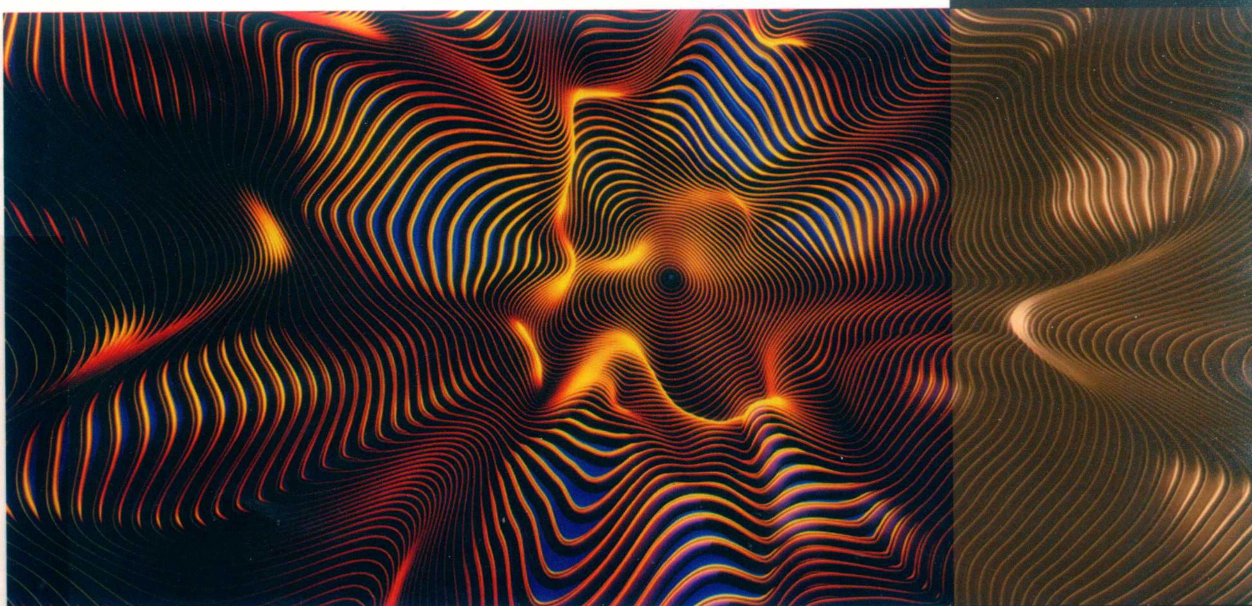
Totally Asymmetric Simple Exclusion Process on Traffic Flow

完全非对称简单排他过程在交通流的研究

肖 松 孙成通 石建辉 著



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Preface

The phenomenon of nonequilibrium traffic has been found from macroscopic objects (such as cars, pedestrians, ants) to microscopic molecular motors. The system can be recognized as a nonequilibrium system and presents many properties such as phase transitions and phase separation which can be due to the interaction of particles. Because of the variety and complexity exhibited in nonequilibrium system, one way to investigate these phenomena is to build an appropriate simplified model. It is common to abstract motile objects into particles by neglecting their sizes and underlying structures, and then taking the traffic as a nonequilibrium system of interacting particles. At present, many models have been applied to study nonequilibrium phenomena, for example, Brusswlator and Oregonator models which are used to simulate chemical oscillation and the formation of spatial pattern, and Schlögl model which is used to stimulate the phenomenon of multiple steady state.

As a typical nonequilibrium model, exclusion process which investigates the particles hopping behavior on lattice has been widely studied. It means that the interaction of particles show as only one particle can be accommodate in one lattice which is called as exclusive interaction. If the hopping rates to two directions are different, it is called asymmetric simple exclusion processes (ASEPs). When particles move only in one direction and the model is the simplest limit of an ASEP, it is called a totally asymmetric simple exclusion processes (TASEP). Therefore, TASEP has been widely used in chemistry, physics and biology such as proteins synthesis, surface growth, the motion of motor proteins along cytoskeleton filaments and vehicular traffic.

