydraulic drop	253
6.3.6 Surface profile of gradually varied flow in prismatic open channel	259
6.3.7 Computation of surface profiles in steady gradually varied flow	268
6.4 Weir flow and underflow of sluice gates	271
6.4.1 Types and basic formula of weir flow	272
6.4.2 Fundamental formula of underflow of a sluice gate	278
Chapter summary	280
Multiple-choice questions (one option)	282
Problems	283
Chapter 7 Seepage flow	287
7.1 The phenomenon of seepage and the seepage model	287
7.1.1 Seepage phenomenon	287
7.1.2 State of water in soil	288
7.1.3 The characteristics of soil seepage	289
7.1.4 Seepage models	289
7.2 The basic law of seepage flow	290
7.2.1 Darcy's Law	290
7.2.2 The limitations of Darcy's law	291
7.2.3 The coefficient of permeability	292

7.3 Dupuit's formula of steady gradually varied seepage flow	293
7.3.1 The velocity distribution in steady uniform and non-uniform seepage flows ...	293
7.3.2 The basic differential equation and the seepage curve of steady gradually varied seepage flow	295
7.4 Seepage calculation of wells and catchment corridors	300
7.4.1 Catchment corridors	300
7.4.2 Fully penetrating open wells	300
7.4.3 Fully penetrating artesian wells	302
7.4.4 The drainage of large-diameter well and foundation ditch	303
7.4.5 Well group	304
7.5 Graphical solution by drawing flow net	306
7.5.1 Drawing of flow net for the planar confined seepage	307
7.5.2 Seepage calculation by flow net	308
Chapter summary	310
Review questions	311
Multiple-choice questions (one option)	311
Problems	312
Answers to selected problems	314
References	317

Chapter 1 Introduction

1.1 What is about hydraulics?

Hydraulics is a branch of fluid mechanics that studies the engineering aspects of the behaviour of fluids, mainly water. It deals with the mechanical properties and laws of motion of fluids and their engineering applications. There are two major aspects of hydraulics, which differ from solid mechanics; one is the nature and properties of the liquid itself, which are very different from those of a solid; the other is that we are frequently concerned with the behaviour of a continuous stream of liquid rather than individual bodies or elements of known mass.

The basic principle of hydraulics includes two parts: hydrostatics and hydrodynamics. Hydrostatics is the study of fluids at rest, i. e. the mechanical characteristics of a stationary fluid. Hydrodynamics is the study of fluids in motion, i. e. the relationship between the force and motion of a moving fluid. Hydrostatics is the fundamental basis, whereas hydrodynamics describes the general principles of fluids in motion. However, it is extremely difficult to specify either the precise movement of a stream of fluid or that of individual particles within it. Therefore, it is often necessary to assume ideal, simplified conditions and patterns of the flow of fluid. The results so obtained for a basic analysis of the flow system may then be modified by introducing appropriate coefficients or factors determined experimentally in order to be applied in engineering practice. This approach has proved to be reasonably satisfactory so far.

Hydraulics is widely used in many engineering disciplines, for example, hydraulic engineering, civil engineering, water supply and distribution, port and shipping engineering, mechanical engineering, petroleum and chemical engineering, mining and metallurgical engineering, energy engineering, and environmental engineering. Therefore, hydraulics is an important fundamental subject required for many engineering programmes of universities, especially for civil engineering.

The pre-requisite knowledge for hydraulics is higher mathematics, physics, theoretical mechanics and materials mechanics.

1.2 Fluids and their properties

1.2.1 The basic characteristic of fluid

A substance has three states: solid, liquid and gas.

The molecules of a solid are usually close each other, and the attractive forces between the molecules are very large, so that a solid can retain its shape and volume. A solid can sustain certain degree of stretching, pressuring and shear forces. In contrast, the molecules of a liquid are less close so that their attractive forces are relatively smaller. A liquid can hardly bear stretching and resist tensile deformations, so the liquid will easily be deformed or flow if any small shear force acts on it. This property of flowing easily is defined as fluidity. Under certain pressure and temperature, a liquid cannot maintain a constant shape, but it will retain a definite volume for a given mass. If a liquid is filled into a container, a clearly defined free surface of liquid will be established.

Due to the molecules of a gas being much farther apart, the intermolecular force of a gas is much smaller than that of a liquid, so a gas has the similar characteristics of deformation as a liquid, i. e. easy change of its shape. In this respect, a liquid or a gas is called a fluid.

