Error Theory and Foundation of Surveying Adjustment

误差理论与测量平差基础

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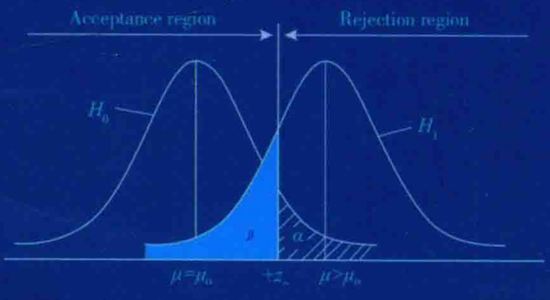

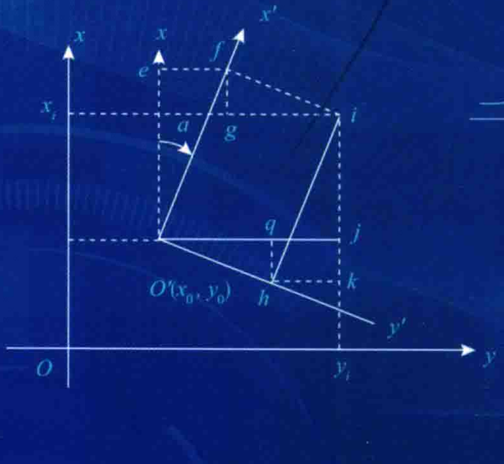

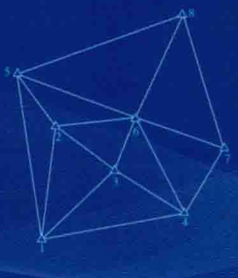

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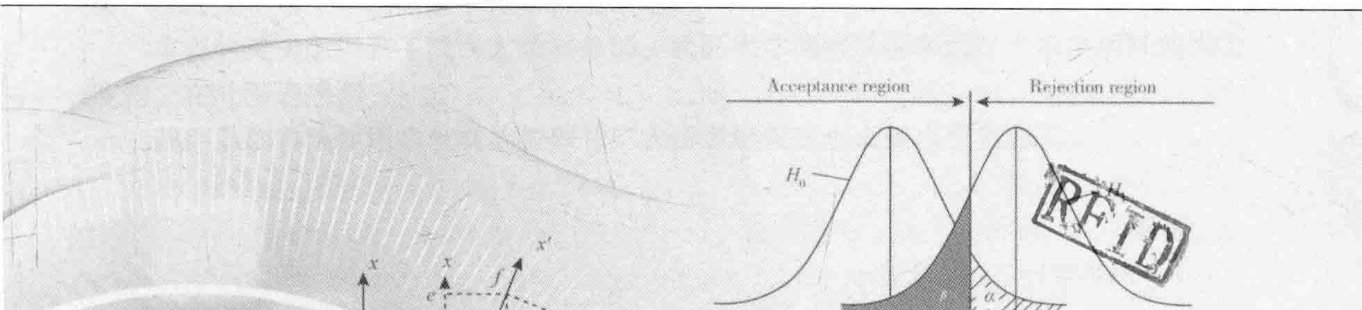

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前　言

《误差理论与测量平差基础》(英文版)是根据《误差理论与测量平差基础》(第三版)翻译而成的。

由武汉大学测绘学院测量平差学科组编写的《误差理论与测量平差基础》是测绘工程专业本科的专业基础核心课程的通用教材。该教材(第三版)由陶本藻教授、邱卫宁教授、姚宜斌教授进行修订，是国家级精品资源共享课程“误差理论与测量平差基础”指定教材。目前，全国开设有工程测量专业的近200所院校大多采用该教材，自2014年5月出版以来，已发行8万余册。近年来，不仅国内有部分院校将“误差理论与测量平差基础”列为双语课程，而且国外也有相关院校把该教材作为教学参考书。鉴于国内外对《误差理论与测量平差基础》英文译本的迫切需求，武汉大学测量平差学科组和加拿大约克大学联合翻译了该教材，以馈读者。