As a based model, the normal TASEP has been used to study some typical traffic flow via mathematical analysis or numerical methods in this book. The theoretical results will be verified by extensive Monte Carlo simulations. The vehicular traffic, such as one-lane TASEPs system with open boundary conditions, two-channel and multi-lane TASEPs traffic models, crossroad on TASEPs, ramp on TASEPs, shortcut on TASEPs and roundabout on TASEPs will be explored in this book.

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XIAO Song
October, 2018

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Chapter 1

Theoretical Investigation of the Synchronous Totally Asymmetric Simple Exclusion Process with a Roundabout

1.1 Introduction

As an important research tool, the totally asymmetric simple exclusion process (TASEP) has primarily been used to describe the kinetics of biopolymerization[1] and recently extended to chemistry, physics and biology[2-6]. More recently, TASEP has been used to investigate the flow of road traffic, gel electrophoresis and problems in many other areas of research[7-15].

In the case of road traffic, most research and applications of the TASEP have focused on the effects of different lattice structures on traffic density. For example, the parallel-lattice TASEP[16-18], parallel-lattice TASEP with junctions[19-22] and coupling in the parallel-lattice TASEP[23-26]. The present paper explores the synchronous TASEP with a roundabout in a one-dimensional system. The model can be used to investigate the roundabout as an alternative vehicular traffic system to a crossroad structure. This system can be described by a one-dimensional lattice that is divided into three segments (segments 1, 2 and 3) by two special sites. At these special sites, cars can attach to (on-ramp) or detach from (off-ramp) the system irreversibly. For a simple one-dimensional roadway model, the boundary conditions will decide the transition from free flow to congestion. When ramps are introduced, the traffic conditions are much more complicated. The effects of attachment and detachment on traffic have been explored in great detail[27-31]. A study of a single on-ramp has found that the system goes through five phases when the on-ramp rate p is less than 0.5 but only four phases when the parameter p exceeds 0.5 and the low-density (LD)/LD phase thus does not occur[28]. Subsequent exploration of TASEPs with attachment and detachment under random update has revealed eight possible phases, which can be used to describe biological traffic[30]. A recent study on TASEPs with detachment under a parallel update shows that the system goes through five stationary phases, and only the regions of the LD/LD and maximal-current (MC)/LD phases increase[29]. The present paper investigates a synchronous TASEP with a roundabout, this model is more complex than a model with a single on-ramp (off-ramp). In addition, compared with TASEPs having attachment and detachment under a random update model, our model describes well vehicle traffic.

1.2 Model and Theoretical Analysis

The synchronous TASEP with a roundabout is sketched in Fig. 1.1. There are $3N$ sites in the one-dimensional lattice. Each site has two stations, namely occupy and empty. At the first site of the system, particles are injected into the lattice with probability α if the site is empty. When a particle reaches the last site, it can leave the lattice at the rate β. For site $i(1<i<3N)$, a particle can hop to site $i+1$, as site $i+1$ is empty. At the first special site ($i=k_1$) and the second special site ($i=k_2$), a particle attaches to and detaches from the system irreversibly with probabilities p and q, respectively. Both special sites are far from boundaries (Fig. 1.1). For simplicity, we assume that the number of lattice sites $3N$ is a large number and that the special sites are equally spaced; i.e., $k_1=N$ and $k_2=2N$. The whole lattice can be divided into three segments, each of which is described by a normal TASEP; there are thus 27 possible steady stations in the system. For these three segments, the effective enhancement rates are respectively α, α_{eff2} andα_{eff3} and the effective exit rates are β_{eff1}, β_{eff2} and β, respectively. The attachment and detachment at sites k_1 and k_2 lead to the MC state being impossible for segments 1 and 3, since the maximal current in the system is 0.5. If the MC phase exists in segment 1 and particle flow can be added at site k_1, the current of segment 2 exceeds 0.5 because some particles will enter segment 2 by attachment. Similarly, the current of segment 2 must exceed 0.5 when the current of segment 3 can be in the MC phase and the particle flow can be reduced at site k_2, because some particles will leave segment 2 by detachment. For segment 2, however, particle can only add to the flow at site k_1 and can only exit from the roundabout at site k_2, and the current must therefore be a maximum in segment 2. There are therefore only 12 possible steady stations in the system. The model undergoes a parallel update. For the normal TASEP with a parallel update, the exact results are as follows.

