

钱学森

力学手稿

6

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西安交通大学出版社

XIAN JIAOTONG UNIVERSITY PRESS

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出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长,学习他的科学思想和卓越风范,展示其丰功伟绩和人格魅力,西安交通大学举办了“纪念钱学森诞辰100周年”系列活动:作为制片方之一,参与西部电影集团摄制传记故事片《钱学森》;与中央电视台合作,出品纪录片《实验班的故事——沿着钱学森走过的路》;扩建钱学森生平业绩展馆,向校内外开放;举办钱学森科学与教育思想研讨会;出版发行《钱学森力学手稿》、《钱学森年谱(初编)》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿,这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风,是钱老勇攀科学高峰和严谨治学的集中体现。这里,我们将部分原稿整理汇集成册,出版《钱学森力学手稿》,作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷,包含两部分内容。第一部分是草稿,包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时,为了求得圆柱壳体的临界压力,在有关的五百多页草稿中,对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算,最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表,每个数字有效位数都长达八位,在手摇机械式计算机作为主要计算工具的年代,这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿,以航空航天工程为核心,涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下,由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助,在此深表感谢。

*Preliminary Calculation of
Circular Cylinder (V)*

$$\frac{w}{R} = (f_0 + \frac{1}{4}f_1) + \frac{1}{2}f_1 \left[\cos \frac{mx}{R} \cos \frac{my}{R} + \frac{1}{4} \cos \frac{2mx}{R} + \frac{1}{4} \cos \frac{2my}{R} \right] \\ + \frac{1}{4}f_2 \left[\cos \frac{2mx}{R} + \cos \frac{2my}{R} \right]$$

$$\frac{\partial w}{\partial x} = (f_0 + \frac{1}{4}f_1) + \frac{1}{2}f_1 \cos \frac{mx}{R} \cos \frac{my}{R} + \frac{1}{4}(\frac{1}{2}f_1 + f_2) \cos \frac{2mx}{R} + \frac{1}{4}(\frac{1}{2}f_1 + f_2) \cos \frac{2my}{R}$$

$$\frac{\partial w}{\partial y} = -m \left[\frac{1}{2}f_1 \cos \frac{mx}{R} \sin \frac{my}{R} + \frac{1}{2}(\frac{1}{2}f_1 + f_2) \sin \frac{2my}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2 \left[\frac{1}{2}f_1 \cos \frac{mx}{R} \cos \frac{my}{R} + (\frac{1}{2}f_1 + f_2) \cos \frac{2mx}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2 \left[\frac{1}{2}f_1 \cos \frac{mx}{R} \cos \frac{my}{R} + (\frac{1}{2}f_1 + f_2) \cos \frac{2my}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x \partial y} = +\left(\frac{m}{R}\right)^2 \left[\frac{1}{2}f_1 \sin \frac{mx}{R} \sin \frac{my}{R} \right]$$

$$\Delta SF = E \left(\frac{m}{R}\right)^2 \left[m^2 \left\{ -\frac{1}{8}f_1^2 \left(\cos \frac{2mx}{R} + \cos \frac{2my}{R} \right) - \frac{1}{4}f_1 \left(\frac{1}{2}f_1 + f_2 \right) \frac{\left(\cos \frac{my}{R} + \cos \frac{3my}{R} \right)}{\cos \frac{mx}{R}} \right\} \right. \\ \left. - \frac{1}{4}f_1 \left(\frac{1}{2}f_1 + f_2 \right) \cos \frac{mx}{R} \left(\cos \frac{my}{R} + \cos \frac{3my}{R} \right) - \left(\frac{1}{2}f_1 + f_2 \right)^2 \cos \frac{2mx}{R} \cos \frac{2my}{R} \right]$$

$$+ \frac{1}{2}f_1 \cos \frac{mx}{R} \cos \frac{my}{R} + (\frac{1}{2}f_1 + f_2) \cos \frac{2mx}{R} \right]$$

$$= E \left(\frac{m}{R}\right)^2 \left[- \left\{ \frac{1}{8}f_1^2 m^2 - \left(g - \frac{1}{2}f_1 \right) \right\} \cos \frac{2mx}{R} - \frac{1}{8}f_1 m^2 \cos \frac{2my}{R} \right.$$

