

钱学森

力学手稿

Application of Tschupig's Transformator
Two Dimensional Flow

10

钱学森

The equations of two dimensional motion of compressible fluids without rotation and assuming that the pressure is only a function of density, can be reduced to a single non-linear equation of the velocity potential. In the supersonic case the problem is solved by Prandtl, Meyer and Busemann by means of the powerful method of characteristics (3). The essential difficulty of the problem in the subsonic case, especially when the velocity is near to the velocity of sound, is to find a function $\phi(x, y)$ which satisfies the equation $\nabla^2 \phi = \frac{E_0}{\rho} F(\rho)$ where E_0 is a constant and $F(\rho)$ is a function of density. This is a non-linear equation and the disturbance surface must be referred to zero as $E_0 \rightarrow 0$. The equation is sufficiently complicated that when the velocity of disturbance is small and high an example of this may be multiplied by $\frac{E_0}{\rho}$ and the presence of stagnation point in the use of the surface makes the application of the described theory very questionable, at least near this region, because when the

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出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长,学习他的科学思想和卓越风范,展示其丰功伟绩和人格魅力,西安交通大学举办了“纪念钱学森诞辰100周年”系列活动:作为制片方之一,参与西部电影集团摄制传记故事片《钱学森》;与中央电视台合作,出品纪录片《实验班的故事——沿着钱学森走过的路》;扩建钱学森生平业绩展馆,向校内外开放;举办钱学森科学与教育思想研讨会;出版发行《钱学森力学手稿》、《钱学森年谱(初编)》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿,这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风,是钱老勇攀科学高峰和严谨治学的集中体现。这里,我们将部分原稿整理汇集成册,出版《钱学森力学手稿》,作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷,包含两部分内容。第一部分是草稿,包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时,为了求得圆柱壳体的临界压力,在有关的五百多页草稿中,对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算,最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表,每个数字有效位数都长达八位,在手摇机械式计算机作为主要计算工具的年代,这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿,以航空航天工程为核心,涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下,由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助,在此深表感谢。

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Section 1

Interorbital Acceleration

$$\zeta(\bullet) = \sqrt{gm}$$

1

Intersatellite Acceleration

Let g = gravitational acceleration at starting radius r_0 .
 $(\cdot)_0$ = initial

$$\frac{d^2 r}{dt^2} = R + r \left(\frac{d\theta}{dt} \right)^2 - g \left(\frac{r_0}{r} \right)^2$$

$$r \frac{d^2 \theta}{dt^2} = 0 - 2 \left(\frac{dr}{dt} \right) \left(\frac{d\theta}{dt} \right)$$

1) Case $\theta = 0$, radial thrust, $R = \text{constant}$

$$r \frac{d^2 \theta}{dt^2} + 2 \left(\frac{dr}{dt} \right) \left(\frac{d\theta}{dt} \right) = 0$$

$$\frac{d}{dt} \left(r^2 \frac{d\theta}{dt} \right) = 0$$

$$r^2 \frac{d\theta}{dt} = r_0^2 \left(\frac{d\theta}{dt} \right)_0$$

$$\frac{d\theta}{dt} = \left(\frac{r_0}{r} \right)^2 \left(\frac{d\theta}{dt} \right)_0$$

$$\frac{d^2 r}{dt^2} = R + \frac{r_0^4 \left(\frac{d\theta}{dt} \right)_0^2}{r^3} - \frac{g r_0^2}{r^2}$$

If the start is made from a satellite,

$$r_0 \left(\frac{d\theta}{dt} \right)_0^2 = g$$

$$\frac{d^2 r}{dt^2} = R + \frac{g r_0^3}{r^3} - \frac{g r_0^2}{r^2}$$

$$\int_0^t M \Theta dt = M \int_0^t \Theta dt - \frac{dM}{dt} \int_0^t \Theta dt$$

$$\frac{dr}{dt} = v_r$$

$$\frac{1}{2} \frac{dv_r^2}{dr} = R + g r_0^3 \left\{ \frac{r_0}{r^3} - \frac{1}{r^2} \right\}$$

Since $v_r = 0$ at $r = r_0$, $t = 0$,

$$\frac{1}{2} v_r^2 = R(r - r_0) + g r_0^2 \left\{ \frac{1}{2} \frac{1}{r_0} - \frac{1}{2} \frac{r_0}{r^2} + \frac{1}{r} - \frac{1}{r_0} \right\}$$

$$\boxed{v_r^2 = 2R(r - r_0) + g r_0^2 \left\{ \frac{2}{r} - \frac{r_0}{r^2} - \frac{1}{r_0} \right\}}$$

