RECENT RESEARCH IN PSYCHOLOGY

Michael Smithson

Fuzzy Set Analysis for Behavioral and Social Sciences

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This book is dedicated to Susan. Unambiguously.

Michael Smithson
Townsville, 1986

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Introduction: Why Fuzzy Sets?

The Case for Fuzziness

Like many intuitively appealing concepts, the idea of a fuzzy set is simple. In classical set theory, an element either belongs inside a set or it does not. In fuzzy set theory, on the other hand, an element may belong <u>partially</u> to a set. Fuzzy sets have gradations of set membership and blurred boundaries, and so they resemble at first glance the kinds of categories ordinary people use in natural thought or communication.

Where classical set theory permits only "red" or "not red", for instance, English-speakers may use "reddish", "sort of red", a "tinge of red", and other similar hedges to indicate degrees of redness. There is ample evidence that color categories are fuzzy in most, if not all, cultures (cf. Kay and McDaniel 1975, and Burgess et. al. 1983). Rosch and her colleagues have demonstrated that many people treat categories of even tangible objects as fuzzy, and learning that certain categories are fuzzy is apparently part of a child's cognitive development (Rosch 1973a and b, Rosch and Mervis 1975). Parikh (1977) has pointed out that in an "observationally connected" space (that is, a space whose elements cannot be split into sets such that all elements in one set are perceptibly distinguishable from all elements in the others), the only possible observational predicates are not classical but "vague" instead. Indeed, there is circumstantial evidence that people may even fuzzily modify words whose original meanings were unfuzzy (e.g., the currently common phrase "rather unique").

The importance of graded or blurry sets extends beyond mere matters of degree. Marginal members of a set may not possess the same properties as prototypical members, thereby invoking complex internal structures for such sets. Indirect evidence and some philosophical discussions have pointed to this for some time. For instance, some social psychological studies of stereotyping refer to the use of residual categories by bigots to handle exceptions to their stereotypes. More recently, Cantor and her colleagues (Cantor and Mischel 1979, Niedenthal and

Cantor 1984) have investigated the nature of prototypicality judgments and found that prototypes indeed may possess different attributes from other elements in the same category. The evidence for the psychological reality of fuzziness will be more systematically reviewed in the first chapter of this book, but it suffices to say that some human thoughts and actions appear to require explanations using fuzziness in at least an intuitive sense.

Another argument for considering fuzziness stems from the notion that social and behavioral scientists not only study fuzzy phenomena, but also use concepts and theoretical schema which themselves are fuzzy (even in a nonpejorative sense). Many popular scales, indexes, and measures may be thought of as fuzzy sets denoting such graded concepts as degree of arousal, agreement with a political statement, identification with one's mother, and so forth. These fuzzy sets, in turn, often are embedded in fuzzy hypotheses. Consider, for instance, Homans' claim that "loss in competition tends to arouse anger" (1961:123) or Brehmer and Hammond's hypothesis that "when the task has a high degree of uncertainty, the level of conflict is higher than when the task...has a low degree of uncertainty" (1977:90). These verbal statements contain not only fuzzy nouns, but fuzzy predicates, adjectives, and comparatives as well.

Taken only this far, the concept of a fuzzy set does not seem to offer anything fundamentally new. The notion that people think and communicate vaguely or ambiguously has occupied psychologists and philosophers for some time. That lay categories are better characterized by a "family resemblance" model than a classical set-theoretic model has been forcefully articulated by Wittgenstein (1953) and his followers. The uncertainty of human thought and language has been formally represented in probability and statistical theories, and when behavioral scientists want to represent a graded construct they usually find the concept of a continuous variable adequate. Why, therefore, bother with fuzziness at all?

Critics of fuzzy set theory usually begin their arguments with observations like these, and they are worth debating. Fuzzy set proponents make two claims in this connection. First, they argue that fuzziness is a distinctive kind of uncertainty that is not probabilistic and also may be distinguished from various other kinds of uncertainty, vagueness, or ambiguity. If true, this is an interesting claim to make since we obviously do have difficulty thinking systematically about uncertainty when we rely solely on probability, and several philosophers have attempted typologies of uncertainty. The second argument is that fuzziness opens the way to a fuzzy set theory, which expands our capacity to deal with uncertainty and vagueness in individuals and systems. This is a more powerful claim, and one which identifies fuzzy set theory with a long tradition in the use of formal languages like mathematics to aid analysis and provide novel insights or understandings. Both claims will be subjected to close scrutiny throughout this book, and indeed they form the main motivations for writing it.