A study shows that 1 cm^3 of water contains 3.3×10^{23} molecules, which have the average distance apart of about $3 \times 10^{-8} \text{ cm}$, and consequently liquid molecules each undertake complex microscopic motion.

Hydraulics is not concerned with the study of the micro-motion of liquid molecules, but their macroscopic mechanical movements as a whole. Therefore, the continuum concept of liquid is used in hydraulics, which assumes that no gap exists between the liquid particles that therefore they continuously fill the space occupied. Thus, the physical and motion properties of liquid particles are continuously distributed. It should be noted that the liquid particle refers to an infinitely small point inside the liquid with a corresponding mass. With the continuum concept of liquid, we can use mathematically continuous functions to study the liquid, and also meet the overall requirements of most practical engineering problems, because the distance between molecules is so small that it can be ignored when compared with the scale of flow in engineering problems.

Based on the continuum concept, liquids are generally considered to be uniform isotropic, i. e. the liquid is homogeneous, so the physical properties of each particle are the same in all directions. In a nutshell, the liquid in hydraulics is a free-flowing, incompressible, homogeneous isotropic continuum.

1.2.2 International system of units (SI) and engineering units

The international system of units (SI) was formally adopted for quantities and units in China, 1977. However, engineering units were used in some earlier published technical books

and literature, and are still used by some senior technical professionals. In order to facilitate learning, it is necessary to clarify the differences and conversion relations between the two systems of units.

1.2.2.1 Dimensions and units

Dimension is a measure of a physical quantity, usually expressed in capital letters within square brackets, such as the length dimension in $[L]$, the time dimension in $[T]$, the mass dimension in $[M]$, the force dimension in $[F]$, and so on. The units of the same nature have the same dimension, such as year, month, day, hour, minute and second in $[T]$. However, the units of different natures have different dimensions, such as seconds, meters, Newton denoted by $[T]$, $[L]$, $[F]$, respectively.

Unit is a specific measure of physical quantity values. Different physical quantities can have different units, and the same physical quantities may also have different units. For example, units of length can be used by centimetres, meters, kilometres, etc., whereas gravity units can be expressed in Newton, kN, tons, etc.

Dimensions can be divided into primary dimensions and derived dimensions.

Primary dimensions are independent, which cannot be deduced from other fundamental dimensions. This is to say that primary dimensions are not dependent on other fundamental dimensions. For example, length dimension $[L]$, time dimension $[T]$, and mass dimension $[M]$ are independent of each other, so $[M]$ can neither be derived from $[L]$ and $[T]$, nor $[L]$ from $[T]$ and $[M]$, or $[T]$ from $[L]$ and $[M]$.

Derived dimensions are the dimensions that can be derived from the primary dimensions. In mechanics, dimension of any physical quantity can be expressed as the index product of three primary dimensions. Any physical quantity x , may therefore have its dimension expressed as:

$$[x] = [L^\alpha T^\beta M^\gamma] \quad (1-1)$$

where the x dimension depends on the indexes α , β , γ . When $\alpha \neq 0$, $\beta = \gamma = 0$, x is the quantity of geometry; when $\alpha \neq 0$, $\beta \neq 0$, $\gamma = 0$, it is the quantity of kinematics; when $\alpha \neq 0$, $\beta \neq 0$, $\gamma \neq 0$, it is the quantity of dynamics. When $\alpha = \beta = \gamma = 0$, that is $[x] = [L^0 T^0 M^0] = [1]$, x is a dimensionless quantity, called pure number.

1.2.2.2 The international system of units (SI) and system of engineering units

The difference between the two systems is in the choice of different fundamental dimensions, which leads to the different derived dimensions. Take Newton's second law $F = ma$ for example, which is often used in fluid mechanics.

International system of units (SI):

If the primary dimensions are $[L]$, $[T]$, $[M]$, the derived dimension of a force is

$$[F] = \left[\frac{ML}{T^2} \right] = [MLT^{-2}]$$

If the units of length, time and mass are used by m, s, kg, respectively, then the unit of force is $\text{kg} \cdot \text{m}/\text{s}^2$, i. e. 1 N (1 N = 1 $\text{kg} \cdot \text{m}/\text{s}^2$).

Engineering units:

For the primary dimensions chosen as $[L]$, $[T]$, $[F]$, the derived dimension of a mass is

$$[M] = \left[\frac{FT^2}{L} \right] = [FT^2L^{-1}]$$

If the units of length, time and force are used by m, s, kgf, respectively, then the engineering unit of mass is $\text{kgf} \cdot \text{s}^2/\text{m}$.

