普通高等教育“十二五”规划教材

高等数学（上册）

Advanced Mathematics

（Ⅰ）

王立冬　周文书　主编

中国电力出版社
CHINA ELECTRIC POWER PRESS

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（I）

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内 容 提 要

本书为普通高等教育"十二五"规划教材。本书把数学教学与外语学习有机结合，使学生在学到数学的相关概念、公式和结论的同时了解到数学的思想、方法和精神实质，在不增加课时的情况下，学会数学专业术语的英文表达，使学生获得用英语进行数学思维获取知识的能力，使教师和学生在教学中学习国外先进的教学理念、方法和方式，进一步提高教学质量，弥补大学英语学习与专业脱节的不足，提高学生的英语应用能力，进而达到学生综合素质的全面提高。

本书可作为普通高等院校学生高等数学课程的教材，也可作为科技英语专业相关课程的教材和参考书。

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沟通中西科技文化的人才

徐利治

二〇〇九年九月大连

前　言

随着科学技术的飞速发展，数学不仅被广泛深入地应用于自然科学、信息技术和工程技术，而且已渗透到诸如生命科学、社会科学、环境科学、军事科学、经济科学等领域，它已成为表达严格科学思想的媒介，人们越来越深刻地认识到，没有数学就难以取得当代的科学成就。正是由于自然科学各学科数学化的趋势以及社会科学各部门定量化的要求，许多学科都或直接或间接、或先或后地经历着数学化的进程。现在已经没有哪一领域能够抵御得住数学的渗透，体现了马克思所说："一门科学只有当它达到能够成功地运用数学时，才算真正发展了"的精辟论述。所以在科学王国中，数学有一个特殊的位置。它既是一门专业领域，又是基础（思维）工具；既是语言，又是文化；既能与经管科学交叉，又能与理工结合，且能向文科渗透。

数学这种特殊的位置和应用的广泛性，加之英语作为信息交流的一种重要工具，确定了数学的语言英文表达有着极为重要的意义，它已成为科学技术交流和传播的重要基础工具之一。数学教学与外语有机的结合，有利于学生综合素质的全面提高，顺应时代发展方向。

因此，编写适合双语教学的，同时又与国内数学课程内容相适应的教材已势在必行。

目前，双语教学的模式基本有两方面的选择。关于教材，或直接采用原版教材，或采用中文版教材，加外语补充材料。关于授课，则采用全外语授课，或部分外语授课，或在使用原版教材的基础上采用全中文授课。各高校大多根据学生的外语水平及教师的外语特长在上述几种情况中选择。近年来，学生的外语水平有了明显的提高，师资的外语及专业能力也有了本质上的变化。因此，双语教学的模式也面临真正意义上的提升。

高等数学课程实施双语教学的目的在于提高数学教育教学质量。通过高等数学双语教学，学生可以学习利用原版教材，学习国外先进的学科体系、教学理念和丰富的数学逻辑内涵以及高等数学在其他学科领域中的基本应用，以弥补中文教材及翻译教材的不足。然而，原版教材一般内容体系庞杂，与国内教学要求难以完全符合；如果采用中文版教材，再提供外语补充材料，则双语教学体现不充分，效果不明显。最好的选择是请既懂专业又有良好外语写作能力的教师（或中方和外方直接合作）按国内的教学要求有针对性地编写教材。这是我们努力的方向，本教材无疑是满足时代要求的一种有益尝试。

本教材引文内容以高等学校高等数学课程教学大纲为依据，参考多种国外原版教材，以使语言表述准确、地道；在章节顺序和内容上参照大连理工大学应用数学系编写的《工科微积分》中文教材进行编写，涵盖了高等数学课程中的主要数学概念、常见的专业词汇、重要的定理；为使学生习惯用英文进行专业思考的同时，又熟悉相应的中文用语，

在每章节列出了中文书写的内容概述、关键词和短语。本书的另一个特色是书中几乎所有的定理都有清晰的证明。

编写本书的直接目的是为讲授高等数学的教师和学习高等数学的学生提供掌握相关内容的英文描述服务，进而使得学生学过本课程后，能够独立阅读相关的英文教材和文献。它可作为学生的配套教材和扩大知识领域的参考书，也可作为科技英语专业高等数学课程的参考书。