$$- \left\{ \frac{1}{2}f_1 \left(g - \frac{1}{2}f_1 \right) m^2 - \frac{1}{2}f_1 \right\} \cos \frac{mx}{R} \cos \frac{my}{R} - \frac{1}{4}f_1 \left(g - \frac{1}{2}f_1 \right) m \cos \frac{3mx}{R} \cos \frac{3my}{R}$$

$$- \frac{1}{4}f_1 \left(g - \frac{1}{2}f_1 \right) m \cos \frac{mx}{R} \cos \frac{3my}{R} - \left(g - \frac{1}{2}f_1 \right) m^2 \cos \frac{2mx}{R} \cos \frac{2my}{R} \right]$$

$$F = E \left(\frac{h}{m} \right)^2 \left[-\frac{1}{16} \left\{ \left(\frac{f}{g} f_1^2 m^2 - (g - \frac{f}{g} f_1) \right) m^2 \cos \frac{2\pi k}{R} - \frac{1}{12} f f_1^2 m^2 \cos \frac{2\pi k}{R} - \frac{1}{4} \left(\frac{f}{g} f_1 (g - \frac{f}{g} f_1) \right) m^2 \cos \frac{2\pi k}{R} \cos \frac{2\pi k}{R} \right\} \right.$$

$$\left. - \frac{1}{400} f f_1 (g - \frac{f}{g} f_1) m^2 \cos \frac{3\pi k}{R} \cos \frac{3\pi k}{R} - \frac{1}{400} f f_1 (g - \frac{f}{g} f_1) m^2 \cos \frac{\pi k}{R} \cos \frac{3\pi k}{R} - \frac{1}{400} f f_1 (g - \frac{f}{g} f_1) m^2 \cos \frac{2\pi k}{R} \cos \frac{2\pi k}{R} \right]$$

$$G_x + G_y = E \left[\frac{1}{4} \left\{ \left(\frac{f}{g} f_1^2 m^2 - (g - \frac{f}{g} f_1) \right) \left\{ m \frac{2\pi k}{R} + \frac{1}{32} f f_1^2 m^2 \cos \frac{2\pi k}{R} + \frac{1}{2} \left(\frac{f}{g} f_1 (g - \frac{f}{g} f_1) \right) m^2 \cos \frac{2\pi k}{R} \cos \frac{2\pi k}{R} \right\} \right. \right.$$

$$\left. \left. + \frac{1}{40} f f_1 (g - \frac{f}{g} f_1) m^2 \cos \frac{3\pi k}{R} \cos \frac{3\pi k}{R} + \frac{1}{40} f f_1 (g - \frac{f}{g} f_1) m^2 \cos \frac{\pi k}{R} \cos \frac{3\pi k}{R} + \frac{1}{40} f f_1 (g - \frac{f}{g} f_1) m^2 \cos \frac{2\pi k}{R} \cos \frac{2\pi k}{R} \right\} \right]$$

$$\lambda + \sqrt{\frac{C}{E}} - \frac{1}{2} m^2 \left[\frac{1}{16} f_1^2 + \frac{1}{g} (g - \frac{f}{g} f_1)^2 \right] + (f_0 + \frac{f}{g} f_1) = 0$$

$$K = -4 \left(\frac{C}{E} \right)^2 - m^2 \frac{C}{E} \left[\frac{1}{16} f_1^2 + \frac{1}{g} (g - \frac{f}{g} f_1)^2 \right]$$

$$O_1 = \frac{1}{g} \left\{ \left(\frac{f}{g} f_1^2 m^2 - (g - \frac{f}{g} f_1) \right)^2 + \frac{1}{512} f_1^4 m^4 + \frac{1}{16} \left\{ f_1 (g - \frac{f}{g} f_1) m^2 - f_1^2 \right\}^2 + \frac{1}{320} f_1^2 m^4 (g - \frac{f}{g} f_1)^2 \right. \\ \left. + \frac{1}{64} m^4 (g - \frac{f}{g} f_1)^4 \right\}$$

