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# 高等数学 上

# ADVANCED MATHEMATICS Ⅰ

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主编

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# PREFACE

Advanced mathematics ( Calculus ) is a part of modern mathematics education. It has two major branches, differential calculus (concerning instantaneous rates of change and slopes of curves) and integral calculus (concerning accumulation of quantities and the areas under and between curves). These two branches are related to each other by fundamental theorem of calculus.

Due to the extensive application of Advanced Mathematics in Science, Engineering and Economics, this course has become one of the most important foundation courses for college students. Not only is it critical that the text be accurate, but readability and even page design also play an important role. The authors have the following purposes when writing the book:

1. To write a book that students can easily comprehend.

2. To enable students to apply what they have learned to solve practical problems.

3. To develop students' meticulous logical thinking skills.

And the features of the book are as follows:

1. The book is not only well defined, comprehensive, but also rigorously structured to improve readability.

2. Almost all knowledge points are accompanied by corresponding examples, which can help students gain solid knowledge of the basic topics.

3. The exercises after each section can help students gain better insight into the mathematical concepts and assess their skills.

4. At the end of each chapter, there exist a summary of the main contents and basic requirements.

This book aims to enable students to construct the links between what they are learning and how they may apply the knowledge. To cultivate students' logical thinking ability is helpful for them to think about mathematical problems from a multi-dimensional perspective, so as to develop a good habit of thinking. It is advisable for students to flexibly apply what they have learned in class and what they have expanded after class to the modeling and solving of practical problems. The book is suitable for students majoring in engineering, economics, life sciences, as well as mathematics and applied mathematics (Chinese-foreign cooperation). For any errors in the book, the authors would be grateful if they were sent to: dudo999@126.com.

# CONTENTS

# Chapter 1 Function and limit

Basically, the object of research in elementary mathematics is a constant quantity, but in advanced mathematics is a variable. In this chapter, we will introduce three fundamental concepts, which are functions, limits and continuity, and some of their properties.

## 1.1 Function

### 1.1.1 Set

**1) Definition of set**

A set is defined as the collection of all objects having some specified property. For example, all students of a class constitute a set, lines passing through a fixed point on a plane constitute a set, natural numbers can be divided by three constitute a set and so on. Each object belonging to a set is called an element of the set.

Sets are usually denoted by capital letters $A$, $B$, $X$, $Y$, $\cdots$. Elements are usually denoted by small letters $a,b,x,y,\cdots$. If $a$ is a member (or element) of $A$, then we call $a$ belongs to $A$, the notation $a \in A$ is used; if $a$ is not a member (or element) of $A$, we call $a$ doesn't belong to $A$, denoted as $a \notin A$. An element either belongs to $A$ or not.

There are two methods to represent a set: one is the method of citation and the other is description.

The method of citation, that is, to list all the elements of the set. For example, a set consisting of finite elements $a_1$, $a_2$, $\cdots$, $a_n$ is written as $A=\{a_1,a_2,\cdots,a_n\}$. A set consisted of infinite elements $b_1$, $b_2$, $\cdots$, $b_n$, $\cdots$ is denoted by $B=\{b_1, b_2, \cdots, b_n, \cdots\}$.

The method of description is to point out the determining property held in common by all the elements of a set, that is $X=\{x \mid x \text{ has property } p(x)\}$. For instance, solution set of inequality $x^2-5x+6>0$ can be presented by

$$X=\{x \mid x^2-5x+6>0\}.$$

A set consisting of finite number of elements is called a finite set. A set consisted of infinite elements is called an infinite set. A set containing no element is called the empty set, denoted by $\varnothing$.

Usually **N** stands for the set of natural numbers, **Z** stands for the set of all integers, and **Q** stands for the set of rational numbers.

$$\mathbf{N}=\{0, 1, 2, \cdots, n, \cdots\};$$

$$\mathbf{Z}=\{0, \pm 1, \pm 2, \cdots, \pm n, \cdots\};$$

$$\mathbf{Q}=\left\{\frac{P}{q}\,\middle|\, p\in \mathbf{Z},\ q\in \mathbf{N},\ q\neq 0, \text{ and } p, q \text{ are co-prime numbers}\right\}.$$

In addition, the set of real numbers consists of all real numbers and is denoted by **R**, and **C** stands for the set of complex numbers.

"$*$" marked in the upper right corner of the letter represents 0 is excluded out of the set. And "$+$" indicates that 0 and negative numbers are removed from the set. For example,

$$\mathbf{Z}^{*}=\{\pm 1, \pm 2, \cdots, \pm n, \cdots\};$$

$$\mathbf{Z}^{+}=\{1, 2, \cdots, n, \cdots\}.$$

**2) Operations on sets**

Suppose that $A$ and $B$ are two sets. If each element in $A$ is also an element in $B$, then $A$ is called a subset of the set $B$, denoted by $A\subseteq B$ or $B\supseteq A$, read as "$A$ is contained by $B$ (or $B$ contains $A$)". For any set $A$, because $\varnothing\subseteq A$, $A\subseteq A$, then $\varnothing$ and $A$ are both subsets of $A$. If $A\subseteq B$ but $A\neq B$, then $A$ is called a proper subset of $B$, denoted by $A\subsetneqq B$.

