

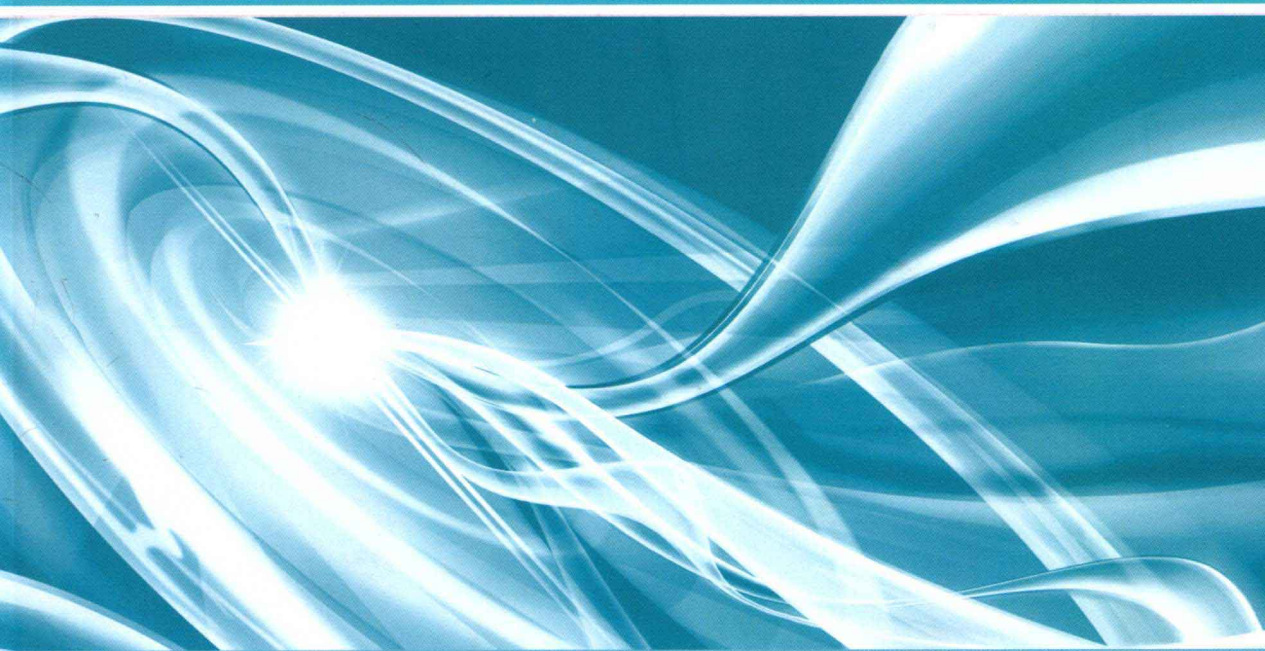


普通高等教育“十二五”规划教材

高等数学 (下册)

Advanced Mathematics (II)

王立冬 周文书 主编



中国电力出版社
CHINA ELECTRIC POWER PRESS



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内 容 提 要

本书为普通高等教育“十二五”规划教材。本书把数学教学与外语学习有机结合,使学生在学到数学的相关概念、公式和结论的同时了解到数学的思想、方法和精神实质,在不增加课时的情况下,学会数学专业术语的英文表达,使学生获得用英语进行数学思维获取知识的能力,使教师和学生在学习中学习国外先进的教学理念、方法和方式,进一步提高教学质量,弥补大学英语学习与专业脱节的不足,提高学生的英语应用能力,进而达到学生综合素质的全面提高。

本书可作为普通高等院校学生高等数学课程的教材,也可作为科技英语专业相关课程的教材和参考书。

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双语教学有利于培养大批
沟通中西科技文化的人才

徐利治

二〇〇九年九月大连

前言

随着科学技术的飞速发展，数学不仅被广泛深入地应用于自然科学、信息技术和工程技术，而且已渗透到诸如生命科学、社会科学、环境科学、军事科学、经济科学等领域，它已成为表达严格科学思想的媒介，人们越来越深刻地认识到，没有数学就难以取得当代的科学成就。正是由于自然科学各学科数学化的趋势以及社会科学各部门量化的要求，许多学科都或直接或间接、或先或后地经历着数学化的进程。现在已经没有哪一领域能够抵御得住数学的渗透，体现了马克思所说：“一门科学只有当它达到能够成功地运用数学时，才算真正发展了”的精辟论述。所以在科学王国中，数学有一个特殊的位置。它既是一门专业领域，又是基础（思维）工具；既是语言，又是文化；既能与经管科学交叉，又能与理工结合，且能向文科渗透。