$$\begin{aligned}
f_1 &= \frac{t}{f} \left\{ \left(\frac{t}{64} f_1^4 m^4 - \frac{t}{4} f_1^2 m^2 \left(f - \frac{1}{2} f_1 \right) + \left(f - \frac{1}{2} f_1 \right)^2 + \frac{t}{64} f_1^6 m^4 + \frac{t}{2} f_1^4 m^4 \left(f - \frac{1}{2} f_1 \right)^2 - \frac{t^2}{4} f_1^2 \left(f - \frac{1}{2} f_1 \right)^4 \right) \right. \\
&\quad \left. + \frac{1}{2} f_1^2 + \frac{1}{100} f_1^2 m^4 \left(f^2 - g f_1 + \frac{1}{4} f_1^2 \right) + \frac{t}{f} m^4 \left(f^4 - 2 g f_1^3 f_1 + \frac{3}{2} g^2 f_1^2 - \frac{1}{2} g f_1^3 + \frac{1}{16} f_1^4 \right) \right\} \\
&= \frac{t}{f} \left\{ \left(\frac{\sqrt{4}}{64} f_1^4 m^4 - \frac{1}{4} f_1^2 \cancel{g^2 m^2} + \frac{1}{2} f_1^2 \cancel{m^2} + \cancel{g^2} - g f_1 + \frac{1}{4} f_1^2 + \frac{1}{64} \cancel{f_1^4 m^4} + \cancel{\frac{1}{2} f_1^2 m^4} - \frac{1}{2} \cancel{f_1^2 m^4} \right. \right. \\
&\quad + \frac{1}{2} \cancel{f_1^2 m^4} - \cancel{\frac{1}{16} f_1^2 m^2} + \cancel{\frac{1}{2} f_1^2 m^2} + \cancel{\frac{1}{2} f_1^2} + \cancel{\frac{1}{100} f_1^2 g^2 m^4} - \cancel{\frac{1}{100} f_1^2 g^2 m^4} + \cancel{\frac{1}{400} f_1^4 m^4} \\
&\quad \left. \left. + \frac{\sqrt{4}}{64} f_1^4 m^4 - \frac{1}{4} f_1^2 \cancel{g^2 m^4} + \frac{1}{16} \cancel{f_1^2 g^2 m^4} - \cancel{\frac{1}{16} f_1^2 g^2 m^4} + \cancel{\frac{1}{16} f_1^4 m^4} \right) \right. \\
&\quad \left. + \frac{t^2}{400} f_1^4 + \frac{t}{120} f_1^4 + \left(-\frac{t}{2} f_1^2 - \frac{t}{100} - \frac{t}{16} \right) f_1^2 f_1^2 + \left(\frac{1}{2} + \frac{t}{100} + \frac{3}{16} \right) f_1^2 f_1^2 - \frac{t}{4} f_1^2 g^2 + \frac{t}{8} f_1^4 \right\} \\
&= \frac{t}{f} \left[m^4 \left\{ \left(\frac{t}{64} f_1^4 + \frac{t}{64} f_1^2 + \frac{t}{f} + \frac{t^2}{400} \right) f_1^4 + \left(-\frac{t}{2} f_1^2 - \frac{t}{100} - \frac{t}{16} \right) f_1^2 f_1^2 \right\} + \left(\frac{3}{4} f_1^2 f_1^2 - g f_1^2 f_1^2 \right) \right] \\
&\quad - m^2 \left\{ \left(-\frac{t}{f} - \frac{1}{2} \right) f_1^3 + \left(\frac{t}{4} + 1 \right) f_1^2 f_1^2 \right\} + \boxed{\left[\begin{array}{l} f_1 = \frac{t}{f} \left[m^4 \left\{ \frac{533}{3200} f_1^4 - \frac{229}{400} f_1^2 g + \frac{279}{400} f_1^2 f_1^2 g^2 - \frac{1}{4} f_1^2 f_1^2 g^3 + \frac{1}{8} f_1^4 g^4 \right\} \\ + m^2 \left\{ \frac{3}{8} f_1^3 - \frac{5}{4} f_1^2 f_1^2 \right\} + \left\{ \frac{3}{4} f_1^2 f_1^2 - g f_1^2 f_1^2 \right\} \end{array} \right]} \right]
\end{aligned}$$