本书主要讲授一元函数的微积分学。主要内容包括：

第 1 章讲授有关函数、极限与连续性的基本概念和基础理论。这些内容构成了微积分学的基础。此外，极限概念在物理和几何上的背景也将在这一章有所介绍。

第 2 章涉及微分学的基本概念和基础理论。在本章，导数及其计算是主要部分。此外，导数的几何意义和应用也将有所介绍。

第 3 章讲授积分学的基本概念和基础理论。主要的注意力放在定积分的计算和定积分的几何意义上。此外，我们还将详述作为微分学和积分学之间联系桥梁的微积分学基本定理。

第 4 章介绍常微分方程的基本解法，将重点介绍一阶线性微分方程和二阶常系数线性微分方程的解法。

第 5 章介绍向量代数与空间解析几何的基本知识。前者涉及向量的概念和向量的运算，而后者着重讨论空间平面、曲线和曲面的方程。

第 6 章讲授多元函数微分学的基本概念和偏导数的几何应用，重点将放在对二元函数的研究上，相应的结果可以平行推广到二元以上的多元函数中。基本概念包括多元函数的定义、极限、连续性、偏导数和全微分。多元复合函数偏导数的运算法则如链式法则、全微分的形式不变形以及隐函数的微分法也将作为重点内容予以介绍。在几何应用部分，主要介绍空间曲线的切线方程、曲面的切平面方程以及解决多元函数极值问题的拉格朗日乘数法。

第 7 章讲授多元数量值函数的积分学。多元数量值函数的积分学是定积分的推广，其多样性的特点使得它比定积分有着更丰富的内容。本章将按照不同几何形体对应的不同积分，分别讨论二重积分、三重积分、对弧长的曲线积分及对面积的曲面积分的计算方法。最后，介绍向量值函数在几何、物理、力学等方面的应用。

第 8 章介绍向量值函数的曲线积分与曲面积分。本章除讨论第二型曲线、曲面积分的性质及计算外，还着重讨论各种积分之间的联系，这些联系体现在格林公式、高斯公式和斯托克斯公式中。最后介绍描述向量场特征的几个重要概念：散度与旋度。

第 9 章介绍有关无穷级数的基本理论。本部分首先介绍常数项级数及其性质，重点讲授判别正项级数收敛的一些常用判别法，如比较判别法、根值判别法和比值判别法。然后，详细介绍有关幂级数的有关理论。最后着重讨论傅里叶级数的概念、收敛定理，以及将函数展开成傅里叶级数的方法。

此外，在每一章里，为了帮助学生更好地理解相关内容，还适当地增加了若干小节——

“解题方法归纳与典型例题”。

大连民族学院开展双语教学已有多年，特别是结合本校师资及学生特点施行的“渗透式双语教学”工作已获得国家级教学成果二等奖。本书是我们进行双语教学的又一次有益尝试，也是一个新的阶段总结，恳请各位专家、同行以及广大读者关注、关心我们的工作，并提出宝贵的建议和意见！

编　者

2012 年 5 月

目 录

前言

Introduction ······ 1
引言 ······ 4
Chapter 1 Functions, limits and continuity 函数、极限与连续性 ······ 6
1.0 Cited examples 引例 ······ 6
1.1 Functions 函数 ······ 7
1.2 Limits 极限 ······ 17
1.3 Properties and operations of limits 极限的运算与性质 ······ 25
1.4 Monotone boundedness principle and irrational number e
单调有界原理与无理数 e ······ 41
1.5 Comparison between infinitesimals 无穷小之间的比较 ······ 46
1.6 Continuity and discontinuity of function 函数的连续性与间断 ······ 52
1.7 Properties of continuous functions on closed interval 闭区间上连续函数的性质 ······ 57
Introduction to Cauchy 柯西简介 ······ 64
Exercises 习题 ······ 65
Chapter 2 Differential calculus of one variable functions and its applications
一元函数的微分学及其应用 ······ 66
2.0 Cited examples 引例 ······ 67
2.1 Derivative 导数 ······ 67
2.2 Rules of finding derivative 求导法则 ······ 76
2.3 Derivatives of higher order and relative rates of change 高阶导数与相关变化率 ······ 92
2.4 Differential and local linear approximation of functions 函数的微分与线性逼近 ······ 97
2.5 Finding limits by using derivative——L'Hospital rule
用导数求极限——罗必达法则 ······ 100
2.6 Mean value theorem of differential 微分中值定理 ······ 107
2.7 Approximation to functions by using polynomial——Taylor formula
用多项式逼近函数——泰勒公式 ······ 116
2.8 Properties of functions by derivatives 用导数研究函数的性质 ······ 124
2.9 Curvature of plane curves 平面曲线的曲率 ······ 136
Introduction to Lagrange 拉格朗日简介 ······ 138