数学这种特殊的位置和应用的广泛性，加之英语作为信息交流的一种重要工具，确定了数学的语言英文表达有着极为重要的意义，它已成为科学技术交流和传播的重要基础工具之一。数学教学与外语有机的结合，有利于学生综合素质的全面提高，顺应时代发展方向。

因此，编写适合双语教学的，同时又与国内数学课程内容相适应的教材已势在必行。

目前，双语教学的模式基本有两方面的选择。关于教材，或直接采用原版教材，或采用中文版教材，加外语补充材料。关于授课，则采用全外语授课，或部分外语授课，或在使用原版教材的基础上采用全中文授课。各高校大多根据学生的外语水平及教师的外语特长在上述几种情况中选择。近年来，学生的外语水平有了明显的提高，师资的外语及专业能力也有了本质上的变化。因此，双语教学的模式也面临真正意义上的提升。

高等数学课程实施双语教学的目的在于提高数学教育教学质量。通过高等数学双语教学，学生可以学习利用原版教材，学习国外先进的学科体系、教学理念和丰富的数学逻辑内涵以及高等数学在其他学科领域中的基本应用，以弥补中文教材及翻译教材的不足。然而，原版教材一般内容体系庞杂，与国内教学要求难以完全符合；如果采用中文版教材，再提供外语补充材料，则双语教学体现不充分，效果不明显。最好的选择是请既懂专业又有良好外语写作能力的教师（或中方和外方直接合作）按国内的教学要求有针对性地编写教材。这是我们努力的方向，本教材无疑是满足时代要求的一种有益尝试。

本教材引文内容以高等学校高等数学课程教学大纲为依据，参考多种国外原版教材，以使语言表述准确、地道；在章节顺序和内容上参照大连理工大学应用数学系编写的《工科微积分》中文教材进行编写，涵盖了高等数学课程中的主要数学概念、常见的专业词汇、重要的定理；为使学生习惯用英文进行专业思考的同时，又熟悉相应的中文用语，在每章节列出了中文书写的内容概述、关键词和短语。本书的另一个特色是书中几乎所有的定理

都有清晰的证明。

编写本书的直接目的是为讲授高等数学的教师和学习高等数学的学生提供掌握相关内容的英文描述服务,进而使得学生学过本课程后,能够独立阅读相关的英文教材和文献。它可作为学生的配套教材和扩大知识领域的参考书,也可作为科技英语专业高等数学课程的参考书。

本书主要讲授一元函数的微积分学。主要包括:

第1章讲授有关函数、极限与连续性的基本概念和基础理论。这些内容构成了微积分学的基础。此外,极限概念在物理和几何上的背景也将在这一章有所介绍。

第2章涉及微分学的基本概念和基础理论。在本章,导数及其计算是主要部分。此外,导数的几何意义和应用也将有所介绍。

第3章讲授积分学的基本概念和基础理论。主要的注意力放在定积分的计算和定积分的几何意义上。此外,我们还将详述作为微分学和积分学之间联系桥梁的微积分学基本定理。

第4章介绍常微分方程的基本解法,将重点介绍一阶线性微分方程和二阶常系数线性微分方程的解法。

第5章介绍向量代数与空间解析几何的基本知识。前者涉及向量的概念和向量的运算,而后者着重讨论空间平面、曲线和曲面的方程。

第6章讲授多元函数微分学的基本概念和偏导数的几何应用,重点将放在对二元函数的研究上,相应的结果可以平行推广到二元以上的多元函数中。基本概念包括多元函数的定义、极限、连续性、偏导数和全微分。多元复合函数偏导数的运算法则如链式法则、全微分的形式不变形以及隐函数的微分法也将作为重点内容予以介绍。在几何应用部分,主要介绍空间曲线的切线方程、曲面的切平面方程以及解决多元函数极值问题的拉格朗日乘数法。