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$$f_{\rho_2} = \frac{1}{2(1-v^2)} \left(\frac{\dot{t}}{R}\right)^2 m^4 \left[f_1^2 + 4 \left(g - \frac{1}{2} f_1\right)^2 \right] = \frac{1}{2(1-v^2)} \left(\frac{\dot{t}}{R}\right)^2 m^4 \left[2f_1^2 + 4g^2 - 4fg \right]$$

$$\beta_{\rho_2} = \frac{1}{6(1-v^2)} \left(\frac{\dot{t}}{R}\right)^2 m^4 \left[f_1^2 - 2f_1 g + 2g^2 \right]$$

$$\gamma = \left(\frac{\dot{t}}{E}\right) = \frac{\dot{t}}{(E)}$$

$$\beta = \frac{\dot{t}}{E}$$

$$\kappa = -4 \left(\frac{\dot{t}}{E}\right)^2 - m^2 \frac{\dot{t}}{E} \left[\frac{3}{2} f_1^2 - \frac{1}{2} fg_f + \frac{1}{2} g^2 \right]$$

$$\frac{\partial R}{\partial t} \gamma \left(\frac{3}{4} \beta - \frac{1}{2} \right) = \frac{1}{\beta} \left[(\gamma \eta)^2 \left(\frac{533}{60} \beta^3 - \frac{667}{400} \beta^2 + \frac{249}{200} \beta - \frac{1}{4} \right) + (\gamma \eta) \left(\frac{15}{\beta} \beta^2 - \frac{5}{2} \beta \right) + \left(\frac{3}{2} \beta - 1 \right) \right] + \frac{1}{3(1-v^2)} \beta^2 \left(\beta - 1 \right)$$

$$\frac{\partial R}{\partial t} \gamma \left(\frac{1}{2} \beta - 1 \right) = \frac{1}{\beta} \left[(\gamma \eta)^2 \left(\frac{229}{400} \beta^3 - \frac{249}{200} \beta^2 + \frac{3}{4} \beta - \frac{1}{2} \right) + (\gamma \eta) \left(\frac{5}{\beta} \beta^2 + (\beta - 2) \right) + \frac{1}{3(1-v^2)} \beta^2 \left(\beta - 2 \right) \right]$$

$$\frac{10}{16}$$

$$\frac{9}{4}$$

$$\frac{\partial^2}{E\partial t} \gamma(\beta\rho - 2) = (\beta\rho)^2 \left(\frac{533}{1600} \rho^3 - \frac{667}{800} \rho^2 + \frac{219}{400} \rho - \frac{1}{8} \right) + (\beta\rho) \left(\frac{15}{16} \rho^2 - \frac{5}{4} \rho \right) + \left(\frac{3}{4} \rho^2 - \frac{1}{2} \right) + \frac{2}{3(1-\rho)} \partial^2 \gamma(\beta\rho - 2)$$

$$\frac{\partial^2}{E\partial t} \gamma(\beta\rho - 2) = (\beta\rho)^2 \left(\frac{229}{1600} \rho^3 - \frac{279}{800} \rho^2 + \frac{15}{400} \rho - \frac{1}{8} \right) + (\beta\rho) \left(\frac{15}{16} \rho^2 + 0 \right) + \left(\frac{1}{4} \rho^2 - \frac{1}{2} \right) + \frac{2}{3(1-\rho)} \partial^2 \gamma(\beta\rho - 2)$$

$$\begin{aligned} 0 &= (\beta\rho)^2 \left(\frac{154}{1600} \rho^4 - \frac{150}{800} \rho^3 - \frac{54}{400} \rho^2 - \frac{25}{100} \rho \right) + (\beta\rho) \left(\frac{15}{16} \rho^2 + 0 \right) + (-\beta\rho + 0) + \frac{2}{3(1-\rho)} \partial^2 \gamma(\beta\rho - 4\rho) \\ &\quad + (\beta\rho)^2 \left(\frac{304}{1600} \rho^3 - \frac{409}{800} \rho^2 + \frac{102}{400} \rho \right) + (\beta\rho) \left(\frac{15}{16} \rho^2 - \frac{5}{2} \rho \right) + (\beta\rho) + \frac{2}{3(1-\rho)} \partial^2 \gamma(+4\rho) \end{aligned}$$