Total energy at r

$$E = \left\{ \frac{1}{2} \left[v_r^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \right] - \frac{g r_0^2}{r} \right\} M$$

$$= \left\{ \frac{1}{2} \left[v_r^2 + \frac{1}{r^2} r_0^3 g \right] - \frac{g r_0^2}{r} \right\} M$$

$$= \left\{ \frac{1}{2} \left[2R(r - r_0) + g r_0^2 \left(\frac{2}{r} - \frac{1}{r_0} \right) \right] - \frac{g r_0^2}{r} \right\} M$$

$$= \left\{ R(r - r_0) - \frac{1}{2} g r_0 \right\} M$$

$$\Delta E = \left\{ R(r - r_0) - \frac{1}{2} g r_0 \right\} M - \left\{ \frac{1}{2} g r_0 - g r_0 \right\} M_0$$

$$= MR(r - r_0) + \frac{1}{2} g r_0 (M_0 - M)$$

Huust $RM = -c \frac{dM}{dt}$

$$\frac{dr}{dt} = \left\{ 2Rr - (2Rr_0 + g r_0) + 2g r_0^2 \frac{1}{r} - g r_0^3 \frac{1}{r^2} \right\}^{1/2}$$

$$dt = \frac{dr}{\sqrt{2Rr - (2Rr_0 + g r_0) + 2g r_0^2 \frac{1}{r} - g r_0^3 \frac{1}{r^2}}}$$

$$dt = -\frac{c}{R} \frac{dM}{M}$$

$$t = \frac{c}{R} \log_e \frac{M_0}{M}$$

$$R = ng$$

$$t = \int_{r_0}^r \frac{dr}{\sqrt{2Rr - (2Rr_0 + g r_0) + g r_0^2 \frac{1}{r} - g r_0^2 \frac{1}{r^2}}}$$

$$= \frac{1}{ng} \int_{r_0}^r \frac{r dr}{\sqrt{2nr^3 - (2nr_0 + r_0)r^2 + 2r_0^2 r - r_0^3}}$$

$$= \frac{1}{ng} \int_{r_0}^r \frac{r dr}{\sqrt{r-r_0} \sqrt{2nr^2 - r_0 r + r_0^2}}$$

$$2nr^2 - r_0 r + r_0^2 > 0.$$

$$2nr^2 > r_0(r - r_0)$$

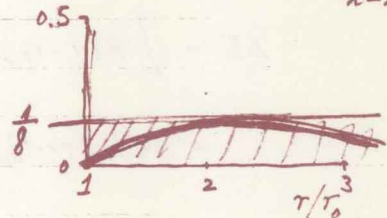
$$2n > \frac{r_0}{r} - \frac{r_0^2}{r^2}$$

$$n > \frac{1}{2} \frac{r_0}{r} \left(1 - \frac{r_0}{r}\right)$$

$$\frac{1}{x} \left(1 - \frac{1}{x}\right)$$

$$-\frac{1}{x^2} + 2\frac{1}{x^3} = 0$$

$$x = 2$$



$$R(r - r_0) - \frac{1}{2} g r_0 = 0$$

$$Rr = (R + \frac{1}{2} g) r_0$$

$$r = \left(1 + \frac{1}{2n}\right) r_0$$

4

$$\eta = 1 + \xi$$

$$t \sqrt{\frac{2}{r_0}} = \int_1^{1+\frac{1}{2n}} \frac{\eta d\eta}{\sqrt{\eta-1} \sqrt{2n\eta^2 - \eta + 1}}$$

$$= \int_0^{\frac{1}{2n}} \frac{(1+\xi) d\xi}{\sqrt{\xi} \sqrt{2n\xi^2 + 4n\xi + 2n - 1 - \xi + 1}}$$

$$= \int_0^{\frac{1}{2n}} \frac{(1+\xi) d\xi}{\sqrt{\xi} \sqrt{2n\xi^2 + (4n-1)\xi + 2n}}$$

$$= \int_0^{\frac{1}{2n}} \frac{d\xi}{\xi^{\frac{1}{2}} [2n\xi^2 + (4n-1)\xi + 2n]^{\frac{1}{2}}} + \int_0^{\frac{1}{2n}} \frac{\xi^{\frac{1}{2}} d\xi}{[2n\xi^2 + (4n-1)\xi + 2n]^{\frac{1}{2}}}$$

$$\int_0^t R dt = \int_{2n-1}^{2n\xi^2 + (4n-1)\xi + 2n} \frac{2n\xi^2 + (4n-1)\xi + 2n}{2n-1}$$