Note that, strictly speaking, kg and kgf are different; kg represents the mass, but kgf denotes the weight or force; however, the differences between the kg and kgf are not clearly identified in some books and materials.

The weight in the two units systems has the following basic relationship

$$1 \text{ kgf} = 9.8 \text{ N} \tag{1-2}$$

The basic relationship of mass conversion is

$$1 \text{ kgf} \cdot \text{s}^2/\text{m} = 9.8 \text{ kg} \tag{1-3}$$

i. e. 1 kgf force = 9.8 Newton and 1 engineering unit of mass = 9.8 kg.

1. 2. 3 The main physical properties of liquids

1. 2. 3. 1 Inertia, mass and density

Inertia is a property of an object, by which it maintains its existing state of rest or uniform motion. Inertia is measured by mass. The greater the mass of an object, the greater is the inertia. When the motion state of a liquid changes as a result of external forces, due to the inertia of the liquid, the resistance to the external forces that attempts to maintain the original state of motion is called an inertia force. Let the mass of a liquid be m , the acceleration a , then the inertia force is

$$F = - ma \tag{1-4}$$

where the negative sign indicates that the direction of the inertia force of liquid is opposite to the direction of acceleration.

The mass of a liquid per unit volume is called the density, denoted by the symbol ρ .

For homogenous liquid

$$\rho = \frac{m}{V} \tag{1-5}$$

where m is the mass of liquid, and V is the volume of liquid.

For heterogeneous liquid

$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V} \tag{1-6}$$

In the international system of units (SI), the unit of density is kg/m^3 . In general, the density of water has little variation with temperature and pressure. Usually we take it as a constant. At the temperature of 4°C and atmospheric pressure of 101.3 kPa, the maximum density of water is $\rho_w = 1000 \text{ kg}/\text{m}^3$.

The density of water at different temperatures is shown in Table 1-1.

Table 1 - 1 Values of the physical properties of water at different temperatures

Temperature $t/^{\circ}\text{C}$	Specific weight $\gamma/\text{kN}\cdot\text{m}^{-3}$	Density $\rho/\text{kg}\cdot\text{m}^{-3}$	Kinetic viscosity coefficient μ $/10^{-3}\text{N}\cdot\text{s}\cdot\text{m}^{-3}$	Dynamic viscosity coefficient $\nu/10^{-6}\text{m}^2\cdot\text{s}^{-1}$	Bulk modulus $K/10^9\text{N}\cdot\text{m}^{-2}$	Surface tension coefficient $\sigma/\text{N}\cdot\text{m}^{-1}$
0	9.805	999.9	1.781	1.785	2.02	0.0756
5	9.807	1000.0	1.518	1.519	2.06	0.0749
10	9.804	999.7	1.307	1.306	2.10	0.0742
15	9.798	999.1	1.139	1.139	2.15	0.0735
20	9.789	998.2	1.002	1.003	2.18	0.0728
25	9.777	997.0	0.890	0.893	2.22	0.0720
30	9.764	995.7	0.798	0.800	2.25	0.0712
40	9.730	992.2	0.653	0.658	2.28	0.0696
50	9.689	988.0	0.547	0.553	2.29	0.0679
60	9.642	983.2	0.466	0.474	2.28	0.0662
70	9.589	977.8	0.404	0.413	2.25	0.0644
80	9.530	971.8	0.354	0.364	2.20	0.0626
90	9.466	965.3	0.315	0.326	2.14	0.0608
100	9.399	958.4	0.282	0.294	2.07	0.0589

1. 2. 3. 2 Gravity, weight and specific weight

The attractive force between matter is called universal gravitation. Earth's gravitational force on an object is called gravity, or the weight, denoted as the symbol G . For the mass of a liquid m , the gravitational force is

$$G = mg \quad (1-7)$$

where g is the acceleration of gravity, taken as 9.8 m/s^2 .

The gravitational force per unit volume of fluid, or simply the weight per unit volume, is defined as specific weight, given by the symbol γ .

For homogeneous liquid

$$\gamma = \frac{G}{V} \quad (1-8)$$

For non-homogeneous liquid

$$\gamma = \lim_{\Delta V \rightarrow 0} \frac{\Delta G}{\Delta V} \quad (1-9)$$

In the international system of units, the unit of specific weight is N/m^3 or kN/m^3 . The specific weight of water has little variation with temperature and pressure, generally regarded as constant. The specific weight of water (γ_w) is taken 9800 N/m^3 at 4°C and $101\,325\text{ Pa}$ (a standard atmospheric pressure).