$$0 = (\beta\rho)^2 \left(\frac{22}{800} \rho^3 + \frac{22}{400} \rho^2 - \frac{231}{200} \rho + \frac{11}{100} \right) + (\beta\rho) \left(\frac{25}{16} \rho - \frac{5}{2} \right) + \frac{2}{3(1-\rho)} \partial^2 \gamma(\beta\rho - 2)$$

$$\boxed{\begin{aligned} & \left\{ \frac{22}{800} (\beta\rho)^2 \right\} \rho^3 + \left\{ \frac{22}{400} (\beta\rho) \right\}^2 \rho^2 + \left[\frac{231}{200} (\beta\rho) \right]^2 + \frac{2}{3(1-\rho)} \partial^2 \gamma(\beta\rho) \\ & + \left\{ \frac{11}{100} (\beta\rho) \right\}^2 - \frac{5}{2} (\beta\rho) - \frac{4}{3(1-\rho)} \partial^2 \gamma(\beta\rho) = 0 \end{aligned}}$$

$$\boxed{\gamma = 0.10, \quad \eta = 10, \quad \zeta = 10.5051 \quad \gamma\eta = 1}$$

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$$\lambda = -\underline{0.0480814}$$

$$0.09625 s^3 + 0.19250 s^2 + 1.352326 s - 1.744652 = 0$$

$$F(s) = s^3 + 2.00000 s^2 + 14.05014 s - 18.12625 = 0$$

$$F'(s) = 3s^2 + 4.00000 s + 14.05014$$

$$\begin{array}{l} F(1.05) = -0.010978 \\ \quad \quad \quad \underline{0.00051} \\ F'(1.05) = 21.558 \end{array}$$

$$F(1.05051) = 0$$

$$s^2 + 3.05051 s + 17.2547 = 0 \quad \text{No Real Root !!}$$

$$\boxed{s = 1.05051, \quad s^2 = 1.10357, \quad s^3 = 1.15931}$$

$$\begin{aligned} \frac{OR}{Et} &= \frac{2}{3(1-\gamma^2)} \gamma + \frac{1}{\gamma(s-1)} \left\{ (17)^2 \left(0.143125 s^3 - 0.34825 s^2 + 0.1875 s - 0.1250 \right) \right. \\ &\quad \left. + (\gamma\eta) 0.3125 s^2 \right\} + \frac{1}{4\gamma} \end{aligned}$$

For this particular case

$$\begin{aligned} \frac{OR}{Et} &= 0.07326 + 10 \left\{ \frac{0.143125 s^3 - 0.03625 s^2 + 0.1675 s - 0.1250}{-0.94949} + 0.25 \right\} \\ &= 0.07326 + 0.41580 = \underline{\underline{0.4891}} \end{aligned}$$

$$(f_3) = 1.05051$$

$$(f_3)^+ = 1.103571$$

$$E = 0.23919 + (10.5051)^2 \left[0.004310824(x) - 0.008621648(-x)^3 \right. \\ \left. + 0.056220149(-x)^2 - 0.010873779(-x) + 0.009121777 \right]$$

$$= \left(\underline{1.2024} \right) \quad (0.9502359)$$

$$\Theta = f \quad \underline{0.2653712}$$

Check !!!