本书第一、二、三、四、五、十二章由王建国教授翻译，第七章由邱卫宁教授翻译，第九、十章由教授姚宜斌翻译，第八、十一章由吴云副教授翻译。全书由王建国统稿，武汉大学测绘学院测量平差学科组审定。

本书的英文版得到了武汉大学教务部、武汉大学测绘学院和武汉大学出版社的大力支持，在此深表感谢。

我们恳切希望使用本教材的教师和广大读者继续对本书提出宝贵意见。

编者

2018年6月

Forward

This English edition of *Error Theory and Foundation of Surveying Adjustment* is based on the third edition of the textbook,《误差理论与测量平差基础》, in Chinese.

The book, 误差理论与测量平差基础 (*Error Theory and Foundation of Surveying Adjustment*) authored by the *Teaching and Research Section of Surveying Adjustment* at School of Geodesy and Geomatics Engineering of Wuhan University, has been the textbook for one of the core undergraduate courses in Geomatics Engineering (a. k. a. Surveying Engineering). The third edition of the book were compiled and revised under the leadership of Prof. Benzao Tao, Prof. Weining Qiu and Prof. Yibin Yao. It became the specified textbook for the subject of "Error Theory and Surveying Adjustment", and has been honored as a national level quality resource sharing course. So far, it has been taken as the standard textbook in *Surveying Engineering* by the majority of the about 200 universities and schools in China. Since the third edition was published in May 2014, more than 80,000 copies have been sold.

Nowadays, the course "Error Theory and Foundation of Surveying Adjustment" (误差理论与测量平差基础) has been offered as a bilingual course in Chinese and English by a number of the schools in China. Moreover, the textbook has been used as a reference book by some foreign professors in Geomatics Science and Geomatics Engineering or Surveying Engineering. Hence, there exists an urgent demand for publishing its English edition. In order to comfort readers' desire, a group of the professors at School of Geodesy and Geomatics Engineering of Wuhan University and at Lassonde School of Engineering of York University have together translated and compiled the textbook into English during the last two years. Now, its edition in English, titled as *Error Theory and Foundation of Surveying Adjustment*, is finally published.

Among the professors, the division of the work is as follows: Prof. Jianguo Wang in charge of Chapter 1, Chapter 2, Chapter 3, Chapter 4, Chapter 5 and Chapter 12; Prof. Weining Qiu in charge of Chapter 7; Prof. Yibin Yao in charge of Chapter 9 and Chapter 10, and Prof. Yun Wu in charge of Chapter 8 and Chapter 11. Prof. Wang has unified the book manuscript. The final reviewing has been performed by the Teaching and Research Section of Surveying Adjustment at School of Geodesy and Geomatics Engineering of Wuhan University.

We sincerely acknowledge the Division of Undergraduate Studies, School of Geodesy and Geomatics Engineering of Wuhan University, and Wuhan University Publish Press for their full support for preparing the publication of the textbook in English.

As usual, we greatly appreciate the valuable advices and feedbacks from students, instructors and readers at large on this textbook.

Editors, Wuhan, Toronto
June 2018

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Chapter 1
Introduction

Measurements or observations mean the data that represent the Earth or contain information of spatial distribution of other objects acquired by using certain surveying instruments, tools, or other means. Surveying and observation are synonym that can be interchangeably used. The observed data, the measurements, can be the direct results of a surveying, or the results of specific preliminary process or transformation. Any surveying measurement contains not only information, but also disturbance which needs to be removed or reduced by taking appropriate measures. This chapter explains that measurements are contaminated with errors, outlines the contents of the study of surveying adjustment, and further introduces the tasks and contents of this subject.

§1-1 Observation Errors

Through a series of repeated observations of a specific quantity, one can find that differences exist among the individual measurements. For instance, the sum of the measured three internal angles of a triangle may not be equal to 180°. The existence of the difference between the repeated observations of the same quantity, or between the measurements and the true values is a common phenomenon in surveying practice. Why does the difference occur? Measurements containing observation errors or measurements are contaminated with observation errors.

