

CHINESE MAINLAND EDITION

David J. Hunter

# Essentials of Discrete Mathematics

Third Edition

## 离散数学概要 第3版

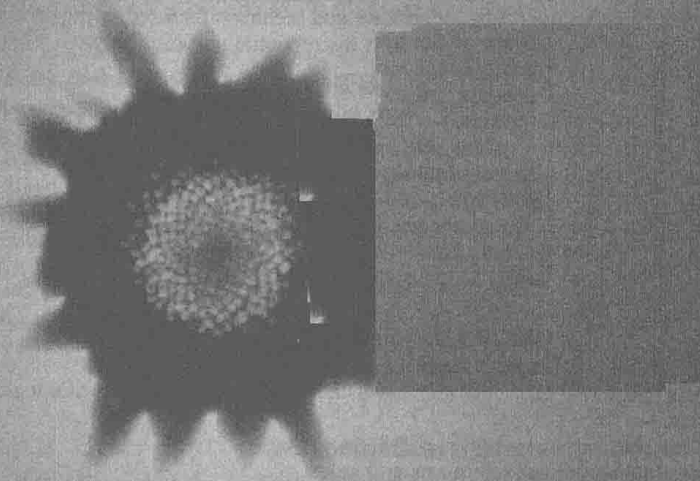


ESSENTIALS  
*of*  
DISCRETE  
MATHEMATICS

— *Third Edition* —

DAVID J. HUNTER

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# Preface

## Introduction

*Essentials of Discrete Mathematics* is designed for first- or second-year computer science and math majors in discrete mathematics courses. This text is also an excellent resource for students in other disciplines.

Unlike most other textbooks on the market, *Essentials* presents the material in a manner that is suitable for a comprehensive and cohesive one-semester course. The text is organized around five types of mathematical thinking: logical, relational, recursive, quantitative, and analytical. To reinforce this approach, graphs are introduced early and referred to throughout the text, providing a richer context for examples and applications.

Applications appear throughout the text, and the final chapter explores several uses of discrete mathematical thinking in a variety of disciplines. Case studies from biology, sociology, linguistics, economics, and music can be used as the basis for independent study or undergraduate research projects. Every section has its own set of exercises, which are designed to develop the skills of reading and writing proofs.

## Synopsis of the Chapters

Chapter 1 introduces the reader to **Logical Thinking**. The chapter explores logic formally (symbolically) and then teaches the student to consider how logic is used in mathematical statements and arguments. The chapter begins with an introduction to formal logic, focusing on the importance of notation and symbols in mathematics, and then explains how formal logic can be applied. The chapter closes with a look at the different ways that proofs are constructed in mathematics.

As most mathematical problems contain different objects that are related to each other, Chapter 2 considers **Relational Thinking**. Finding the relationships among objects often is the first step in solving a mathematical problem. The mathematical structures of sets, relations, functions, and graphs describe these relationships, and thus this chapter focuses on exploring ways to use these structures to model mathematical relationships. Graph theory is introduced early and used throughout the chapter.

Chapter 3 describes **Recursive Thinking**. Many objects in nature have recursive structures: a branch of a tree looks like a smaller tree; an ocean swell has the same shape as the ripples formed by its splashes; an onion holds

a smaller onion under its outer layer. Finding similar traits in mathematical objects unleashes a powerful tool. Chapter 3 begins by studying simple recurrence relations and then considers other recursive structures in a variety of contexts. Students also will cover recursive definitions, including how to write their own, and will extend the technique of induction to prove facts about recursively defined objects.

Chapter 4 engages the reader in **Quantitative Thinking**, as many problems in mathematics, computer science, and other disciplines involve counting the elements of a set of objects. The chapter examines the different tools used to count certain types of sets and teaches students to think about problems from a quantitative point of view. After exploring the different enumeration techniques, students will consider applications, including a first look at how to count operations in an algorithm. Chapter 4 also will practice the art of estimation, a valuable skill when precise enumeration is difficult.

Chapter 5 explores **Analytical Thinking**. Many applications of discrete mathematics use algorithms, and thus it is essential to be able to understand and analyze them. This chapter builds on the four foundations of thinking covered in the first four chapters, applying quantitative and relational thinking to the study of algorithm complexity, and then applying logical and recursive thinking to the study of program correctness. Finally, students will study mathematical ways to determine the accuracy and efficiency of algorithms.

The final chapter, **Thinking Through Applications**, examines different ways that discrete mathematical thinking can be applied: patterns in DNA, social networks, the structure of language, population models, and twelve-tone music.

## What's New in the Third Edition

The third edition incorporates several improvements and additions designed to make teaching a course from this text more rewarding and effective. Thanks to helpful feedback from faculty and students, incremental changes in clarity and accuracy have been made, resulting in a more polished product. The early introduction to graph theory has been expanded, allowing deeper integration with later topics such as functions and relations. Perhaps most significantly, each section now opens with a set of inquiry problems designed to motivate the upcoming material. These open-ended problems can be assigned as independent work prior to a class meeting or as group work to engage students in class discussion. Unlike the end-of-section exercises, these inquiry problems often have a variety of plausible answers, prompting students to construct their own notation and conjectures. The instructor's version of the *Inquiry Problems* supplement contains pedagogical notes on each set of section-opener problems, as well as over 150 additional problems that can be used in a variety of ways.