Introduction to Taylor 泰勒简介 ······ 139
Exercises 习题 ······ 140

Chapter 3 Integral calculus of one variable functions and its application 一元函数的积分及其应用 ······ 144

3.0 Cited examples 引例 ······ 144
3.1 Concepts, properties and integrable rule of definite integral 定积分的概念、性质与可积准则 ······ 145
3.2 Fundamental theorem of calculus 微积分基本定理 ······ 157
3.3 Indefinite integral 不定积分 ······ 170
3.4 Computation of definite integral 定积分的计算 ······ 192
3.5 Applications of definite integral 定积分的应用 ······ 203
3.6 Improper integral 反常积分 ······ 221
Introduction to Newton 牛顿简介 ······ 224
Introduction to Leibniz 莱布尼兹简介 ······ 225
Introduction to Riemann 黎曼简介 ······ 226
Exercises 习题 ······ 227

Chapter 4 Differential Equations 微分方程 ······ 229

4.0 Cited examples 引例 ······ 230
4.1 Basic concepts of differential equations 微分方程的基本概念 ······ 233
4.2 Elementary integration methods for solving some simple differential equations 解简单微分方程的初等积分法 ······ 234
4.3 Introduction to methods for establishing differential equations 建立微分方程的方法简介 ······ 235
4.4 Higher-order differential equations 高阶微分方程 ······ 235
Introduction to Euler 欧拉简介 ······ 255
Exercises 习题 ······ 256

References 参考文献 ······ 259

Introduction

Calculus is a branch of mathematics that includes the study of limits, derivatives, integrals, and infinite series, and constitutes a major part of modern university education. Historically, it was sometimes referred to as "the calculus of infinitesimals", but that usage is seldom seen today. Most basically, calculus is the study of change, in the same way that geometry is the study of space.

Calculus has widespread applications in science and engineering and is used to solve problems for which algebra alone is insufficient. Calculus builds on algebra, trigonometry, and analytic geometry and includes two major branches: **differential calculus** and **integral calculus**, that are closely related by the fundamental theorem of calculus. In more advanced mathematics, calculus is usually called analysis and is defined as the study of functions.

The history of calculus falls into several distinct time periods, most notably the ancient, medieval, and modern periods. The ancient period introduced some of the ideas of integral calculus, but does not seem to have developed these ideas in a rigorous or systematic way. Calculating volumes and areas, the basic function of integral calculus, can be traced back to about 1800 BC, when an Egyptian successfully calculated the volume of a pyramidal frustum. From the school of Greek mathematics, Eudoxus (c. 408-355 BC) used the method of exhaustion, which prefigures the concept of the limit, to calculate areas and volumes while Archimedes (c. 287-212 BC) developed this idea further, inventing heuristics which resemble integral calculus.

The method of exhaustion was later used to find the area of a circle in China by Liu Hui, around 3rd century, a Chinese mathematician who lived in the Wei Kingdom, in the 3rd century AD in order to find the area of a circle. It was also used by Zu Chongzhi (429-500, a prominent Chinese mathematician and astronomer during the Liu-Song and Southern Qi Dynasties) in the 5th century AD, who used it to find the volume of a sphere. The Indian mathematician-astronomer Aryabhata in 499 used a notion of infinitesimals and expressed an astronomical problem in the form of a basic differential equation. Manjula, in the 10th century, elaborated on this differential equation in a commentary. This equation eventually led an Indian mathematician in the 12th century to develop the concept of a derivative representing infinitesimal change, and he described an early form of "Rolle's theorem".

In the modern period, independent discoveries in calculus were being made in the early 17th Japan, such as the method of exhaustion is expanded. In Europe, the second half of the 17th century was a time of major innovation. Calculus provided a new opportunity in mathematical physics to solve long-standing problems. Several mathematicians contributed to these breakthroughs, notably John Wallis (1616-1703, an English mathematician) and Isaac Barrow (1630-1677, an English mathematician). James Gregory (1638-1675, a Scottish mathematician and astronomer) was able to prove a restricted version of the second fundamental theorem of calculus in AD 1668.