1. Error Sources

There are many causes that may lead to measurement errors, which may be summarized into the following three aspects.

1.1 Measuring Instruments

A measuring instrument means a device or tool that is used to collect or acquire measurements. Due to the limited precision of a specific instrument, the measurements from the observation are generally contaminated with errors. For example, a leveling observation using a pair of leveling rods only with the cm graduations cannot ensure an accurate reading of the fractional number within a cm below centimetres. Meanwhile, the instrument itself certainly has some errors, e. g. the error due to the parallel misalignment between the line of sight of the telescope and the leveling axis of the level. Besides, the measurements acquired using a digitizer or a scanner in map digitization, a total station, or a GPS receiver etc. will be contaminated with different instrumental errors.

1. 2 Observer

Due to the limit of resolving power of the sense organs of an observer, instrument operation can bring about errors. At the same time, the working attitude and technical skills of an observer may directly affect the measurement quality.

1. 3 Environment Conditions

The environment conditions such as temperature, humidity, wind, and atmospheric refraction and so on, and their changes will directly have influence on the observations. Especially with the high accuracy measurement acquisition, measurement errors caused by changes of environment conditions should be paid attention to. For example, the received signal by a GPS receiver comes from the satellites about 20km away in the sky. The atmosphere may delay the propagation of satellite signal in general, i. e. bring errors to the received satellite signal.

Instrument, observer and environment condition are the three major factors to bring errors to measurements. Hence, these factors together are called the observation conditions. Obviously, the quality of observation results is closely related to the observation condition. Under good observation condition, errors brought by the observation process are relatively small on average so that the measurement quality can be higher than the one under bad observation condition. Under the same observation condition, the same measurement quality is expected. Thus, measurement quality practically reflects the quality of the observing condition.

However, it does not matter how the observation condition is, the observation results are contaminated with various errors from the entire observation process. Measurement error is inevitable in measuring process. Without doubt, an observer, or a surveyor, can, or has to guarantee to have high quality of measurements as the objective conditions permit.

2. Classification of Measurement Errors

According to the effects of characteristic of measurement errors on results, they can be classified into three kinds: random errors, systematic errors and outliers.

2. 1 Random Errors

With a group of measurements collected under the same observation condition, if the magnitude and sign of the errors on measurements behave in a random manner, i. e., from the perspective of a single error, its magnitude and sign are lack of regularity. However, a large number of errors together as a whole having a certain statistical regularity are accordingly called random errors.

For instance, the subtle changes of the measurements due to not strictly aiming at the target while using a theodolite, the guess of the millimeter readings on a leveling rod while performing leveling observation, and the climatic change during the observation, etc. are random errors. Furthermore, if the error on a measurement is a cumulative sum of various random errors, it still behaves in random manner. The uncertainty on a measured angle could be caused all together, for example, by target aiming error, reading error, instrumental errors, and the environmental variations, etc. Hence, the error on an angular measurement could practically be the sum of many small errors, each of which varies with the change of its corresponding accidental factors and can be small or big, and positive or negative. Therefore, neither its magnitude nor its sign is predictable about this cumulative error sum, which remains random error and is the most general case how the random measurement error occurs.

On the basis of probability and statistic theory, if the effect of each error on the total sum is small, i. e. no error source is dominated by any other, the total error sum is a random variable which follows or approximately follows the normal distribution. Overall, a group of random errors as a whole can statistically be characterized with certain regularities.

2. 2 Systematic Errors

With a group of measurements being collected under the same observation condition, if the magnitude and sign of their errors behave or change in a systematic manner, or are constant within the observing process, this type of errors is called systematic errors.