$$\frac{2\xi^{\frac{1}{2}}}{\sqrt{\frac{1}{2}}} + \int \frac{\xi^{\frac{1}{2}} \cdot [4n\xi + (4n-1)] d\xi}{\sqrt{\frac{1}{2}}}$$

s

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24
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$$\int_0^t M \Theta dt = M \int_0^t \Theta dt + \int_0^t \frac{dM}{dt} dt \int_0^t \Theta dt'$$

$$-\frac{dM}{dt} = \frac{G}{c} M$$

$$M \int_0^t \Theta dt + \frac{1}{c} \int_0^t (\Theta dt \int_0^t \Theta dt')$$

$$\frac{1}{c} \int \frac{\log r}{r} dr$$

$$= \frac{1}{2} (\log r)^2$$

$$\begin{aligned} 1 - \varepsilon + \varepsilon^2 \\ 1 - 2\varepsilon + 3\varepsilon^2 \end{aligned}$$

$$\frac{1}{2} \frac{dv_r^2}{dr} = g \frac{r_0^3}{r^3} - g \frac{r_0^2}{r^2}$$

$$\frac{1}{2} v_r^2 = g r_0^3 \left(\frac{1}{r_0^2} - \frac{1}{r^2} \right) - g r_0^2 \left(\frac{1}{r_0} - \frac{1}{r} \right)$$

$$= g r_0 \left[\frac{1}{2} \left(1 - \frac{r_0^2}{r^2} \right) - \left(1 - \frac{r_0}{r} \right) \right]$$

$$\left(1 - \frac{r_0}{r} \right) \left[\frac{1}{2} + \frac{1}{2} \frac{r_0}{r} - \frac{1}{2} \frac{r_0^2}{r^2} - 1 + \frac{r_0}{r} \right]$$

$$\frac{1}{2} - \frac{1}{2} \frac{r_0^2}{r^2} - 1 + \frac{r_0}{r}$$

$$-\frac{1}{2} \left(1 - \frac{r_0}{r} \right)^2$$

2) Case R=0

$$\frac{d}{dt} \left(r^2 \frac{d\ell}{dt} \right) = \Theta r$$

$$r^2 \frac{d\ell}{dt} = r_0^2 \left(\frac{d\ell}{dt} \right)_0 + \int_0^t \Theta r dt$$

$$\int_0^r \frac{\Theta r}{r^2} dr$$

$$\frac{d^2 r}{dt^2} = \frac{1}{r^3} \left\{ r_0^2 \left(\frac{d\ell}{dt} \right)_0 + \int_0^t \Theta r dt \right\}^2 - g \frac{r_0^2}{r^2}$$

$$\frac{d^2 r}{dt^2} = g \frac{r_0^2}{r^2} \left(\frac{r_0}{r} - 1 \right) + 2 \frac{r_0^2 \left(\frac{d\ell}{dt} \right)_0}{r^3} \int_0^t \Theta r dt + \frac{1}{r^3} \left[\int_0^t \Theta r dt \right]^2$$

$$\text{At } t=0, \quad \frac{d^2 r}{dt^2} = 0, \quad r = r_0, \quad \frac{dr}{dt} = 0.$$

$$\left(r^3 \frac{d^2 r}{dt^2} + g r_0^2 r \right)^{\frac{1}{2}} = r_0^2 \left(\frac{d\ell}{dt} \right)_0 + \int_0^t \Theta r dt$$

$$\Theta r = \frac{1}{2} \frac{3r^2 \frac{dr}{dt} \frac{d^2 r}{dt^2} + r^3 \frac{d^3 r}{dt^3} + g r_0^2 \frac{dr}{dt}}{\left(r^3 \frac{d^2 r}{dt^2} + g r_0^2 r \right)^{\frac{1}{2}}}$$

$$\Theta r = \frac{1}{2} \frac{\frac{1}{2} 3r^2 \frac{dv_r^2}{dr} + \frac{1}{2} r^3 \frac{d^2 v_r^2}{dr^2} + g r_0^2}{\left(r^3 \frac{1}{2} \frac{dv_r^2}{dr} + g r_0^2 r \right)^{\frac{1}{2}}} v_r$$

$$r = \frac{\alpha t^3}{1 + \beta t^2} + r_0$$

$$v_r \frac{d}{dr} \left(r^3 \frac{1}{2} \frac{dv_r^2}{dr} \right)$$

$$mg r_0 \eta = \frac{1}{2} \frac{\left(r_0^4 / T^3 \right) \left[3 \eta^2 \frac{d\eta}{dt} \frac{d^2 \eta}{dt^2} + \eta^3 \frac{d^3 \eta}{dt^3} \right] + g \frac{r_0^3}{T} \frac{d\eta}{dt}}{\left[\frac{r_0^4}{T^2} \eta^3 \frac{d\eta}{dt^2} + g r_0^3 \eta \right]^{\frac{1}{2}}}$$