## About the Cover

The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . . is a famous example of recursion; each number in this sequence is the sum of the two numbers that precede it. The *Asteraceae* specimen pictured on the front cover of this book exhibits 21:34 Fibonacci phyllotaxis: there are 21 yellow spirals in one direction and 34 in the other. This flower continues the theme of the cover illustrations for the first and second editions of this text: an 8:13 pine cone and a 13:21 *Santolina* flower, respectively. You can read more about Fibonacci numbers in Section 3.1.2.

## Supplements

- *Instructor's Solutions Manual*
- *Lecture slides in PowerPoint format*
- *WebAssign™*, developed by instructors for instructors, is a premier independent online teaching and learning environment, guiding several million students through their academic careers since 1997. With WebAssign, instructors can create and distribute algorithmic assignments using questions specific to this textbook. Instructors can also grade, record, and analyze student responses and performance instantly; offer more practice exercises, quizzes, and homework; and upload additional resources to share and communicate with their students seamlessly, such as the lecture slides in PowerPoint format supplied by Jones & Bartlett Learning.
- *eBook format*. As an added convenience, this complete textbook is now available in eBook format for purchase by the student through WebAssign.
- *Inquiry Problems*. Available in both student and instructor versions, this sequence of problems is designed to lead students through the main topics of the book using inquiry-based learning.

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Finally, I am deeply thankful to my wife Patti, whose support, patience, and encouragement sustained me throughout this project.

*David Hunter*  
*Westmont College*

# How to Use This Book

This book is designed to present a coherent one-semester course in the essentials of discrete mathematics for several different audiences. Figure 1 shows a diagram describing the dependencies among the sections of this book. Regardless of audience, a course should cover the sections labeled “Core” in the diagram: 1.1–1.5, 2.1–2.4, 3.1–3.4, 4.1–4.3, 4.5, and 5.1.

Beyond these 18 core sections, instructors have many options for additional sections to include, depending on the audience. A one-semester course

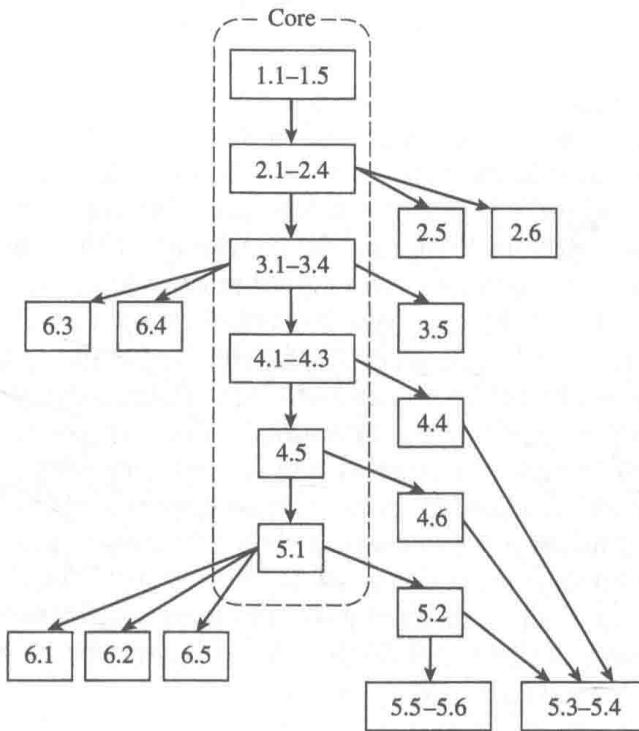


Figure 1 Dependencies among sections of this book.

| Computer Science Emphasis | Mathematics Emphasis | Interdisciplinary |
|---------------------------|----------------------|-------------------|
| 1.1–1.5                   | 1.1–1.5              | 1.1–1.5           |
| 2.1–2.4                   | 2.1–2.6              | 2.1–2.4           |
| 3.1–3.5                   | 3.1–3.4              | 3.1–3.4           |
| 4.1–4.6                   | 4.1–4.5              | 4.1–4.3, 4.5      |
| 5.1–5.2                   | 5.1                  | 5.1               |
| 5.3–5.4 and/or 5.5–5.6    | 6.3, 6.4, 6.5        | 6.1–6.5           |

**Table 1** Three possible course outlines.

should be able to cover approximately 5–8 additional sections. Table 1 shows three possible course outlines, each with a different focus.

Some subsections in the core (3.2.2, 4.2.3, and 4.3.4) have been marked with a double-dagger symbol (‡) to indicate that these subsections may safely be omitted without disrupting the continuity of the material. Answers and hints to selected problems can be found at the end of the book. Exercises requiring extra effort or insight have been marked with an asterisk (\*).

At the beginning of each section, you will find some questions labeled “Inquiry.” These problems introduce and motivate the material in the section that follows. As such, they can be attempted before the corresponding material is presented in class. Instructors may elect to use these problems in a variety of ways: as group exercises, as independent student investigations, or as the basis for student presentations in class. Often these problems are open ended and can inspire class discussions. The *Inquiry Problems* supplement to this text contains all of these inquiry problems and many more, and the instructor’s version of this supplement includes several notes with additional pedagogical suggestions.

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