Assume to measure a distance using a steel tape, the distance error caused by the length of the steel tape is proportional to the length of measured distance. The longer the distance is measured, the greater the error on the distance measurement is. Thus, the length error of a steel tape is a kind of systematic errors. The length error of a steel tape is constant and is also called the constant systematic error. Its impact on the total length of the being measured distance is a linear error. If the length error of a steel tape is ε, and the total distance to be measured is m times of the length of the steel tape, the linear error on the measured distance is $m\varepsilon$. Further, between two specific points, if the leveling observation is repeated daily in terms of vertical deformation monitoring, it can be found out that there exists certain cyclic error on the leveling results that varies annually with the changes of environmental conditions such as temperature. Such systematic error with linear and cyclic phenomena is a type of regular systematic error. Some other sources of systematic errors may only affect some part of the observation regularly, and the effect of the signs may be different on different measurement groups, i. e., it may behave in a random manner. Thus, this type of systematic errors is called random systematic errors or semi-systematic errors. Besides, there are also other systematic errors whose causes are still unknown.

Systematic errors and random errors are always generated at the same time during an observation process. When apperent systematic errors appear in the observation, random errors become secondary so that the measurement errors appear systematically; conversely, they appear randomly.

Systematic errors have impacts on measurements in an accumulative way and can significantly affect the quality of observation results. In practice, one should take appropriate measures to eliminate or reduce the influence of systematic errors to an ignorable level, which means the leftover systematic errors on measurements become less or at least be equal to the influence brought by random errors. In order to make it possible, one way is to introduce appropriate field operation procedures. For example the systematic error on the measured height difference due to the misalignment between the line of sight and the axis of the tabular leveling bubble can be eliminated by taking the equal distance to the backsight and foresight leveling rods. Another is to apply correction using formulas. An example is how to deal with the length error of steel tapes. One can calibrate the steel tapes for their length errors in advance, and then apply corrections to distance measurements so that this systematic error can be reduced to the certain extend.

In case that the influence of systematic errors on measurements are removed, or reduced to a secondary role compared with the effect of random errors, one can consider the measurement errors to be dominated by random errors.

However, the existence of systematic errors and their effects on observation cannot be eliminated using such simple approaches as mentioned above, but by taking steps in data processing.

2. 3 Outliers

Outliers are gross errors, which mean that the errors are greater than the maximum possible errors under the normal observationcondition. Literally, an outlier can be several times as big as a random error. Several examples can be enumerated: a mistake with the initial data of a control network, a wrong reading of an observation of large numbers, a mistake of computer input data, and a wrong aerial photo interpretation, etc. They are human mistakes and can be avoided to a certain degree. During the use of today's high-tech in Geomatics, such as GPS (Global Positioning System), GIS (Geographic Information System), RS (Remote Sensing) and other high accurate automatic data acquisition, outliers are often interfused into information. One may not be able to recognize outliers by using simple methods, but needs to identify them and eliminate their impact on special data processing methods.

§ 1-2 Subjects of Study in Surveying Adjustment

Since observation results are inevitably contaminated with errors, how to process the measurements with errors and how to reach the optimal estimation of unknown parameters become the study contents of surveying adjustment in Surveying and Mapping.

In Surveying Engineering and other engineering disciplines, measurements affected only by random errors are in the majority and more common situations. Hence, they are the fundamental contents studied in surveying adjustment, and also the essential basis that has been most widely applied and theoretically developed, which belongs to the category of classic surveying adjustment.

In order to determine a side length, if it is measured only once, the error on the measurement is unknown and data processing is not needed accordingly. However, one can repeatedly measure the side length for n times to obtain n measured distances and takes their average as the final side length. In this way, the effect of the random errors on these n measured distances can be eliminated or reduced, i. e., the accuracy of the measured side length is improved and one can also check if any mistake exists. This is the benefit gained through these $n-1$ redundant measurements. Averaging is a method to process the measurements contaminated with random errors. The $n-1$ extra measurements are called redundant measurements. The number of redundant measurements is equal to the difference between the number of the measurements and the number of the unknowns. In Surveying Engineering, certain redundant measurements are made in order to improve the quality of the estimated unknowns and identify outliers and/or mistakes. Due to the measurement errors, redundant measurements will result conflicts among themselves. For example, one can determine the shape of a plane triangle by measuring two of its internal angles. Now if all of its three internal angles are measured, their sum will not be equal to 180° in general due to measurement errors. Therefore, a misclosure, or a closure error is generated accordingly. Indeed, how to optimally estimate unknown parameters and evaluate their accuracy on the basis of the conflicts, or misclosures resulted from the redundant measurements is the fundamental task of surveying adjustment.