Some basic operations on sets are as follows.

Suppose that $A$ and $B$ are two sets. Union of the sets $A$ and $B$, denoted by $A\cup B$, is the set of all subjects that are a member of $A$, or $B$, or both, namely

$$A\cup B=\{x\,|\,x\in A \text{ or } x\in B\}.$$

Intersection of the sets $A$ and $B$, denoted $A\cap B$, is the set of all objects that are members of both $A$ and $B$. A set consisted of all elements belonging to both $A$ and $B$ is called intersection of $A$ and $B$, namely

$$A\cap B=\{x\,|\,x\in A \text{ and } x\in B\}.$$

Set difference of $A$ and $B$, denoted $A\setminus B$, is the set of all members of $A$ that are not members of $B$, namely

$$A\setminus B=\{x\,|\,x\in A \text{ but } x\notin B\}.$$

If all the sets are subsets of $I$, then $I$ is called a universal set, and the set difference $I\setminus A$ is also called the complement of $A$ in $I$, the notation $A^C$ is sometimes used instead of $I\setminus A$, namely

$$A^C=I\setminus A=\{x\,|\,x\in I \text{ but } x\notin A\}.$$

The operations on sets are shown in Figure 1-1 (the shaded region in the figure are the results).

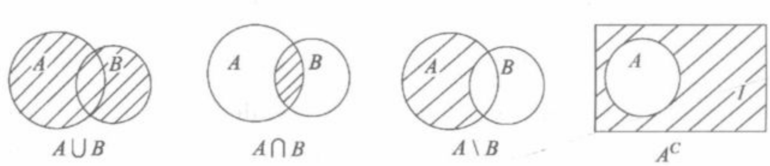


**Figure 1-1**

It is easily seen from the geometric meaning that operations on sets satisfy the following rules. Let $A$, $B$, $C$ be any three sets, then

(1) Commutative law $A\cup B=B\cup A$, $A\cap B=B\cap A$;

(2) Associative law $A\cup(B\cup C)=(A\cup B)\cup C$, $A\cap(B\cap C)=(A\cap B)\cap C$;

(3) Distribution law $(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$,
$(A\cap B)\cup C=(A\cup C)\cap(B\cup C)$;

(4) Idempotent law $A\cup A=A$, $A\cap A=A$;

(5) Absorption law $A\cup(A\cap B)=A$, $A\cap(A\cup B)=A$;

(6) Dualization law (De Morgan law)

$$(A\cup B)^C=A^C\cap B^C,\ (A\cap B)^C=A^C\cup B^C.$$

**3) Interval**

Intervals are a class of important number sets. Let $a$ and $b$ are real numbers, $a<b$. The number set $\{x|a<x<b\}$ is called an open interval, denoted by $(a, b)$, that is

$$(a,b)=\{x|a<x<b\},$$

$a$ and $b$ are called the end points of the open interval $(a,b)$. The number set $\{x|a\leqslant x\leqslant b\}$ is said to be a closed interval, denoted by $[a,b]$, that is

$$[a,b]=\{x|a\leqslant x\leqslant b\}.$$

$a$ and $b$ are called the end points of the closed interval $[a,b]$. Similarly left-open right-closed interval is $(a,b]$, and left-closed right-open interval is $[a,b)$,

$$(a,b]=\{x|a<x\leqslant b\},\ [a,b)=\{x|a\leqslant x<b\}.$$

The above intervals are finite intervals. Notations $+\infty$ (read as positive infinity) and $-\infty$ (read as negative infinity) can be used to define infinite intervals:

$$(a,+\infty)=\{x|a<x<+\infty\},$$
$$(-\infty,\ b)=\{x|-\infty<x<b\},$$
$$[a,+\infty)=\{x|a\leqslant x<+\infty\},$$
$$(-\infty,\ b]=\{x|-\infty<x\leqslant b\},$$
$$(-\infty,+\infty)=\{x|-\infty<x<+\infty\}=\mathbf{R}.$$