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$$\frac{ER}{Et} = \frac{2}{3(1-\gamma)} \left\{ \frac{2\rho-2}{3\rho-2} + \frac{1}{\gamma(3\rho-2)} \right\} \left\{ (\gamma)^2 \left(0.333125\rho^3 - 0.65875\rho^2 + 0.6925\rho - 0.125 \right) \right. \\ \left. + (\gamma)(0.9375\rho^2 - 1.250\rho) + (0.25\rho - 0.5) \right\}$$

$$= \frac{0.2}{3(1-\gamma)} \left\{ \frac{0.10102}{1.15153} + \frac{10}{1.15153} \right\} \left\{ 0.333125\rho^3 + 0.07875\rho^2 + 0.1975\rho - 0.625 \right\} \\ = \underline{\underline{0.48907}} \quad O.K. \quad \left| \begin{array}{l} \left(\frac{ER}{t} \right) = \\ = 0.9748 \end{array} \right. \quad \underline{\underline{\lambda = + 0.124455}}$$

$$\gamma = 10 \quad \boxed{\gamma = 0.144} \quad \boxed{\gamma = 1.44} \quad \xi = 10.7961, \quad \lambda = 0.0733316$$

$$0.199584\rho^3 + 0.399168\rho^2 + 1.220163\rho - 2.033710 = 0$$

$$F(\rho) = \rho^3 + 2.00000\rho^2 + 6.113631\rho - 10.16974 = 0$$

$$F'(\rho) = 3\rho^2 + 4.00000\rho + 6.113631$$

$$F(1) = -1.07611$$

$$F'(1) = 13.113631$$

$$F(1.0762) = +0.03338$$

$$F'(1.0762) = 13.954$$

$$\frac{.00239}{1.07961}$$

$$F(1.07961) = +0.00006, \quad O.K.$$

$$\rho^2 + 3.07961\rho + 9.43841 = 0$$

No more real Root

$$f^2 = 0.020736 \quad (f\beta) = 1.5546384$$

$$(f\beta)^2 = 2.4169006$$

$$\begin{aligned} E &= 0.0461446 + (10.795)^2 \left\{ 0.009441018(-\lambda)^4 - 0.016882036(-\lambda)^3 \right. \\ &\quad \left. + 0.085829781(-\lambda)^2 - 0.015660708(-\lambda) + 0.006603078 \right\} \\ &= \underline{0.7347181} \quad (0.847512) \end{aligned}$$

$$\Theta = -0.1503705$$

$$s = 1.07961, \quad s^2 = 1.16555, \quad s^3 = 1.25834$$

645

$$\frac{OR}{Et} = 0.105495 + 6.944444 \left\{ \frac{0.296784s^3 - 0.273168s^2 + 0.388800s - 0.2592}{-0.92039} + 0.25 \right\}$$

$$= 0.105495 + 0.109257 = \underline{\underline{0.2148}}$$

Check

$$\frac{OR}{Et} = \frac{0.288}{2.73} \frac{0.15922}{1.23883} + \frac{6.94444}{1.23883} \left\{ 0.690768s^3 - 0.430704s^2 + 0.396336s - 0.2592 \right\}$$

$$= 0.0135587 + 0.201254 = \underline{\underline{0.214813}}$$

$$\left(\frac{ER}{t} \right) = 0.092992 \quad \phi = +0.167600$$

s = 0.144	n = 15,	ny = 2.16	S = 17.0025,
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$$0.449064s^3 + 0.898128s^2 + 0.0264230s - 1.837870 = 0 \quad \lambda = \underline{\underline{0.1177768}}$$

$$F(s) = s^3 + 2.00200s^2 + 0.0588402s - 4.092668 = 0$$

$$F'(s) = 3s^2 + 4.000s + 0.0588402$$

$$\begin{array}{rcl} F(1.134) = +0.004243 & & F'(1.134) = 8.4527 \\ \hline & 50 & \\ & 1.13350 & \end{array}$$

$$F(1.13350) = 0, \quad s^2 + 3.13350s + 3.61066 = 0 \quad \text{No more real root.}$$

$$y^2 = 0.020736, \quad (y^3) = 2.4483600 \\ (y^3)^2 = 5.9944667$$

$$\begin{aligned} E &= 0.3350094 + (17.0025)^2 \left\{ 0.02341566 (-1)^4 - 0.046831771 (-1)^3 \right. \\ &\quad \left. + 0.163809542 (-1)^2 - 0.044754594 (-1) + 0.007769092 \right\} \\ &= \underline{1.6932169} \quad (5.1066960) \end{aligned}$$

$$\theta = -0.668432$$