While some of the ideas of calculus were developed earlier, in Greece, China, India, and Japan, the modern use of calculus began in Europe, during the 17th century, when Newton (1643-1727, an English physicist, mathematician and astronomer) and Leibnitz (1646-1716, a Germany mathematician) built on the work of earlier mathematicians to introduce the basic principles of calculus. This work had a strong impact on the development of physics. Newton and Leibnitz pulled these ideas together into a coherent whole, and they are usually credited with the independent and nearly simultaneous invention of calculus. Newton was the first to apply calculus to general physics and Leibnitz developed much of the notation used in calculus today; he often spent days determining appropriate symbols for concepts. The basic insight that both Newton and Leibnitz had was the fundamental theorem of calculus. When Newton and Leibnitz first published their results, there was great controversy over which mathematician (and therefore which country) deserved credit. Newton derived his results first, but Leibnitz published first. A careful examination of the papers of Leibnitz and Newton shows that they arrived at their results independently. Today, both Newton and Leibnitz are given credit for developing calculus independently. It is Leibnitz, however, who gave the new discipline its name. Newton called his calculus “the science of fluxions”.

Since the time of Leibnitz and Newton, many mathematicians have contributed to the continuing development of calculus. For example, Sweden mathematician Euler (1707-1783), by using calculus as a tool, solved many problems in the fields of astronomy, physics and mechanics, and also founded many new subjects such as differential equation, infinite series and calculus of variations. And the first systematically integrated book on analysis, *The Infinitesimal Analysis,* was published in 1748. But no efficient solution to the loose base of calculus had been found. In the 19th century, calculus was put on a much more rigorous footing by mathematicians such as Cauchy (1789-1857, a French mathematician), Riemann (1826-1866, a Germany mathematician), and Weierstrass (1861-1897, a French mathematician).

Applications of differential calculus include computations involving velocity and acceleration, the slope of a curve, and optimization. Applications of integral calculus include computations involving area, volume, arc length, center of mass, work, and pressure. More advanced applications include power series and Fourier series. Calculus is also used to gain a more precise understanding of the nature of space, time, and motion. For centuries, mathematicians and philosophers wrestled with paradoxes involving division by zero or sums of infinitely many numbers. These questions arise in the study of motion and area. The ancient Greek philosopher Zeno gave several famous examples of such paradoxes. Calculus provides tools, especially the limit and the infinite series, which resolve the paradoxes.

Calculus is rooted in different subjects of natural science and social science, which not only supplies all mathematical methods and algorithms with methodology, but also cultivate people's thinking mode. Calculus is a ubiquitous topic in most modern high schools and universities, and mathematicians around the world continue to contribute to its development.

引　　言

微积分学是数学的一个分支，主要研究极限、导数、积分和无穷级数等内容。很久以前，人们曾把微积分学看作"无穷小量的微积分学"。总体说来，微积分学是关于变量的研究，而几何学是关于空间的研究。

微积分在科学与工程中有着广泛的应用，常常能够解决代数学无法解决的问题。微积分建立在代数学、三角学以及解析几何的基础之上，包含两个部分：微分学和积分学，联系这两个分支的桥梁是微积分学基本定理。在高等数学中，微积分通常被称为分析学，研究的对象是函数。

微积分的发展历史经历了几个不同的阶段，大致可分为古代、中世纪和现代。定积分的思想起源于古代，但那时这个思想并没有得到很好的发展。对积分学能够用于计算体积和面积这些基本功能的较早认识要追溯到大约公元前 1800 年，那时一个埃及人成功地计算出了金字塔平截头体部分的体积。在希腊的一所数学学校里教书的欧德克斯（大约公元前 408—355）用一种被称为"穷竭法"的思想来计算体积和面积，而这正是极限概念的萌芽。后来，阿基米得（大约公元前 287—212）进一步提出了对后人更富有启发性的思想，而这个思想与积分学思想已经十分接近。