第7章讲授多元数量值函数的积分学。多元数量值函数的积分学是定积分的推广,其多样性的特点使得它比定积分有着更丰富的内容。本章将按照不同几何形体对应的不同积分,分别讨论二重积分、三重积分、对弧长的曲线积分及对面积的曲面积分的计算方法。最后,介绍向量值函数在几何、物理、力学等方面的应用。

第8章介绍向量值函数的曲线积分与曲面积分。本章除讨论第二型曲线、曲面积分的性质及计算外,还着重讨论各种积分之间的联系,这些联系体现在格林公式、高斯公式和斯托克斯公式中。最后介绍描述向量场特征的几个重要概念:散度与旋度。

第9章介绍有关无穷级数的基本理论。本部分首先介绍常数项级数及其性质,重点讲授判别正项级数收敛的一些常用判别法,如比较判别法、根值判别法和比值判别法。然后,详细介绍有关幂级数的有关理论。最后着重讨论傅里叶级数的概念、收敛定理,以及将函数展开成傅里叶级数的方法。

此外,在每一章里,为了帮助学生更好地理解相关内容,还适当地增加了若干小节——“解题方法归纳与典型例题”。

大连民族学院开展双语教学已有多年，特别是结合本校师资及学生特点施行的“渗透式双语教学”工作已获得国家级教学成果二等奖。本书是我们进行双语教学的又一次有益尝试，也是一个新的阶段总结，恳请各位专家、同行以及广大读者关注、关心我们的工作，并提出宝贵的建议和意见！

编 者

2012 年 5 月



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Introduction

Calculus is a branch of mathematics that includes the study of limits, derivatives, integrals, and infinite series, and constitutes a major part of modern university education. Historically, it was sometimes referred to as “the calculus of infinitesimals”, but that usage is seldom seen today. Most basically, calculus is the study of change, in the same way that geometry is the study of space.

Calculus has widespread applications in science and engineering and is used to solve problems for which algebra alone is insufficient. Calculus builds on algebra, trigonometry, and analytic geometry and includes two major branches: **differential calculus** and **integral calculus**, that are closely related by the fundamental theorem of calculus. In more advanced mathematics, calculus is usually called analysis and is defined as the study of functions.

The history of calculus falls into several distinct time periods, most notably the ancient, medieval, and modern periods. The ancient period introduced some of the ideas of integral calculus, but does not seem to have developed these ideas in a rigorous or systematic way. Calculating volumes and areas, the basic function of integral calculus, can be traced back to about 1800 BC, when an Egyptian successfully calculated the volume of a pyramidal frustum. From the school of Greek mathematics, Eudoxus (c. 408-355 BC) used the method of exhaustion, which prefigures the concept of the limit, to calculate areas and volumes while Archimedes (c. 287-212 BC) developed this idea further, inventing heuristics which resemble integral calculus.

The method of exhaustion was later used to find the area of a circle in China by Liu Hui, around 3rd century, a Chinese mathematician who lived in the Wei Kingdom, in the 3rd century AD in order to find the area of a circle. It was also used by Zu Chongzhi (429-500, a prominent Chinese mathematician and astronomer during the Liu-Song and Southern Qi Dynasties) in the 5th century AD, who used it to find the volume of a sphere. The Indian mathematician-astronomer Aryabhata in 499 used a notion of infinitesimals and expressed an astronomical problem in the form of a basic differential equation. Manjula, in the 10th century, elaborated on this differential equation in a commentary. This equation eventually led an Indian mathematician in the 12th century to develop the concept of a derivative

representing infinitesimal change, and he described an early form of “Rolle’s theorem”.

In the modern period, independent discoveries in calculus were being made in the early 17th Japan, such as the method of exhaustion is expanded. In Europe, the second half of the 17th century was a time of major innovation. Calculus provided a new opportunity in mathematical physics to solve long-standing problems. Several mathematicians contributed to these breakthroughs, notably John Wallis (1616-1703, an English mathematician) and Isaac Barrow (1630-1677, an English mathematician). James Gregory (1638-1675, a Scottish mathematician and astronomer) was able to prove a restricted version of the second fundamental theorem of calculus in AD 1668.

