

H K Versteeg
W Malalasekera

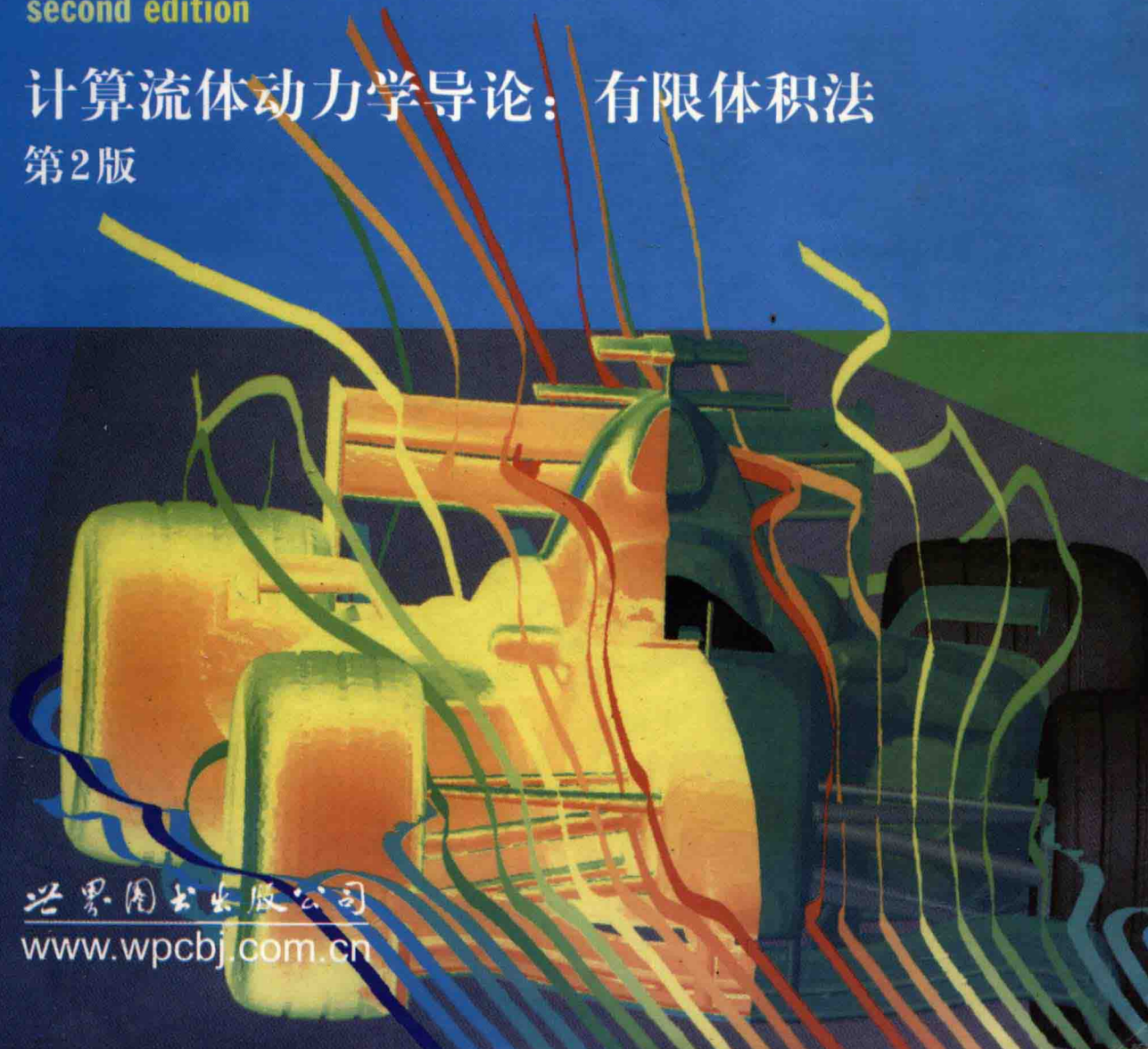
An Introduction to
**COMPUTATIONAL FLUID
DYNAMICS**

The Finite Volume Method

second edition

计算流体力学导论：有限体积法
第2版

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计算流体力学

有限体积法

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An Introduction to Computational Fluid Dynamics

THE FINITE VOLUME METHOD

Second Edition

H K Versteeg and W Malalasekera



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Preface

We were pleasantly surprised by the ready acceptance of the first edition of our book by the CFD community and by the amount of positive feedback received over a period of 10 years. To us this has provided justification of our original plan, which was to provide an accessible introduction to this fast-growing topic to support teaching at senior undergraduate level, post-graduate research and new industrial users of commercial CFD codes. Our second edition seeks to enhance and update. The structure and didactic approach of the first edition have been retained without change, but augmented by a selection of the most important developments in CFD.

In our treatment of the physics of fluid flows we have added a summary of the basic ideas underpinning large-eddy simulation (LES) and direct numerical simulation (DNS). These resource-intensive turbulence prediction techniques are likely to have a major impact in the medium term on CFD due to the increased availability of high-end computing capability.

Over the last decade a number of new discretisation techniques and solution approaches have come to the fore in commercial CFD codes. To reflect these developments we have included summaries of TVD techniques, ? which give stable, higher-order accurate solutions of convection-diffusion problems, and of iterative techniques and multi-grid accelerators that are now commonly used for the solution of systems of discretised equations. We have also added examples of the SIMPLE algorithm for pressure-velocity coupling to illustrate its workings.

At the time of writing our first edition, CFD was firmly established in the aerospace, automotive and power generation sectors. Subsequently, it has spread throughout engineering industry. This has gone hand in hand with major improvements in the treatment of complex geometries. We have devoted a new chapter to describing key aspects of unstructured meshing techniques that have made this possible.

Application of CFD results in industrial research and design crucially hinges on confidence in its outcomes. We have included a new chapter on uncertainty in CFD results. Given the rapid growth in CFD applications it is difficult to cover, within the space of a single introductory volume, even a small part of the submodelling methodology that is now included in many general-purpose CFD codes. Our selection of advanced application material covers combustion and radiation algorithms, which reflects our local perspective as mechanical engineers with interest in internal flow and combustion.

Finally, we thank colleagues in UK and overseas universities who have encouraged us with positive responses and constructive comments on our first edition and our proposals for a second edition. We are also grateful to several colleagues and postgraduate researchers who have given help in the

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H.K. Versteeg
W. Malalasekera

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