As the above-mentioned misclosuresare due to random observation errors, one has to study the probability and statistics theory about random errors inclusive of distribution of random errors, indexes of accuracy evaluation, laws of error propagation, test statistics of errors and error analysis and so on.

Random errors are not avoidable in any observation. If there also exist systematic errors or outliers, or both together with random errors, the data processing becomes difficult, which is considered to belong to the scope of modern surveying adjustment. Under the condition that systematic errors and outliers are removed or their impacts are substantially reduced, what remains is still the basic task: optimal estimation of unknown parameters and their accuracy evaluation.

The theory and methods of surveying adjustment belongs to the essential component of measurement processing and its quality control in surveying and mapping, and have widely been applied in the fields of modern GNSS (Global Navigation Satellite Systems), GIS

(Geographical Information System), RS (Remote Sensing) and modern high-tech surveying technologies, for instance the high accuracy automatic digital data acquisition and processing systems.

In summary, surveying adjustment means measurement adjustment. It is basically defined as the theory and methods that determine optimal estimates of unknown parameters and their accuracy after certain optimization criteria using a series of measurements. Surveying Adjustment is a special terminology used in Surveying and Mapping and possesses a long history. From its definition given above, the theory and methods of Surveying Adjustment are applicable to any other discipline, in whichever the measurements need to be processed. Hence, it is clear that applications of Surveying Adjustment are very broad.

§1-3 Brief History and Development of Surveying Adjustment

As with other disciplines, the emergence of Surveying Adjustment was the result of the demand in scientific and technological practice. It has been developed with the progress made in sciences and technologies. At the end of 18th century, the demand for solving the conflicts among measurements, i.e., how to estimate unknowns based on the measurements contaminated with errors, was put forward in Surveying and Astronomic Geodesy. In 1794, C. F. Gauss, at the age of 17, first proposed a method of solving the problem — Least-Squares Method. He derived the probability distribution of random errors by taking the arithmetic mean as the most probable value of the unknown on the ground of four characteristics followed by random errors, and gave the calculation method for the most probable values of unknowns under the Principle of Lease-Squares. However, Gauss did not publish his theory at that time. In 1801, the Italian astronomers performed a series of observations of an arc of the Ceres' orbit, which was just found by them. This was a practice to determine the orbit of a star based on measurements with errors. Unfortunately, the observation was interrupted. Gauss used his own method, the Least-Squares Method, and solved this difficult problem at that time so that the orbit of the Ceres could be predicted and astronomers could find it in time. In 1809, Gauss first published his method in *Theoria motus corporum coelestium in sectionibus conicus solem ambientium*, or, *Theory of the motion of heavenly bodies moving about the sun in conic sections.* Before that time, A. M. Legendre published his paper *Nouvelles Méthodes pour la Détermination des Orbites des Comètes*, which put forward Least-Squares Method, also referred to as Gauss-Legendre Method.

Over about one hundred and fifty years from the beginning of the 19th century to 1960s, the geodesists contacted plenty of studies on Surveying Adjustment after Least-Squares Principle and published a series of adjustment methods to solve different types of problems, also categorized as classic adjustment methods. Due to the overall situation of computing tools at that time, many simplified methods were developed to be capable of solving linear equations in groups.