While some of the ideas of calculus were developed earlier, in Greece, China, India, and Japan, the modern use of calculus began in Europe, during the 17th century, when Newton (1643-1727, an English physicist, mathematician and astronomer) and Leibnitz (1646-1716, a Germany mathematician) built on the work of earlier mathematicians to introduce the basic principles of calculus. This work had a strong impact on the development of physics. Newton and Leibnitz pulled these ideas together into a coherent whole, and they are usually credited with the independent and nearly simultaneous invention of calculus. Newton was the first to apply calculus to general physics and Leibnitz developed much of the notation used in calculus today; he often spent days determining appropriate symbols for concepts. The basic insight that both Newton and Leibnitz had was the fundamental theorem of calculus. When Newton and Leibnitz first published their results, there was great controversy over which mathematician (and therefore which country) deserved credit. Newton derived his results first, but Leibnitz published first. A careful examination of the papers of Leibnitz and Newton shows that they arrived at their results independently. Today, both Newton and Leibnitz are given credit for developing calculus independently. It is Leibnitz, however, who gave the new discipline its name. Newton called his calculus “the science of fluxions”.

Since the time of Leibnitz and Newton, many mathematicians have contributed to the continuing development of calculus. For example, Sweden mathematician Euler (1707-1783), by using calculus as a tool, solved many problems in the fields of astronomy, physics and mechanics, and also founded many new subjects such as differential equation, infinite series and calculus of variations. And the first systematically integrated book on analysis, *The Infinitesimal Analysis*, was published in 1748. But no efficient solution to the loose base of calculus had been found. In the 19th century, calculus was put on a much more rigorous footing by mathematicians such as Cauchy(1789-1857, a French mathematician), Riemann(1826-1866, a Germany

mathematician), and Weierstrass(1861-1897, a French mathematician).

Applications of differential calculus include computations involving velocity and acceleration, the slope of a curve, and optimization. Applications of integral calculus include computations involving area, volume, arc length, center of mass, work, and pressure. More advanced applications include power series and Fourier series. Calculus is also used to gain a more precise understanding of the nature of space, time, and motion. For centuries, mathematicians and philosophers wrestled with paradoxes involving division by zero or sums of infinitely many numbers. These questions arise in the study of motion and area. The ancient Greek philosopher Zeno gave several famous examples of such paradoxes. Calculus provides tools, especially the limit and the infinite series, which resolve the paradoxes.

Calculus is rooted in different subjects of natural science and social science, which not only supplies all mathematical methods and algorithms with methodology, but also cultivate people's thinking mode. Calculus is a ubiquitous topic in most modern high schools and universities, and mathematicians around the world continue to contribute to its development.

引言

微积分学是数学的一个分支,主要研究极限、导数、积分和无穷级数等内容。很久以前,人们曾把微积分学看作“无穷小量的微积分学”。总体说来,微积分学是关于变量的研究,而几何学是关于空间的研究。

微积分在科学与工程中有着广泛的应用,常常能够解决代数学无法解决的问题。微积分建立在代数学、三角学以及解析几何的基础之上,包含两个部分:微分学和积分学,联系这两个分支的桥梁是微积分学基本定理。在高等数学中,微积分通常被称为分析学,研究的对象是函数。

微积分的发展历史经历了几个不同的阶段,大致可分为古代、中世纪和现代。定积分的思想起源于古代,但那时这个思想并没有得到很好的发展。对积分学能够用于计算体积和面积这些基本功能的较早认识要追溯到大约公元前 1800 年,那时一个埃及人成功地计算出了金字塔平截头体部分的体积。在希腊的一所数学学校里教书的欧德克斯(大约公元前 408—355)用一种被称为“穷竭法”的思想来计算体积和面积,而这正是极限概念的萌芽。后来,阿基米得(大约公元前 287—212)进一步提出了对后人更富有启发性的思想,而这个思想与积分学思想已经十分接近。