The Case for a Theory of Fuzzy Sets

For many people, fuzzy sets become more interesting when we introduce such concepts as union, intersection, inclusion, and logical implication, all of which are well-defined in classical set theory. These relationships are not easily defined or operationalized, however, if we are dealing with conventional methods of analyzing continuous (or graded) variables. For instance, how do people combine fuzzy or graded traits to arrive at judgments on global attributes? Do they use weighted sums of the traits, as in N.H. Anderson's early (1968) "cognitive algebra"? Are "big" people "tall + heavy", "tall and heavy", or "tall or heavy"? To answer this question empirically requires a way of aggregating the fuzzy traits "tall" and "heavy" in the sense of the conjunctions "and" and "or". Clearly just adding them together will not do.

Researchers who want to test hypotheses based on substantive theories which use "and" and "or" face a similar problem. Smelser's (1962) theory of hostile outbursts requires the simultaneous effect of "prior strain" and "structural conduciveness". Conventional wisdom on so-called "interaction" terms in regression notwithstanding, this hypothesis is not necessarily adequately tested via the usual linearizable models using multiplicative terms. When the social psychologist J. Antill (1983) dichotomized Bem's femininity scale at the median and then performed an ANOVA to determine its impact on the happiness of married couples, visual inspection of the means revealed that marital happiness increased only if both husband and wife scored high on femininity. All that the conventional ANOVA procedures reveal in this instance is that an interaction has occurred. What if Antill wished to leave Bem's scale in its interval scaled form and test this finding by a regression-type approach? No adequate framework in conventional regression exists, but in chapter 6 I will demonstrate that fuzzy set theory enables such a test to be undertaken.

Consider the matter of set inclusion. Intuitively, we all know that some words embrace "broader" meanings than others, but how might this notion be operationalized in cases where the membership of a referent in the category denoted by the word is fuzzy? Smithson et. al. (1983) had respondents rate the extent to which various helping episodes were good examples of some 40 terms such as "rescuing", "donating", "aiding", or "succouring". The researchers wished to arrange these terms in a taxonomy which reflected their similarity and breadth of meaning. A definition of inclusion was required for such a taxonomy to be possible, and this particular case is discussed in chapters 3 and 5.

Finally, let us examine some examples of prediction in social science theories: "A high level of education comes close to being a necessary condition for democracy" (Lipset 1960: 57) and "...incompatibility [of authority systems in organizations] is...a sufficient but not necessary condition for instability" (Scott et. al. 1967: 113). Both predictions use logical terms (necessity and sufficiency), but they also use graded or fuzzy variables, and in Lipset's case a fuzzy hedge on necessity. Social scientists are not alone in framing "if-then" predictions using continuous variables. Indeed, it could argue that these comparatively weak predictions are entirely compatible with any multicausal theory. For instance, medical researchers might well predict that a high level of animal-fat in one's diet increases the likelihood of cancer of the colon, but other factors may increase that risk as well. The medical and social scientists all face the same dilemma, however. There is no established technique for testing the extent to which a necessity or sufficiency relationship holds between two continuous variables. Correlation and regression simply will not do. What is needed instead is a definition of fuzzy necessity or sufficiency, and indeed a fuzzy logic.

The case for a theory of fuzzy sets stems from two claims. The first claim is that fuzzy set theory may provide models or metaphors which mirror the rules by which people manipulate fuzzy categories in thought and language. In this sense, fuzzy set theory is hardly a "theory" at all, but instead a language. The second claim is that fuzzy set mathematics can augment and, in some cases, surpass conventional techniques for analyzing data. Both claims are open to debate since fuzzy set applications in the human sciences are still in infancy. However the concepts, evidence, and arguments in themselves are creatively stimulating for both researchers and theorists. If you are a researcher or student who has been frustrated by the restrictions of traditional statistical methods, then fuzzy sets may have something to offer you.

The Case for This Book

Fuzzy set theory made its first appearance in two nearly simultaneous publications by Zadeh (1965) and Klaua (1965), although a considerable body of mathematical and philosophical work on vagueness and multi-valued logic had preceded it. Zadeh's work was oriented toward computer and engineering applications, and therefore received much more attention than Klaua's pure mathematical treatment. Since then, hundreds of articles on fuzzy set theory and applications have been published in more than a dozen fields. In 1978, the journal <u>Fuzzy Sets and Systems</u> was inaugurated, and the late 70s and early 80s have seen several books and monographs introducing fuzzy sets and their applications (see annotated bibliography at the end of chapter 1). Nor does the flood seem to be abating. Fuzzy sets have found their way into several fields adjacent to the behavioral and social sciences (e.g., artificial intelligence, management science, ergonomics, decision theory, and economics).