For $\alpha<\beta<1$, the system is in an LD phase, and the current and bulk density are given by

$$J=\rho,\ \rho=\rho_1,\ \rho_1=\frac{\alpha}{1+\alpha},\ \rho_N=\frac{\alpha}{\beta(1+\alpha)} \tag{1.1}$$

where J and ρ are respectively the system current and bulk density and ρ_1 and ρ_N are respectively the densities at the entrance and exit.

For $\beta<\alpha<1$, the system is in a high-density (HD) phase and the current and bulk density are given by

$$J=1-\rho,\ \rho=\rho_N,\ \rho_1=1-\frac{\beta}{\alpha(1+\beta)},\ \rho_N=\frac{1}{1+\beta} \tag{1.2}$$

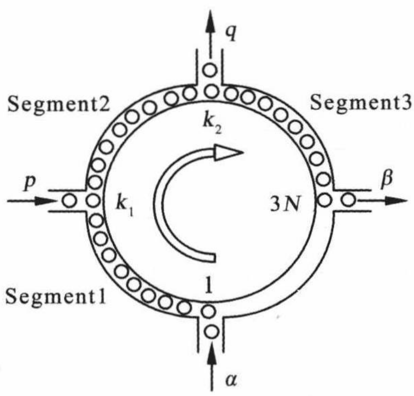

Fig. 1.1 Sketch of a TASEP with a roundabout

the model can be mapped as three normal TASEPs connected at sites k_1 and k_2

When $\alpha=\beta=1$, the system is in an MC phase and the current and bulk density are given by

$$J=0.5,\ \rho=\rho_1=\rho_N=0.5 \tag{1.3}$$

For the system, current conservation is important and we get

$$J_A+J_L=J_M=J_D+J_R \tag{1.4}$$

where J_L, J_M and J_R are the currents of segments 1, 2 and 3, respectively. J_A and J_D are the currents of the attachment to and detachment from the system.

The following is the investigation and analysis of the existence and behavior of different stationary phases. The LD/LD/LD phase is determined by the conditions

$$\alpha<\beta_{eff1}<1;\ \alpha_{eff2}<\beta_{eff2}<1;\ \alpha_{eff3}<\beta<1 \tag{1.5}$$

from Eq.(1.1), the stationary properties of this phase are obtained as

$$J_L=\frac{\alpha}{1+\alpha},\ J_A=\frac{p\alpha_{eff2}}{1+\alpha_{eff2}},\ J_M=\frac{\alpha_{eff2}}{1+\alpha_{eff2}},\ J_D=\frac{q\alpha_{eff2}}{1+\alpha_{eff2}},\ J_R=\frac{\alpha_{eff3}}{1+\alpha_{eff3}} \tag{1.6}$$

from Eq.(1.4), we get

$$\alpha_{eff2}=\frac{\alpha}{(1+\alpha)(1-p)-\alpha},\ \alpha_{eff3}=\frac{(1-q)\alpha}{(1+\alpha)(1-p)-(1-q)\alpha} \tag{1.7}$$

the system is thus in the LD/LD/LD phase when

$$\frac{(1-q)\alpha}{(1+\alpha)(1-p)-(1-q)\alpha}<\beta,\ \alpha<\frac{1-p}{1+p} \tag{1.8}$$

similar calculations can be performed for the LD/LD/HD phase, which exists when

$$\alpha<\beta_{eff1}<1;\ \alpha_{eff2}<\beta_{eff2}<1;\ \beta<\alpha_{eff3}<1 \tag{1.9}$$