在公元 3 世纪,我国的刘辉(约公元 3 世纪,是魏朝时期的一个数学家)用这种所谓的“穷竭法”思想计算了圆周的面积;在公元 5 世纪,我国的祖冲之(429—500,是南北朝时期南朝宋齐之间的一位杰出的数学家和天文学家)也用这种思想求出了球的体积。公元 499 年,印度数学家和天文学家阿亚巴哈塔用无穷小的概念将一个天文学问题表示成了微分方程的形式。公元 10 世纪,玛纽拉在一篇注释中详尽地阐述了这个微分方程,而它最终导致一位印度数学家在公元 12 世纪发展了表示无穷小变化的导数的概念,并且他还描述了罗尔定理的早期形式。

在 17 世纪初期,日本数学家做出了许多独立的发现,例如他们拓展了穷竭法。在欧洲,17 世纪下半叶是创新的主要时期。微积分给人们提供了一个新的机遇,以解决在数学物理中长期悬而未决的问题,几位数学家在此领域做出了突破性的进展,比较著名的有英国数学家约翰·沃利斯(1616—1703)和艾萨克·巴罗(1630—1677)。在公元 1668 年,苏格兰数学家和天文学家詹姆斯·乔治(1638—1675)已经能够给出微积分学第二基本定理的严格证明。

如果说微积分的某些思想发源于中国、印度和日本,那么微积分的应用则开始于 17 世纪的欧洲。基于早期数学家的工作,牛顿(1643—1727,英国物理学家、数学家和天文学家)和莱布尼茨(1646—1716,德国数学家)建立了微积分学中的基本原理,他们的工作对物理学的发展产生了强烈的影响。他们把那些分散的想法有机地融合成一个整

体,通常认为他们是各自独立、几乎是同时发明了微积分。牛顿首先把微积分应用到物理学中,而莱布尼茨则发展了沿用至今的许多微积分的符号。牛顿和莱布尼茨的基本贡献是建立了微分学和积分学之间的桥梁,即微积分学基本定理。就在牛顿和莱布尼茨都各自发表了自己的结果时,在就谁(甚至牵扯到是哪个国家)应当值得享有发明微积分的这个莫大荣誉的问题上出现了巨大的分歧和争论。牛顿首先推导出了这个结果,而莱布尼茨则首先发表了这个结果。仔细推敲他们的文章后,人们发现是他们各自独立地发明了微积分。所以他们应当共同分享发明微积分这一殊荣。最初牛顿把他的发明称作“流数术”,而莱布尼茨则把他的发明称做微积分,这一称谓沿用至今。

自从牛顿和莱布尼茨发明了微积分以后,许多数学家都对微积分的发展作出了贡献。例如,瑞士数学家欧拉以微积分为工具解决了天文学、物理学和力学中的许多问题,并且他还由此创立了几个新的数学分支如微分方程、无穷级数及变分法等。1748年,欧拉出版了分析学中的第一部系统而完整的著作《无穷小分析》。不过,那时人们也发现微积分的基础并不牢固。直到19世纪,柯西(1789—1857,法国数学家)、黎曼(1826—1866,德国数学家)和魏尔施特拉斯(1861—1897,德国数学家)等数学家才建立了微积分的严格数学理论基础。

微分学的应用包括计算速度、加速度以及曲线的斜率和最优化等。定积分的应用包括计算面积、体积、弧长以及质心、功和压力等。幂级数和傅里叶级数等则是微积分的更高层次的应用。微积分也常常被用来精确理解空间、时间和运动的本性。几个世纪以来,数学家和哲学家经常就被零除以及无穷多个数的和等问题发生争论。这些问题源自于对运动和面积的研究。例如,古希腊哲学家芝诺就曾给出了几个著名悖论。不过,用微积分特别是以极限和无穷级数为工具,这些矛盾都能被轻易化解。

如今微积分学在自然科学和社会科学中的不同领域已经根深蒂固,它不仅提供了许多数学方法、计算方法和方法论,而且也培养了人们的思考模式。现在,微积分已经走进大多数大、中学校的课堂,而且许多国内外的数学家都在为它的发展而努力。



Chapter 5

Vector Algebra and Analytic Geometry in Space

向量代数与空间解析几何

Vectors are the description and abstraction of a class of quantities with both size and direction in nature and engineering technology. As an important mathematical tool, vector algebra has extensive applications in many fields.