在公元 3 世纪，我国的刘辉（约公元 3 世纪，是魏朝时期的一个数学家）用这种所谓的"穷竭法"思想计算了圆周的面积；在公元 5 世纪，我国的祖冲之（429—500，是南北朝时期南朝宋齐之间的一位杰出的数学家和天文学家）也用这种思想求出了球的体积。公元 499 年，印度数学家和天文学家阿亚巴哈塔用无穷小的概念将一个天文学问题表示成了微分方程的形式。公元 10 世纪，玛纽拉在一篇注释中详尽地阐述了这个微分方程，而它最终导致一位印度数学家在公元 12 世纪发展了表示无穷小变化的导数的概念，并且他还描述了罗尔定理的早期形式。

在 17 世纪初期，日本数学家做出了许多独立的发现，例如他们拓展了穷竭法。在欧洲，17 世纪下半叶是创新的主要时期。微积分给人们提供了一个新的机遇，以解决在数学物理中长期悬而未决的问题，几位数学家在此领域做出了突破性的进展，比较著名的有英国数学家约翰·沃利斯（1616—1703）和艾萨克·巴罗（1630—1677）。在公元 1668 年，苏格兰数学家和天文学家詹姆斯·乔治（1638—1675）已经能够给出微积分学第二基本定理的严格证明。

如果说微积分的某些思想发源于中国、印度和日本，那么微积分的应用则开始于 17 世纪的欧洲。基于早期数学家的工作，牛顿（1643—1727，英国物理学家、数学家和天文学家）和莱布尼茨（1646—1716，德国数学家）建立了微积分学中的基本原理，他们的工作对物理学的发展产生了强烈的影响。他们把那些分散的想法有机地融合成一个整

体，通常认为他们是各自独立、几乎是同时发明了微积分。牛顿首先把微积分应用到物理学中，而莱布尼茨则发展了沿用至今的许多微积分的符号。牛顿和莱布尼茨的基本贡献是建立了微分学和积分学之间的桥梁，即微积分学基本定理。就在牛顿和莱布尼茨都各自发表了自己的结果时，在就谁（甚至牵扯到是哪个国家）应当值得享有发明微积分的这个莫大荣誉的问题上出现了巨大的分歧和争论。牛顿首先推导出了这个结果，而莱布尼茨则首先发表了这个结果。仔细推敲他们的文章后，人们发现是他们各自独立地发明了微积分。所以他们应当共同分享发明微积分这一殊荣。最初牛顿把他的发明称作“流数术”，而莱布尼茨则把他的发明称做微积分，这一称谓沿用至今。

自从牛顿和莱布尼茨发明了微积分以后，许多数学家都对微积分的发展作出了贡献。例如，瑞士数学家欧拉以微积分为工具解决了天文学、物理学和力学中的许多问题，并且他还由此创立了几个新的数学分支如微分方程、无穷级数及变分法等。1748 年，欧拉出版了分析学中的第一部系统而完整的著作《无穷小分析》。不过，那时人们也发现微积分的基础并不牢固。直到 19 世纪，柯西（1789—1857，法国数学家）、黎曼（1826—1866，德国数学家）和魏尔施特拉斯（1861—1897，法国数学家）等数学家才建立了微积分的严格数学理论基础。

微分学的应用包括计算速度、加速度以及曲线的斜率和最优化等。定积分的应用包括计算面积、体积、弧长以及质心、功和压力等。幂级数和傅里叶级数等则是微积分的更高层次的应用。微积分也常常被用来精确理解空间、时间和运动的本性。几个世纪以来，数学家和哲学家经常就被零除以及无穷多个数的和等问题发生争论。这些问题源自于对运动和面积的研究。例如，古希腊哲学家芝诺就曾给出了几个著名悖论。不过，用微积分特别是以极限和无穷级数为工具，这些矛盾都能被轻易化解。

如今微积分学在自然科学和社会科学中的不同领域已经根深蒂固，它不仅提供了许多数学方法、计算方法和方法论，而且也培养了人们的思考模式。现在，微积分已经走进大多数大、中学校的课堂，而且许多国内外的数学家都在为它的发展而努力。

Chapter 1

Functions, limits and continuity

函数、极限与连续性

The object of calculus is to study functions which formed in the 17th century. With the unceasing development of science and technology, people's understanding to functions also deepens and develops gradually.

Limit is a basic operation of calculus, and limit approach is a main tool to study functions. Calculus consists of two parts, namely, differential calculus and integral calculus, in which numerous important concepts are defined by using limit approach. Limit theory is the foundation and "soul" running through the whole calculus.