Yet the dialogue between the human sciences and fuzzy set theory has been scattered, unsystematic, and slow to develop. There seem to be two main reasons for this. First, fuzzy set mathematics are couched in foreign and rather abstruse notation which is forbidding even to the more mathematically sophisticated behavioral scientist. There is no work available which introduces the concepts, symbols, and theory in a manner oriented towards research in the human sciences. Instead, virtually all texts on the topic assume either a mathematical or computer science and engineering orientation. Were this the sole reason, then the remedy would be simple enough. But it is also true that some of the conceptual machinery in fuzzy set theory is not sufficiently developed for immediate application to behavioral and social science problems. A number of statistical, measurement, and conceptual issues must be addressed before the full potential of fuzzy sets can be assessed.

No single book can definitively solve these problems, but undoubtedly a need exists for a work which introduces fuzzy sets clearly, integrates the current literature on applications in the human sciences, and presents at least some promising approaches to existing problems. This book has been written with these three goals in mind, and accordingly is written on two levels. The main text assumes no more than a knowledge of high-school algebra and introductory statistics, although it is aimed at advanced students and researchers. Some chapters include sections on "advanced" topics for the more mathematically adventurous (or devoted), and these have an asterisk in front of their headings. These sections provide a more in-depth conceptual grounding for concepts covered in the text, as well as discussions of the algorithms used in computer programs whose listings are given in appendices to certain chapters. However, even these sections do not rise above simple calculus and matrix algebra, and certainly are no more difficult than material found in second-level statistics courses for behavioral scientists.

Since this is a book on applications, every major concept or technique is illustrated with real-world research examples taken from cognitive psychology, social psychology, sociology, social anthropology, political science, and evaluation research. The reader is advised to follow these examples carefully, since they often aid immeasurably in making sense of abstractions. A glossary of technical terms and a list of mathematical symbols have been provided near the back of the book for convenient reference, should the reader become disoriented or forgetful. Finally, the computer program listings (in standard FORTRAN) are intended for readers to use as they will. Nothing aids understanding more than using the techniques and concepts to solve real problems or analyze real data.

The book begins with the basic elements of fuzzy set theory and then progresses from topics in univariate measurement to discussions of multivariate analysis. Chapter 1 addresses definitional and conceptual issues in the foundations of fuzzy set theory, introduces the theory itself, and outlines the scope for applications. In chapter 2, fuzzy set theory is reality-tested from both cognitive psychological and philosophical standpoints. Chapter 3 discusses certain measurement problems associated with fuzzy sets, the construction of fuzzy indexes and scales, scale transformations, a fuzzy generalization of Guttman scaling, and compound logical indexes. In chapter 4, measures of fuzziness and other aspects of internal category structure are introduced. Chapter 5 applies the concepts of fuzzy set overlap, inclusion, association, and similarity to problems in clustering, taxonomic analysis, and data reduction. Chapter 6 employs fuzzy intersection and union as a basis for multivariate "and" and "or" models in ANOVA and regression setups, and these in turn yield insights into the murky area of nonlinear models and interactions. Chapter 7 returns to the bivariate level to develop a framework for testing "if-then" and other one-to-many predictive models, by combining fuzzy logic with proportional reduction of error and uncertainty frameworks. The discussion then moves from simple to compound bivariate and multivariate predictions. In chapter 8, fuzzy set applications in several adjacent fields are briefly outlined, and the book concludes with an assessment of the contributions fuzzy set concepts may make to theory construction and to the hiatus between qualitative and quantitative research methods.

I do not intend this book to be a definitive work on fuzzy set theory, nor will I argue that fuzzy sets are some kind of methodological panacea. There are no such cure-alls. Research is essentially creative, and our methods should assist and reflect that creativity rather than stifle it in the name of convention. Fuzzy set theory has a role to play in the construction of new and more sensitive analytical techniques, as well as in the enrichment of theory. Conversely, human scientists can contribute to fuzzy set theory, not only empirically but conceptually as well. After all, fuzzy set theory is a linguistic framework which embodies a model of human thought. This framework should be modified in any way that enhances its capacity to empower our understanding of thought and behavior. Accordingly, I hope this book serves a wider purpose than merely importing still more mathematical gadgetry into the human sciences. I want to stimulate constructive dialogue, critical debate, and creative work on a large set of conceptual and methodological problems that have been with us for too long.

Chapter 1: Fuzzy Set Theory: The Basics

1.1 What Is Fuzziness?

A set is fuzzy when an element can belong partly to it, rather than having to belong completely or not at all. Fuzzy set theory, therefore, begins with the assignment of **membership values** to elements which are not restricted to 0 (nonmembership) or 1 (full membership), but which may lie somewhere in the interval from 0 to 1. This verbal statement, and the mathematics which formalize it, have some intuitive appeal since ordinary common sense presents us with sets which fit this description. Fuzzy set theorists themselves have not been forthcoming about what fuzziness is, and some even eschew definitional issues altogether. Mathematicians, to some extent, can do this because they do not necessarily deal in substantive questions about empirical reality. We in the behavioral and social sciences, however, cannot afford to ignore these issues. After all, psychologists, philosophers, linguists, and social scientists have used terms which might be related to or even synonymous with fuzziness. Is fuzziness a type of ambiguity or vagueness? How is it related to uncertainty? Is it a kind of probability (and would probability do just as well)? But before moving into a full-blown discussion of terms, it is worth considering what the founders of fuzzy set theory have had to say about fuzziness. While they are reluctant to say what it is, they have been quite vocal on what it is not.