**4) Neighborhood**

The definition of neighborhood is useful and widely used. An open interval with center $a$ is said to be a neighborhood of point $a$, denoted by $U(a)$. Let $\delta$ be any positive number, then the open interval $(a-\delta,\ a+\delta)$ is a neighborhood of point $a$, which is called $\delta$-neighborhood of $a$, denoted by $U(a,\ \delta)$. That is

$$U(a,\ \delta)=\{x|a-\delta<x<a+\delta\}=\{x\,\|\,x-a|<\delta\},$$

where $a$ is the center of neighborhood, $\delta$ is called the radius of the neighborhood. For example

$$U\left(-2,\frac{1}{2}\right)=\left\{x\,\middle|\,|x-(-2)|<\frac{1}{2}\right\}=\left(-\frac{5}{2},-\frac{3}{2}\right).$$

The neighborhood is called a deleted neighborhood, if the center is removed from $U(a,\ \delta)$, denoted by

$$\mathring{U}(a, \delta)=\{x \mid 0<|x-a|<\delta\}.$$

The open interval $(a-\delta, a)$ is called the left $\delta$-neighborhood of $a$, and $(a, a+\delta)$ is called the right $\delta$-neighborhood of $a$.

### 1.1.2 Function

**1) Concept of function**

In observing natural and social phenomena, there are many different quantities, some of which are constant quantity; and others are variables. Usually $a$, $b$, $c$ are used to represent constants, $x$, $y$, $z$ are used to represent variables.

In the same process, there are often several variables that are interrelated and interact with one another and follow certain objective laws. If the causal relationship of these changes can be accurately described, this will help us really understand the trend and the law of development of things. The function is the most basic mathematical tool to reflect the relationship of variables.

**Definition 1** Let $x$, $y$ are two variables, $D$ is a nonempty set. If for every $x \in D$, there exists one and only one $y$ corresponding to $x$ according to some determined rule $f$, then $y$ is called a function of $x$ on set $D$, denoted by $y=f(x)$, where $x$ is called the independent variable, and $y$ is called the dependent variable. The set $D$ is called domain of $f$, denoted by $D(f)$. The set consists of all functional value $y$ is called the range of $f$, denoted by $W(f)$, namely

$$W(f)=\{y \mid y=f(x),\ x \in D\}.$$

In the definition of function, there are two essential factors, namely the domain $D$ and the corresponding rule $f$. If the domain and the corresponding rule are determined, then the function and the range of function are determined. If two functions have the same domain $D$ and the same corresponding rule, then the two functions are the same. Therefore the symbols, which are used to denote the independent variable and dependent variable, are insignificant.

For example, the two functions $f(x)=x^2\ (D(f)=[0,1])$ and $g(t)=t^2\ (D(g)=[0,1])$ are the same, because they have the same domain and corresponding rule. The two functions $y=\ln x^2\ (x \neq 0)$ and $y=2\ln x\ (x>0)$ are different, because their domains are different.

Suppose the domain of definition of $y=f(x)$ is $D$, in the $xOy$ plane, for every $x \in D$, there corresponds to a unique point $(x, y)=(x, f(x))$ corresponding to it. When we study a function given by an analytic representation, sketching its graph will often help us to obtain some intuitive information about the function. In the coordinate system, the point set $G=\{(x, y) \mid y=f(x),\ x \in D\}$ often a curve, is called the graph of the function $y=f(x)$ (Figure 1-2).

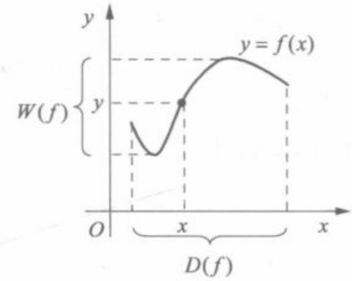

Figure 1-2

It should be noted that the analytic representation of a function sometimes consists of several components on different subsets of the domain of the function. A function expressed by this kind of representations is called a piecewise defined function.

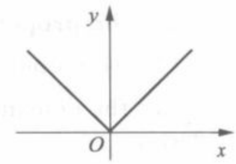

Figure 1-3

**Example 1** Absolute function (Figure 1-3)

$$y=|x|=\begin{cases} x, & x \geqslant 0, \\ -x, & x<0. \end{cases}$$

**Example 2** Sign function(Figure 1-4)

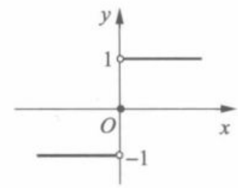

Figure 1-4

$$y=\operatorname{sgn} x=\begin{cases} 1, & x>0, \\ 0, & x=0, \\ -1, & x<0, \end{cases}$$

The domain of the function is $D=(-\infty,+\infty)$, the range of function is $W=\{-1, 0, 1\}$.

**Example 3** The greatest integer function $y=[x]$, where $[x]$ represents the value at any number $x$ is the largest integer smaller than or equal to $x$. The domain of the function is $D=(-\infty,+\infty)=$ **R**, the range of function is $W=\{0, \pm 1, \pm 2, \cdots\}=$ **Z**. Its graph is shown in Figure 1-5. For instance, $[2.5]=2,[\sqrt{2}]=1,[-\pi]=-4$.