From Equation (1-7): $G = mg$, by dividing the volume V on both sides of the equation, we have $\frac{G}{V} = \frac{m}{V}g$, which shows a relationship between specific weight and density:

$$\gamma = \rho g \quad (1-10)$$

or

$$\rho = \gamma/g \tag{1 - 11}$$

Specific weights of some common liquids are given in Table 1 - 2.

Table 1 - 2 γ values of common liquids(at a standard atmospheric pressure)

Liquid	Gasoline	Pure alcohol	Distilled water	Sea water	Mercury
Specific weight/ $N \cdot m^{-3}$	6 664 ~ 7 350	7 778. 3	9 800	9 996 ~ 10 084	133 280
Temperature/ $^{\circ}C$	15	15	4	15	0

1. 2. 3. 3 Viscosity and viscosity coefficient

Viscosity is of most unique and inherent physical properties of fluids. It is different from the other physics concepts learned in the past.

Fluid at rest cannot resist shearing forces and has shear deformation. However, when in motion, there is a relative motion between the liquid particles or motion layers, which will generate internal friction to resist the relative motion between the fluid particles or flow layers. The internal friction does work and dissipates mechanical energy. This feature is called viscosity of fluid.

Take an example of the motion of liquid in a wide and shallow flume. If the abscissa denotes the velocity u of the liquid, the vertical coordinate y represents the distance of the particle from the bottom of the flume, set measuring points at different depths, and then measure the average velocity of these points in the motion direction. Thus we can obtain the variation of velocity with depth, $u = u(y)$, shown in Fig. 1 - 1.

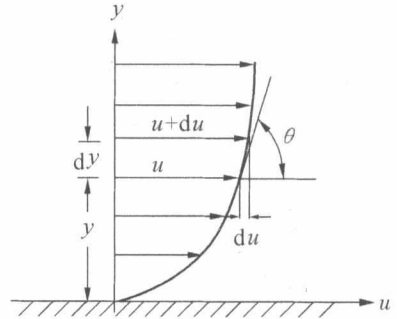


Fig. 1 - 1

The liquid in contact with the bottom of the flume has a zero velocity due to the adhesion of solid boundary, where no relative sliding motion exists. This is called the no-slip condition for the wall in fluid mechanics. The liquid above the bottom has different velocities, which are subject to the impact of retardation of bottom boundary. The closer to the bottom, the greater the impact of the retardation is, and the flow velocity is lower. However, the farther away from the bottom, the less the impact of the retardation is, so the flow velocity is higher.

The velocity of liquid on the free surface is slightly smaller than that of the liquid at the lower layers, which is due to the influence of air resistance. Assume that the velocity is u at the distance y to the bottom boundary, the velocity of liquid at the position of $y + dy$ will be $u + du$, where dy is the distance between the two layers, and du is the difference of their velocities. Due to the different velocities of flow at the two layers, relative motion exists. For a viscous liquid, a resistance takes place to such a relative motion; the liquid of the lower layer retards the liquid of

the upper layer by a frictional force in the opposite direction of motion, trying to slow down the motion of the faster layer; the liquid on the upper layer takes the liquid of the lower forwards by a frictional force in the direction of flow, trying to speed up the velocity of the slower moving layer. The two frictional forces are of the same size but in opposite directions, and both of them resist the relative movement. Note that this type of frictional force is different from the external friction in physics, because it does not exist between the liquid and solid walls, but only in between the liquid particles or flow layers. So this is called the internal friction. Internal friction is an important reason for the formation of the flow resistance. In order to overcome the internal friction and maintain the movement of liquid, it is required to consume efficient mechanical energy. Therefore, the viscosity of the liquid is the source of the mechanical energy loss.

In 1686, Newton proposed the well tested and verified Newton's law of internal friction, described as follows: when liquid has parallel linear motion along a solid surface, internal frictional force is generated along the interface of two adjacent layers in relative motion. The internal friction is related to the property of the liquid, and proportional to both the gradient of velocity and the contact area of liquid, but is nothing to do with the normal pressure on the contact surface. Its mathematical expression is

$$F = \mu A \frac{du}{dy} \tag{1-12}$$

where μ —the dynamic viscosity coefficient of fluid, $N \cdot s / m^2$, i. e. , $Pa \cdot s$;

A —the contact area of fluid, m^2 ;

du/dy —the velocity gradient, i. e. , $grad u$, which is the velocity increment of flow per unit depth, s^{-1} .