Ever since the inception of fuzzy set theory, a debate has raged over whether fuzziness is merely probability in disguise. Zadeh and his colleagues (e.g., Zadeh 1980 and Gaines 1975) have therefore taken considerable pains to distinguish fuzziness from probability. They have done this in two ways: conceptually and mathematically. These two lines of argument are to some extent independent, and so I will deal only with the basic conceptual issues, since the mathematical distinguishability question is best delayed until after introducing fuzzy set theory itself.

The most popular interpretation of probability is based on the concept of randomicity or chance, with a frequentist basis for estimating probabilities of events. Fuzzy set proponents claim that there are uncertainties and vaguenesses which are nonstatistical in nature. Their most popular example is the concept of tallness, as in "John is a tall man". The probability that John is tall may well be equal to 1, but tallness nonetheless is a vague term, at least in relation to a question like "how tall is John?". There may be fuzzy events which are certain (as in "right now we have light to moderate drizzle") and likewise, there may be nonfuzzy events which are only probable (e.g., "will the coin come down heads or tails?").

So far, so good, but there is at least one other concept of probability that has given fuzzy set theorists a more difficult time in establishing their claim for distinguishing fuzziness from probability. Subjective probability usually is defined as the degree of belief a person has that an event will occur, and is the basis for the Bayesian approach to probabilistic reasoning. Indeed, it is the Bayesians who have been most critical of fuzzy set theory (e.g., Stallings 1977 and Lindley 1982). Furthermore, some fuzzy set researchers have used fuzziness in ways which seem close to the sense of a subjective probability. Sugeno (1977), for instance, claims fuzziness is the degree of certainty that may be assigned to the statement "x is a member of set S". In an important series of papers, Giles (1976, 1979, 1982) has attempted to provide a formal Bayesian definition for fuzzy set membership. In his approach, the degree of membership (or truth or possibility) for the proposition "x is S" is the amount of money one would be willing to wager in a "fair bet" against an opponent who has agreed to pay $1 if x is not S. The fuzzy set theory Giles derives from his formulation is quite similar to Zadeh's, but fuzziness becomes a special kind of subjective probability or belief.

While the Bayesian approach to fuzziness seems reasonable for some situations, it does not work equally well under all conditions. One important class of propositions where it fails consists of certain self-referential statements. A common example in the behavioral sciences is the rating a person assigns to his or her degree of agreement with an attitudinal statement. Intuitively, agreement or disagreement are fuzzy concepts, but it makes little sense to bet with oneself on the truth of the propostion "I agree with S". This is an important objection, since a lot of data in behavioral and social science research are collected on individuals' self-reports of attitudes, values, beliefs, and behaviors. More recent statements from both sides of the argument (e.g., the probabilist Natvig 1983 and the pro-fuzzy Blockely et. al. 1983) appear to be moving towards the view that there is some overlap between various versions of probability and fuzziness, but no single approach is likely to handle all types of uncertainty or vagueness.

Zadeh and many others, however, conceive of fuzziness as a nonprobabilistic kind of vagueness. Since vagueness itself is not an immediately obvious concept, Zadeh has attempted to specify how fuzziness differs from other kinds of vagueness. He claims (1976, 1979) that ambiguity is entirely distinct from fuzziness. He gives as an example the statement "Ruth has dark skin and owns a red Porsche" in which the adjectives "dark" and "red" are fuzzy because they are graded, but not ambiguous. On the other hand, the proposition "Ruth lives somewhere near Berkeley" is insufficiently specific to precisely characterize Ruth's location. Zadeh then defines vagueness as a combination of both fuzziness and ambiguity.

These definitions and usages differ somewhat from their more traditional forms in philosophy and semantics. In his classic studies of vagueness, Black (1937) distinguished **vagueness** from both **generality** (or nonspecificity) and **ambiguity**. However, for Black ambiguity refers to the case where a word or phrase has several possible interpretations, each of which might be precise or not. For instance, the term "hot" in the statement "the food is hot" might mean either spicy or having a high temperature. In the absence of further information, the statement is ambiguous. Puns and optical illusions both exploit ambiguity to achieve their effects. Black defines vagueness in terms similar to Zadeh's sense of fuzziness. Even closer is Alston's (1964) definition of **degree vagueness**, whose attributes correspond to those Zadeh claims for fuzziness.

Unfortunately, these conceptual distinctions and examples do