from Eqs.(1.1) and(1.2), the stationary properties of this phase are obtained as

$$J_L=\frac{\alpha}{1+\alpha},\ J_A=\frac{p\alpha_{eff2}}{1+\alpha_{eff2}},\ J_M=\frac{\alpha_{eff2}}{1+\alpha_{eff2}},\ J_D=\frac{q\alpha_{eff2}}{1+\alpha_{eff2}},\ J_R=\frac{\beta}{1+\beta} \tag{1.10}$$

according to Eq.(1.4), we get

$$\alpha_{\text{eff2}}=\frac{\alpha}{(1+\alpha)(1-p)-\alpha},\ \beta=\frac{(1-q)\alpha}{(1+\alpha)(1-p)-(1-q)\alpha} \tag{1.11}$$

the system is thus in the LD/LD/HD phase when

$$\frac{(1-q)\alpha}{(1+\alpha)(1-p)-(1-q)\alpha}=\beta,\ \alpha<\frac{1-p}{1+p} \tag{1.12}$$

the case with the LD/HD/LD phase can be generally described by the conditions

$$\alpha<\beta_{\text{eff1}}<1;\ \beta_{\text{eff2}}<\alpha_{\text{eff2}}<1;\ \alpha_{\text{eff3}}<\beta<1 \tag{1.13}$$

from Eqs.(1.1) and(1.2), the stationary properties of this phase are obtained as

$$J_{\text{L}}=\frac{\alpha}{1+\alpha},\ J_{\text{A}}=\frac{p\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{M}}=\frac{\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{D}}=\frac{q\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{R}}=\frac{\alpha_{\text{eff3}}}{1+\alpha_{\text{eff3}}} \tag{1.14}$$

according to Eq.(1.4), we get

$$\beta_{\text{eff2}}=\frac{\alpha}{(1+\alpha)(1-p)-\alpha},\ \alpha_{\text{eff3}}=\frac{(1-q)\alpha}{(1+\alpha)(1-p)-(1-q)\alpha} \tag{1.15}$$

However, we cannot use Eqs.(1.13) and(1.15) to obtain the conditions of α and β, because the LD/HD/LD phase does not exist.

Similar calculations can be performed for the LD/HD/HD phase, which exist when

$$\alpha<\beta_{\text{eff1}}<1;\ \beta_{\text{eff2}}<\alpha_{\text{eff2}}<1;\ \beta<\alpha_{\text{eff3}}<1 \tag{1.16}$$

from Eqs.(1.1) and(1.2), the stationary properties of this phase are given by

$$J_{\text{L}}=\frac{\alpha}{1+\alpha},\ J_{\text{A}}=\frac{p\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{M}}=\frac{\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{D}}=\frac{q\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{R}}=\frac{\beta}{1+\beta} \tag{1.17}$$

according to Eq.(1.4), we get

$$\beta_{\text{eff2}}=\frac{\alpha}{(1+\alpha)(1-p)-\alpha},\ \beta=\frac{(1-q)\alpha}{(1+\alpha)(1-p)-(1-q)\alpha} \tag{1.18}$$

However, we cannot use Eqs.(1.16) and(1.18) to obtain the conditions of α and β, because the LD/HD/HD phase does not exist.

The LD/MC/LD phase is defined by

$$\alpha<\beta_{\text{eff1}}<1;\ \alpha_{\text{eff2}}=\beta_{\text{eff2}}=1;\ \alpha_{\text{eff3}}<\beta<1 \tag{1.19}$$

from Eqs.(1.1) and(1.3), the stationary properties of this phase are obtained as

$$J_{\text{L}}=\frac{\alpha}{1+\alpha},\ J_{\text{A}}=\frac{p}{2},\ J_{\text{M}}=\frac{1}{2},\ J_{\text{D}}=\frac{q}{2},\ J_{\text{R}}=\frac{\alpha_{\text{eff3}}}{1+\alpha_{\text{eff3}}} \tag{1.20}$$