Since early 19th century to 1950s-1960s, Surveying Adjustment got a great development with the advancement made in computing techniques and the demand of high precision and accuracy in scientific and engineering practice, which is mainly reflected in the following:

(1) Methodological studies have been extended from random errors to systematic errors and outliers. On the basis of random error theory, the adjustment theory and methods have been comprehensively studied. Thus, the studying scope of Surveying Adjustment has been greatly expanded.

(2) In 1947, T. M. Tienstra put forward the Adjustment Theory of Correlated Observations. However, due to the limit of computing tools, it has been widely applied since 1970s. The emergence of Adjustment Theory of Correlated Observations generalized the concept of measurements and extended the classic Least-Squares adjustment to a wider range of applications.

(3) In classic least squares, the chosen parameters (unknowns) are assumed to be deterministic. With the progress of surveying technology, there is a need to solve the problems with both of the measurements and parameters being random variables. The Least-Squares Method under the consideration of random parameters was proposed in the late 1960s and developed in 1970s. It was originated from the adjustment problem of gravity anomalies using Least-Squares Interpolation and Extrapolation given by H. Moritz and T. Krarup, named as Least-Squares Filtering, Prediction and Collocation.

(4) The classic Least-Squares Adjustment deals with the full-rank problems, i. e., the resulted normal equation systems are full rank so that unique solutions can be delivered. In 1960s, P. Meissl established the principle of inner-constrained (or minimum constrained) adjustment. Later on, a complete theory of free network adjustment or rank-defect network adjustment was formed in 1970s and 1980s through the efforts made by many geodetic scientists and widely used in surveying practice.

(5) The use of the electronic distance measuring technique innovated the measurement weighting theory and methods. Many scholars studied how to posteriori determine measurement weights. In 1980s, the theory of variance and covariance component estimation was also formed. No other research topic in Surveying and Mapping had obtained more attention than the variance and covariance component estimation did in terms of the number of publications.

(6) Since there may exist systematic errors on measurements, the theoretical studies on the characteristics, propagation, statistic tests, and analysis have been naturally

carried out, and the relevant methods have been developed, for instance, the adjustment method with systematic parameters etc. In order to identify the existence of systematic errors and analyse their impacts on adjustment results, methods of linear hypotheses from linear statistics has been introduced to surveying adjustment with the consideration of specific objects and characteristics in Surveying and Mapping. The simultaneous execution of statistic tests together with the parameter estimation became a standard in modern measurement adjustment process.

(7) Since there may exist outliers on measurements, the theory of measurement outliers has accordingly been developed. The most well-known work was the data snooping and the reliability theory of surveying systems by W. Baarda in the late 1960s. This laid a good foundation of theoretical and practical studies on measurement outliers. By far, the theoretical system for outlier identification, estimation and statistical hypotheses has been formed. The treatment of measurement outliers may go in two different ways, data snooping or robust estimation instead of using Least-Squares Method. Studies on robust estimation methods and their applications in Surveying Adjustment are still carried on.

In brief, since 1970s, especially during the last two decades, Surveying Adjustment has gained sufficient development. The use of the outcomes is now fairly generalized in routine surveying and mapping techniques. With the continuous emergence and development of new surveying and mapping technologies, how to apply the existing methods and develop novel adjustment theory and methodology to fit to modern data processing is still a problem deserving of study.

§1-4 Tasks and Contents of the Textbook

Fundamentals of Surveying Adjustment are namely the theoretical basis of Surveying Adjustment. This subject as a course mainly offers the fundamental theories and methods of Surveying Adjustment. The objects of data processing are measurements containing random errors along with the relevant basic theoretical knowledge about errors.

The teaching objectives are to enable students to acquire the theory of random errors and the basic principle of Surveying Adjustment, or Least-Squares Principle to lay down a good foundation of Surveying Adjustment, to be capable of further self studying the modern error theory, to learn various classic adjustment methods, and be capable of independently solving problems in practice.

This course mainly focuses on adjustment of measurements with random errors:

(1) Random error theory, which includes characteristics of random errors, error propagation laws, accuracy indexes and their estimates, definition of weights and