Let τ represent the frictional force per unit area, i. e. , the viscous shear stress, then

$$\tau = \frac{F}{A} = \mu \frac{du}{dy} \tag{1-13}$$

【Example 1 - 1】 A Newtonian fluid has the viscosity coefficient of $0.368 N \cdot s/m^2$, and its velocity distribution near the fixed wall is shown in Fig. 1 - 2. Determine the viscous shear stress at the wall (note that d is the height between the layer with the maximum velocity and the wall).

Solution: From Newton's law of internal friction, the viscous shear stress at the wall is

$$\tau_0 = \mu \left. \frac{du}{dy} \right|_{y=0}$$

where

$$\frac{du}{dy} = \frac{\pi}{2} \frac{U}{\delta} \cos\left(\frac{\pi}{2} \frac{y}{\delta}\right)$$

Then

$$\tau_0 = 0.368 \cdot \frac{\pi}{2} \frac{U}{\delta} = 0.578 \frac{U}{\delta}$$

For the fluid, the viscous force is towards the left, opposite to the direction of flow, and retards

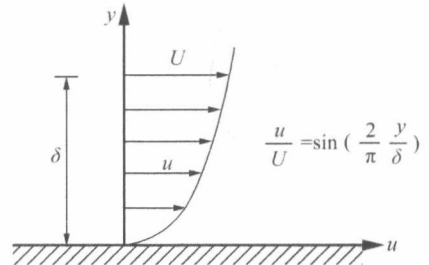


Fig. 1 - 2

the flow; for the walls, the force acts in the right direction, consistent with the flow direction.

【Example 1 - 2】 A movable plate between two fixed parallel plates forms two gaps, which are filled with two different types of Newtonian fluid, as shown in Fig. 1 - 3a. If the moving plate moves at a speed of 4 m/s horizontally, find the shear stress on the fixed plates. Assume that the surface area of the plates is very large, ignore the marginal impact of the plates, and that the fluid between the plates has a linear distribution of velocity.

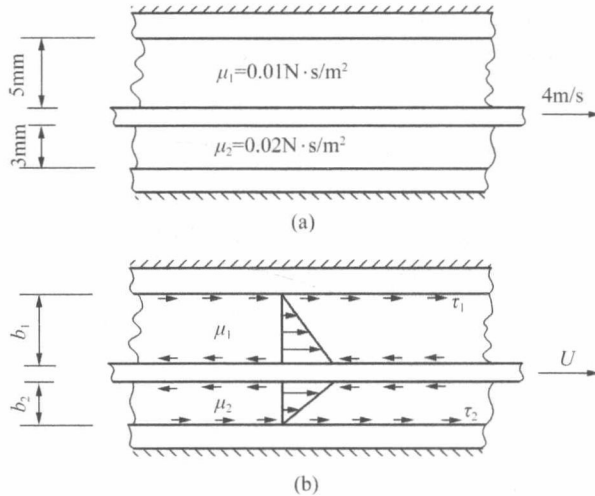


Fig. 1 - 3

Solution: The linear velocity distribution of the fluid between the plates is triangular as shown in Fig. 1 - 3b, where the velocity of the fluid at the fixed wall is zero, the velocity of the fluid at the moving plate is U . Therefore, for the fluid with the viscosity coefficient μ_1 , the velocity gradient is U/b_1 ; for the fluid with μ_2 , its velocity gradient is U/b_2 .

By Newton's law of internal friction, and substituting the known conditions, the viscous shear stress is

$$\tau_1 = \mu_1 \frac{U}{b_1} = 0.01 \times \frac{4}{5 \times 10^{-3}} = 8 \text{ N}$$

Similarly,

$$\tau_2 = \mu_2 \frac{U}{b_2} = 0.02 \times \frac{4}{3 \times 10^{-3}} = 26.7 \text{ N}$$

The directions of the shear stress are shown in Fig. 1 - 3b.

【Example 1 - 3】 A rotating cylinder viscometer has a fixed outer cylinder, whereas the inner cylinder is rotated by a synchronous motor, filled with a liquid sample between the inner and outer cylinders, as shown in Fig. 1 - 4. The radius of the inner cylinder $r_1 = 1.93 \text{ cm}$, with the height of $h = 7 \text{ cm}$, the radius of the outer cylinder $r_2 = 2 \text{ cm}$, the inner cylinder speed $n = 10 \text{ r/min}$, and the torque on the rotating shaft is measured to be $M = 0.004 \text{ N} \cdot \text{m}$. Calculate the viscosity coefficient μ of the liquid.

Solution: Under the impact of the rotating inner cylinder, the liquid in the gap of the two