The basic idea of analytic geometry is to study the geometric problems with algebraic methods. After setting up the Cartesian coordinate in space, one may establish the corresponding relations between the points in space and ternary arrays with orders, surface in space and the equations with ternary variables, curve in space and system of equations with ternary variables, vectors in space & its operations' forms of geometry and coordinate forms. It is the combination of forms and numbers that makes the achievement of geometric aims by mathematical means possible. Conversely, algebraic languages become very intuitive because of the geometric explanations. With the development of modern computer technology, the mathematical methods of combining forms with numbers are used extensively in many fields including science, engineering technology, art in cinema and television and so on.

Both vector algebra and analytic geometry in space are independent system of knowledge as well as the necessary foundation for learning calculus of functions with several variables.

In this chapter, we first introduce the concept of vector, and then demonstrate its operations against practical backgrounds. Following this, by setting up the Cartesian coordinate in space, vectors and its operations will be given a quantitative process with coordinate method. In the part concerning analytic geometry in space, the plane and straight line equations in space, as emphases, will be discussed using vectors as a tool. In the equations of surfaces, we are concerned mainly with the cylinder, surface of revolution and cone, and will use the trace method to study the graphs of the quadric surface.

5.0 Cited examples

引 例

In previous chapters, we have met with many curves expressed by parametric equations and polar coordinate equations. Most of them are very distinctive, and their graphs are very beautiful. Many of these curves have practical application background. For example, asteroid is one of them. Do you know how it is formed?

Swiss mathematician Euler once put forward a question: How to express the volume of a tetrahedron with the length of edge for the tetrahedron?

The above two problems are not difficult to solve if you use the method of vector algebra in this chapter.

You may have such experience: When a supersonic aircraft flies at high altitudes, you always see it fly in the air first and hear the roar of it later. What is the region in which you can hear the sound of the aircraft the moment you see it? What interesting is that the region is a cone with the aircraft as its vertex. Outside this cone, no matter how near you are to the aircraft, you will not hear the roar of the aircraft. This is the famous "Mach Cone". In the end of this chapter, we will use the method of analytic geometry in space to deduce the equation of cone surface for mach cone.

5.1 Vector and its operations

向 量 及 其 运 算

Key points of this section

(1) Understand the concept of vector.

(2) Master and familiarize the linear operations of vector, the concepts and operations of the dot product and the cross product.

(3) Master the method of finding the included angle of two vectors and the necessary and sufficient conditions whether two vectors are parallel or perpendicular.

本节重点

(1) 理解向量概念.

(2) 熟练掌握向量的线性运算, 向量的数量积和向量积的概念及其运算.

(3) 掌握两向量夹角的求法及向量平行、垂直的充分条件.

Words and expressions 单词和短语

vector 向量	projection 投影
directed line segment 有向线段	projective vector 投影向量
initial point 始点	scalar product 数量积
terminal point 终点	dot product 点积
module(norm) 模	inner product 内积
unit vector 单位向量	vector product 向量积
zero vector 零向量	cross product 叉积
negative 负数	outer product 外积
distance 距离	mixed product 混合积
parallel 平行	included angle 夹角
equal to 等于	cosine 余弦
triangle rule 三角形法则	perpendicular 垂直
parallelogram rule 平行四边形法则	collinear 共线
law of commutation 交换律	coplanar 共面
law of association 结合律	tetrahedron 四面体
law of distribution 分配律	If...find... 如果……求……
addition 加法	Given...find... 已知……求……
minus 减	Suppose that...show that... 设……证明……
multiplication 乘法	Determine whether...is true or false 判断……是正确还是错误的
linear operation 线性运算	

5.1.1 Concept of vectors

向量的概念

A **vector** means a quantity with both magnitude and direction, vector is used by scientists to indicate a quantity such as displacement or velocity or force. A vector is usually expressed by a directed line segment, its length denotes the magnitude of the vector and its direction denotes the direction of vector. A vector with initial point A and terminal point B is denoted by \overrightarrow{AB} . Vectors are usually written in boldface letters and drawing small arrows above the letters, for example \mathbf{a} , \mathbf{b} , \mathbf{F} or \vec{a} , \vec{b} , \vec{F} . The magnitude of a vector is called **norm**. A vector that has no any relations with the starting point is called **free vector**, which is discussed in mathematics.

Unit vector is a vector of length 1. **Zero vector** is a vector of length 0, denoted by