$$4\alpha - 3\beta = \alpha - \beta$$

$$3\alpha = 2\beta$$

$$\beta = 1.5\alpha$$

$$\frac{r_0^4}{T^3} = \frac{g r_0^3}{T}$$

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$$m g r_0 \gamma = \frac{1}{2} \frac{r_0^2}{T^2} \frac{x}{\lambda^2}$$

$$\frac{3\gamma^2 \frac{d\gamma}{dt} \frac{d^2\gamma}{dt^2} + \gamma^3 \frac{d^3\gamma}{dt^3} + \frac{d\gamma}{dt}}{[\gamma^3 \frac{d^2\gamma}{dt^2} + \gamma]^{\frac{1}{2}}}$$

$$4\alpha - 2\beta = \alpha$$

$$1.5\alpha + 1 = \alpha - \beta$$

$$= -0.5\alpha$$

$$m g \frac{T^2}{r_0^{\frac{1}{2}}}$$

$$\frac{r_0}{T^2} = f$$

$$T^2 = \frac{r_0}{f}$$

$$T^2 = \frac{1}{f}$$

$$\gamma = \frac{1}{\sqrt{m}}$$

$$T = \frac{1}{\sqrt{m}}$$

$$2\alpha + 1 = 0$$

$$\alpha = -\frac{1}{2}$$

$$2m\gamma \left[\gamma^3 \frac{d^2\gamma}{dt^2} + \gamma \right]^{\frac{1}{2}} = 3\gamma^2 \frac{d\gamma}{dt} \frac{d^2\gamma}{dt^2} + \gamma^3 \frac{d^3\gamma}{dt^3} + \frac{d\gamma}{dt}$$

$$\frac{1}{2} - \frac{1}{2x^2} - 1 + \frac{1}{x}$$

$$\frac{1}{2} - \frac{1}{2x^2} + \frac{1}{x}$$

$$-\frac{1}{2} \left(1 - \frac{2}{x} + \frac{1}{x^2} \right)$$

$$\left(1 - \frac{1}{x} \right)^2$$

$$\int_0^t \Theta dt = F$$

$$\int_0^t \Theta r dt = Fr - \int_{r_0}^r F dr$$

$$2m = \frac{d^3\gamma}{dt^3}$$

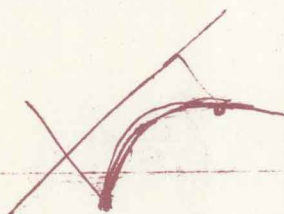
$$\frac{d^3\gamma}{dt^3} = \frac{d}{dt} \frac{d}{d\gamma} \left(\frac{1}{2} \frac{dv^2}{d\gamma} \right)$$

$$\left[r^3 \frac{1}{x} \frac{dx}{dr} + \frac{m}{x} \right]^{\frac{1}{2}} =$$

$$\gamma = 1 + \frac{2m}{6} t^3 + a_4 t^4 + a_5 t^5 + \dots$$

$$\frac{d\gamma}{dt} = \frac{2m}{2} t^2$$

$$\frac{d^2\gamma}{dt^2} = 2mt$$



$$\frac{-2 \pm 3(-4)}{2 \cdot 3}$$

$$1 - \varepsilon$$

$$-1 + \varepsilon - \varepsilon^2$$

$$-g r_0 (\varepsilon - \varepsilon^2)$$

$$-g r_0 \left(1 - \frac{r_0}{r}\right)$$

$$+ \frac{1}{2} g r_0 \left(1 - \frac{r_0^2}{r^2}\right)$$

$$+ g r_0 \left[\varepsilon - \frac{3}{2} \varepsilon^2\right]$$

$$- g r_0 \frac{1}{2} \varepsilon^2$$

$$K^2 \frac{1}{r} + [2K - 1 + 2K^2] \frac{1}{r} + (K-1)^2 \frac{1}{r} = 0$$

$$2K - 1 - 2K + 1 = 0$$

$$\frac{1}{2} v_r^2 = g r_0 \left[K^2 \log \frac{r}{r_0} + [2K(1-K) - 1] \left(1 - \frac{r_0}{r}\right) + \frac{1}{2} (K-1)^2 \left(1 - \frac{r_0^2}{r^2}\right) \right]$$