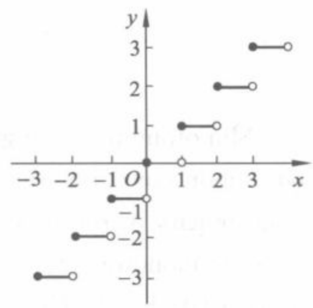

Figure 1-5

In Definition 1, for every $x \in D$, there is a unique $y$ corresponding to $x$ according to some determined rule $f$, then function $y=f(x)$ is called single-valued function. If relax restriction on the uniqueness of dependent variable: there are two or more $y$ corresponding to $x$, then function $y=f(x)$ is called multi-valued function.

For multi-valued functions, we can limit the range of the dependent variable to make it become a single-valued function. For example, the function $y=\pm\sqrt{1-x^2}$ defined by $x^2+y^2=1$ is a multi-valued function. If $y \geqslant 0$, then $y=\sqrt{1-x^2}$. If $y \leqslant 0$, then $y=-\sqrt{1-x^2}$. These two functions are single-valued functions, each of which is called one branch of multiple-valued function. For another example, anti-trigonometric function $y=\operatorname{Arcsin} x$ is a multi-valued function, for any $x \in[-1,1]$ there are infinitely $y$ corresponding to $x$. We define as $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$, then $y=\arcsin x$, it is a single-valued function. And it is called the principal branch of $y=\operatorname{Arcsin} x$. Other anti-trigonometric functions have a similar situation.

The functions to be discussed in the future, if not specified, are single valued functions.

**2) The properties of functions**

(1) Monotone functions

Let the domain of function $y=f(x)$ is $D(f)$ and interval $I \subset D(f)$.

If $f(x_1)<f(x_2)$ for any $x_1$, $x_2 \in I$ with $x_1<x_2$, then $f(x)$ is said to be monotone increasing on the interval $I$ (Figure 1-6a).

If $f(x_1)>f(x_2)$ for any $x_1$, $x_2 \in I$ with $x_1<x_2$, then $f(x)$ is said to be monotone decreasing on the interval $I$ (Figure 1-6b).

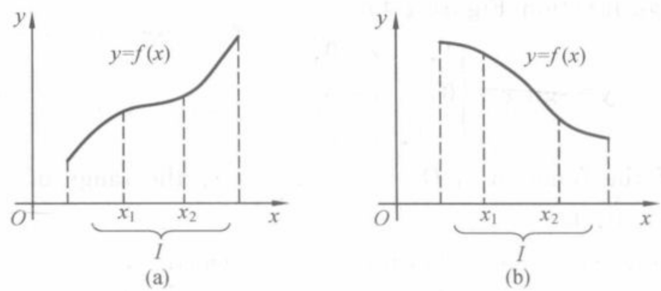


**Figure 1-6**

Monotone increasing functions and monotone decreasing functions are called by a joint name monotone function, $I$ is called monotone interval. It should be noted that the monotonicity of the function is related to the range of independent variable. For example, $y=x^2$ is monotone decreasing on the interval $(-\infty, 0]$ and monotone increasing on the interval $(0, +\infty]$. However, $y=x^2$ is not monotonous in the interval $(-\infty, +\infty)$.

(2) Even (odd) functions

Suppose the domain of function $y=f(x)$ is an interval $D$ that is symmetric about the origin. If $f(-x)=f(x)$ for any $x \in D$, $y=f(x)$ is called an even function; If $f(-x)=-f(x)$ for any $x \in D$, $y=f(x)$ is called an odd function.

The graph of an even function is symmetric about the $y$-axis, and the graph of an odd function is symmetric about the origin (Figure 1-7).

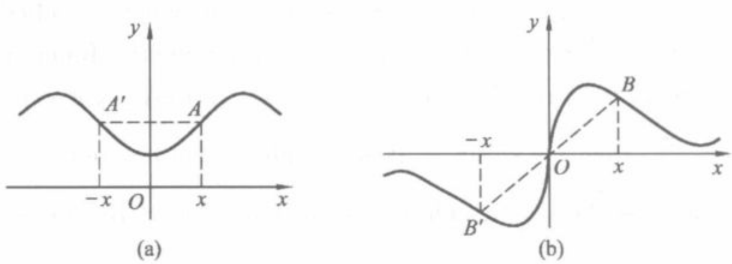


**Figure 1-7**

For example, both $y=x^2$ and $y=\cos x$ are even functions in the interval $(-\infty, +\infty)$. $y=x^3$ and $y=\sin x$ are odd functions in the interval $(-\infty, +\infty)$.

(3) Bounded functions

Let the domain of function $y=f(x)$ is $D(f)$ and interval $I \subset D(f)$. If there exists a constant $M$ such that