according to Eq.(1.4), we get

$$\alpha=\frac{1-p}{1+p},\ \alpha_{\text{eff3}}=\frac{1-q}{1+q} \tag{1.21}$$

the system is thus in the LD/MC/LD phase when

$$\alpha=\frac{1-p}{1+p},\ \frac{1-q}{1+q}<\beta \tag{1.22}$$

similar calculations can be performed for the LD/MC/HD phase, which exists when

$$\alpha<\beta_{\text{eff1}}<1;\ \alpha_{\text{eff2}}=\beta_{\text{eff2}}=1;\ \beta<\alpha_{\text{eff3}}<1 \tag{1.23}$$

from Eqs.(1.1), (1.2) and(1.3), the stationary properties of this phase are given by

$$J_{\text{L}}=\frac{\alpha}{1+\alpha},\ J_{\text{A}}=\frac{p}{2},\ J_{\text{M}}=\frac{1}{2},\ J_{\text{D}}=\frac{q}{2},\ J_{\text{R}}=\frac{\beta}{1+\beta} \tag{1.24}$$

according to Eq.(1.4), we get

$$\alpha=\frac{1-p}{1+p},\ \beta=\frac{1-q}{1+q} \tag{1.25}$$

the system is thus in the LD/MC/HD phase when

$$\alpha=\frac{1-p}{1+p},\ \beta=\frac{1-q}{1+q} \tag{1.26}$$

the HD/LD/LD phase is defined by

$$\beta_{\text{eff1}}<\alpha<1;\ \alpha_{\text{eff2}}<\beta_{\text{eff2}}<1;\ \alpha_{\text{eff3}}<\beta<1 \tag{1.27}$$

from Eqs.(1.1) and(1.2), the stationary properties of this phase are obtained as

$$J_{\text{L}}=\frac{\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}},\ J_{\text{A}}=\frac{p\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},\ J_{\text{M}}=\frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},\ J_{\text{D}}=\frac{q\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},\ J_{\text{R}}=\frac{\alpha_{\text{eff3}}}{1+\alpha_{\text{eff3}}} \tag{1.28}$$

according to Eq.(1.4), we get

$$\alpha_{\text{eff2}}=\frac{\beta_{\text{eff1}}}{(1+\beta_{\text{eff1}})(1-p)-\beta_{\text{eff1}}},\ \alpha_{\text{eff3}}=\frac{(1-q)(1-p)\beta_{\text{eff1}}}{(1+\beta_{\text{eff1}})-(1-q)(1-p)\beta_{\text{eff1}}} \tag{1.29}$$

However, we cannot use Eqs.(1.27) and(1.29) to obtain the conditions of α and β, because the HD/LD/LD phase does not exist.

Likewise, the HD/LD/HD phase and HD/HD/LD phase do not exist for any conditions of the injection rate α and ejection rate β.

The HD/HD/HD phase is determined by the conditions

$$\beta_{\text{eff1}}<\alpha<1;\ \beta_{\text{eff2}}<\alpha_{\text{eff2}}<1;\ \beta_{\text{eff3}}<\alpha_{\text{eff3}}<1 \tag{1.30}$$

from Eq.(1.2), the stationary properties of this phase are obtained as

$$J_{\text{L}}=\frac{\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}},\ J_{\text{A}}=\frac{p\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{M}}=\frac{\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{D}}=\frac{q\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}},\ J_{\text{R}}=\frac{\beta}{1+\beta} \tag{1.31}$$

according to Eq.(1.4), we get

$$\beta_{\text{eff1}}=\frac{(1-p)\beta}{(1+\beta)(1-q)-(1-p)\beta},\ \beta_{\text{eff2}}=\frac{\beta}{(1+\beta)(1-q)-\beta} \tag{1.32}$$

the system is thus in the HD/HD/HD phase when

$$\frac{(1-p)\beta}{(1+\beta)(1-q)-(1-p)\beta}<\alpha,\ \beta<\frac{1-q}{1+q} \tag{1.33}$$

similar calculations can be performed for the HD/MC/LD phase, which exists when

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