$$(1 + \varepsilon)^{-2} = 1 - 2\varepsilon + 3\varepsilon^2$$

$$2K^2 - 2K + 1 = 0$$

$$K^2 - K + \frac{1}{2} = 0$$

$$K = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{2}}$$

$$\int_0^t \Theta r dt = \int_0^r \frac{\Theta r}{v_r} dr$$

$$\text{Let } \frac{\Theta r}{v_r} = K \sqrt{g r_0}$$

$$\int_0^t \Theta r dt = K \sqrt{g r_0} (r - r_0)$$

$$\begin{aligned} \frac{d^2 r}{dt^2} = \frac{1}{2} \frac{dv_r^2}{dr} &= \frac{g r_0^2}{r^3} (r_0 - r) + 2 \frac{K^2 g r_0^2}{r^3} (r - r_0) \\ &\quad + \frac{K^2 g r_0}{r^3} (r - r_0)^2 \end{aligned}$$

$$= g r_0^2 (2K - 1) \frac{r - r_0}{r^3} + \frac{K^2 g r_0}{r^3} (r^2 - 2r r_0 + r_0^2)$$

$$= \frac{K^2 g r_0}{r} + [g r_0^2 (2K - 1) - 2 K^2 g r_0^2] \frac{1}{r^2} + [K^2 g r_0^3 - g r_0^3 (2K - 1)] \frac{1}{r^3}$$

$$2K - 2K^2 - 1 \quad (K - 1)^2$$

$$2K(1 - K) - 1$$

$$- [1 + 2K(K - 1)]$$

$$\begin{aligned} \frac{1}{2} v_r^2 &= K^2 g r_0 \log \frac{r}{r_0} + g r_0^2 [2K - 1 - 2K^2] \left[\frac{1}{r_0} - \frac{1}{r} \right] \\ &\quad + \frac{1}{2} (K - 1)^2 g r_0^3 \left[\frac{1}{r_0^2} - \frac{1}{r^2} \right] \end{aligned}$$

$$\begin{aligned} \int_0^t \Theta dt &= \int_0^r \frac{\Theta}{v_r} dr = \int_0^r \frac{dr}{r} K \sqrt{g r_0} \\ &= K \sqrt{g r_0} \log \frac{r}{r_0} \end{aligned}$$

f

$$\frac{1}{2} r^2 \left(\frac{dr}{dt} \right)^2 = \frac{1}{2} \frac{1}{r^2} \left[r_0^2 \left(\frac{dr}{dt} \right)_0 + K \sqrt{g r_0} (r - r_0) \right]^2$$

$$= \frac{1}{2} \frac{1}{r^2} \left[r_0 \sqrt{g r_0} + K \sqrt{g r_0} (r - r_0) \right]^2$$

$$= \frac{1}{2} \frac{r_0^2}{r^2} g r_0 \left[1 + K \left(\frac{r}{r_0} - 1 \right) \right]^2$$

$$K\varepsilon = \frac{1}{2}$$

$$= \frac{1}{2} \frac{r_0^2}{r^2} g r_0 \left[(1-K) + K \frac{r}{r_0} \right]^2$$

$$= \frac{1}{2} \frac{r_0^2}{r^2} g r_0 \left[(1-K)^2 + 2K(1-K) \frac{r}{r_0} + K^2 \frac{r^2}{r_0^2} \right]$$

$$\frac{1}{2} \left[v_r^2 + r^2 \left(\frac{dr}{dt} \right)^2 \right]$$

$$K\varepsilon - \frac{1}{2} = 0$$

$$= \frac{1}{2} (1-K)^2 g r_0 + K^2 g r_0 \log \left(\frac{r}{r_0} \right) + g r_0 [2K(1-K) - 1]$$

$$+ \frac{1}{2} K^2 g r_0 + g r_0^2 \frac{1}{r} \left[-\frac{1}{2} K + 1 + \frac{1}{2} K^2 \right]$$

$$+ K - K^2$$

$$\frac{r_0}{r} = 1 - \varepsilon$$

$$= \left(\frac{1}{2} K^2 - K + \frac{1}{2} \right) g r_0 + K^2 g r_0 \log \left(\frac{r}{r_0} \right) -$$

$$E = \left(K - K^2 - \frac{1}{2} \right) g r_0 + K^2 g r_0 \log \left(\frac{r}{r_0} \right) + \frac{K(K-1)}{K(K-1)} g r_0 \left(\frac{r_0}{r} \right)$$

$$= 0$$

$$- \frac{g r_0^2}{r}$$

$$K$$

$$K - \frac{1}{2} + K^2 \varepsilon + (K^2 - K)$$

$$- (K^2 - K) \varepsilon = 0$$