As an important property of functions, continuity is the impersonal reflection and the mathematical depiction of the gradually variational phenomena existing widely in the infinite universe. In theoretical researches and practical applications, continuous functions always hold the important status. This course is to mainly study continuous functions.

In this chapter, we first introduce the concepts and properties of functions as well as the notions of elementary functions.

Limit is the emphasis of this chapter. We mainly introduce the definitions of limits of functions, while the limit of sequence of numbers will be dealt as a special case. The content of this part includes the definitions, properties and operations of limits, two important formulas on limit, the concepts of infinity and infinitesimal, and the properties and applications of infinitesimal.

The later part of this chapter is to introduce the concepts of continuous functions, the concepts and classifications of discontinuity points of functions. And then we discuss the properties of continuous functions and the continuity of elementary functions. Finally, we will introduce some important properties of continuous functions on a closed interval in geometry.

1.0 Cited examples

引　　例

From the macroscopical space of universe to the microcosmic world of particle and from the quotidian life to the fast development of high and exact technologies, the word “quantity” exists everywhere. Relations of quantities, changes of quantities, changes of relations of quantities and relations of changes of quantities are just important contents for studying in mathematics.

According to the tax laws of our country, individual wage and stipend should pay for personalincome tax, and the tax rate is different from person to person. Can you list concisely the conversion relation between person income and tax paid?

Let’s put a square stable with four equal legs on an uneven ground. Can you make the four legs contact simultaneously the ground such that the table stands steadily?

Throwing shot is a sport game. What is the quantity relation among the throwing distance, the initial speed and the angle when someone throws the shot away from his hand? Which factors should be considered for achieving a perfect result in daily training?

You may have heard of the Fibonacci’s sequence which is also called the interesting rabbit problem! You can find some similar results from the “pedigree” of honeybees, the arrangement of pianos’ syllables and the branches of trees. The phrase “golden mean” put forward by Da Vinci, a famous artist in middle ages, means a proportion relation between quantities. As the story goes, the relation can make buildings more handsome and make tonalities more harmonious. Interestingly, many blooming flowers and bonny builds of people also possess its traits. Surprisingly, the two problems which are seemingly independent have close immanent connection, would you like to know?

You should have learnt how to calculate the area and the perimeter of a regular polygon when studying plane geometry in middle school. As is well known, the area and the perimeter of any regular n polygon inscribed in a unit circle, however large n is, are always smaller than π and 2π, respectively. A question is that whether you can construct a figure such that it is contained in a unit circle, and its area is not more than π, while its perimeter tends to infinity.

These interesting problems with some application backgrounds are related to the contents of this chapter, i.e., functions, limits and continuity.

1.1 Functions
函　　数

Key points of this section

(1) Understand and master the concept and properties of function, such as the independent variable and dependent variable in the definition of a function, the domain and range of a function, the boundedness, monotonicity, parity, periodicity, and so on.

（2）Understand the concepts of composite function and inverse function; master the conditions for composing the composite function and the existence theorem of inverse functions.

（3）Understand the concepts of elementary function and non-elementary functions.

本节重点

（1）理解并掌握函数的概念和性质，如函数定义中的自变量、因变量，函数的定义域、值域，函数的有界性、单调性、奇偶性、周期性等.

（2）理解复合函数与反函数的概念，掌握构成复合函数的条件以及反函数存在定理.

（3）理解初等函数和非初等函数的概念.

Words and expressions 单词和短语

rational numbers 有理数

irrational numbers 无理数

real number 实数

definition 定义

proposition 命题

variable 变量

constant 常数

function 函数

independent variable 自变量

dependent variable 因变量

domain 定义域

one variable function 一元函数

range 值域

constant function 常量函数

power function 幂函数

exponential function 指数函数

logarithmic function 对数函数

trigonometric function 三角函数

inverse trigonometric function 反三角函数

basic elementary function 基本初等函数

piecewise function 分段函数

boundedness 有界性

monotonicity 单调性

parity 奇偶性

periodicity 周期性

bounded 有界的

unbounded 无界的

monotone increasing 单调增加

monotone decreasing 单调减少

monotonic function 单调函数

even function 偶函数

odd function 奇函数

periodic function 周期函数

composite function 复合函数

intermediate variable 中间变量

inverse function 反函数

mapping 映射

composite mapping 复合映射

surjection 满射

injection 单射

one-to-one mapping 一一映射

inverse mapping 逆映射