牛津大学研究生教材

多体系统的量子场论

文小刚 著

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多体系统的量子场论

从声子的起源到光子和电子的起源

文小刚
Department of Physics, MIT

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To my wife, Xiao-Li Li

影印版前言

本书是作为牛津大学出版社研究生教材系列丛书选定的一部凝聚态理论教科书。是1996年至2002年作者在麻省理工学院讲授量子多体物理课程期间成型，并于2004年面世的。作者的另一本内容相近的教材《量子多体理论》，由胡滨译成中文，也是在2004年由我国高等教育出版社作为《当代科学前沿丛书》之一出版，估计应该是由本书初稿翻译的。本书显然是延续原来思路，扩充了一些最新进展之后重新编排和改写而成。作者在本书中详尽地阐述了量子力学路径积分方法和量子场论方法在凝聚态物理中最重要的一些问题中的应用。涵盖了相当广泛的物理概念和各种计算方法，阐明了许多过去被忽略或未充分强调的方面。讲述了学科发展的进程和尚未解决的问题，旨在帮助学生尽快掌握现代凝聚态理论的前沿知识，是一部独具特色的优秀专著。

对于本书的详细介绍，包括内容的取舍及其特色、选材遵循的指导原则、与同类图书的比较以及使用的建议等，作者在序言中都有详细的说明，这里不打算赘述。我们只强调指出，该书的副标题“从声子的起源到光子和电子的起源”，无疑是本书一个亮点，非常值得关注。按照传统观念，声子是晶体或者超流体中原子集团运动的低能激发，是晶格振动衍生的准粒子。它们不可能成为高能和短距离下物质的基本组分。而光子和电子则完全不同。按照当前高能物理中经受了无数精确的实验检验建立起来的标准模型，电子与夸克同属类点的基本粒子，是我们物质世界的最基本的组分。光子是传递电磁相互作用的零质量的规范玻色子。但是，作者在本书中试图颠覆这些传统观念，认为可以把真空看作是一种极为特殊的凝聚态，根本不存在什么基本粒子。所谓的“基本”粒子实际上都是这种凝聚态的低能集团激发，都属于“衍生”粒子。通过近二十年的研究工作，作者及其合作者们发现光和费米子以及库仑定律都是来自于真空的量子序。所谓的弦网凝聚（string - net condensation）提供了一种把光和费米子统一的方法。对于“光与费米子是什么?”“它们来自何处?”以及”为什么存在光和费米子?”等基本问题，尝试给出非常新颖、有趣的回答。诚如作者指出的，这些个人的或许有些夸张的观点可能并不正确，但它具有非常引人注目的启发价值。读者不妨跟随作者的思路，认真地读一读这本书并深入地思考这些基本问题，应该大有裨益。

本书的作者文小刚，于1977年进入中国科学技术大学物理系。通过1981年度中国－美国联合招考赴美物理研究生（英文简称CUSPEA）招生考试。文小刚以第一名成绩到美国普林斯顿大学深造，现为美国麻省理工学院教授，加利福尼亚理工学院特邀摩尔学者，清华大学长江学者讲座教授。他作为CUSPEA留学生中一直坚守物理学研究的一位杰出代表，在量子霍耳效应理论、高温超导理论、强关联系统中的量子序和拓扑序及新的物质态、基本粒子的起源等等现代凝聚态理论研究方面做出了具有重大影响的贡献，在我国物理学

界具有很高的知名度。

凝聚态理论是我国物理类相关科目研究生的一门重要的必修基础理论课。本书内容非常充实，观点异常新颖，展现了作者的许多新想法。本书的数学推导详尽、而且每章都配有习题，帮助读者理解和掌握讲授的内容。书中每一小节都先列出要点提示，便于内容的取舍，和难点的掌握，对于教学十分有利。但总体而言本书数学难度较大，对于初学者学习起来有一定困难。但对于攻读凝聚态各相关专业的高年级研究生和相关领域的研究人员是一部非常有用的参考书。

中国科学院大学　丁亦兵

前　言

凝聚态物质（即固体和液体）的量子理论过去主要有两个主题。第一个主题是能带理论和微扰理论，它多少基于朗道的费米液体理论。第二个主题是朗道的对称性破缺理论和重整化群理论。凝聚态理论是一个非常成功的理论。它使我们理解了几乎所有形态物质的性质。第一个主题的一项成果是半导体理论，它奠定了各种电子器件的理论基础，带动了当今科技的高速发展。第二个主题也非常重要，它让我们理解了物质的态以及各种态之间的相变，它是液晶显示和磁记录等技术的理论基础。

由于凝聚态理论如此成功，人们开始有大功告成的心态，感觉凝聚态理论的发展即将结束。然而，本书却试图展现一幅不同的图像，我们所看到的不过是它刚刚开始。一个全新的世界有待于我们去探索。

全新世界的一个暗示是分数量子霍尔效应的发现（Tsui et al，1982），另一个则是高温超导体的发现（Bednorz and Mueller，1986）。这两种现象都完全超出了以上提及的两个主题。近 20 年来，分数量子霍尔效应和高温超导方面快速而振奋人心的发展带来了许多新的思想和新的概念。我们目睹凝聚态系统多体理论中一个新的主题正在脱颖而出。对凝聚态物理来说，这是一个令人激动的时刻。凝聚态物理的新范例甚至会影响人们对自然界中一些基本问题的认识。

正是在这样的背景下，我写了这本书[1]。本书的前半部分介绍了两个老的主题，被称为传统凝聚态理论[2]。本书的第二部分窥视了新出现的主题，它被称为现代凝聚态理论。在这第二部分中所包含的内容非常新颖，有些只是几个月前新发现的结果。该理论仍在迅速发展。

读完本书，我希望读者感觉的不是完美，而是渺茫。凝聚态理论发展已经有 100 多年了，我们从中受益良多，但对丰富的自然界仍知之甚少。然而，我希望读者感到的不是灰心，而是被我们的不完备的理解所激励。这意味着凝聚态理论中更有趣和更激动人心的时刻仍然在我们前面，不是已落在我们后面。我还希望读者将会感觉到自信：没有什么不能回答的问题，也没有什么不能理解的谜。尽管世界上还有很多奥秘有待破解，但人们还是理解了很多曾被认为是神秘莫测的谜团。我们已经理解了许多非常很基本的问题，当初这些问题基本到似乎不可能有答案。人脑的想象力是没有极限的[3]。

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1 当我 1996 年开始写这本书时，计划写入量子多体理论中一些新的和令人振奋的进展。当时，尚不清楚这些新的进展是否会变成凝聚态理论中的一个新的主题。现在，经过最近取得的一系列进展后，我确信一个新的主题正在凝聚态理论中形成。然而，该理论仍处于其发展的早期阶段。只有时间会将会告诉我们是否真正得到一个新的主题。

2 有人也将第一个主题称为传统凝聚态理论，而将第二个主题称为现代凝聚态理论。

3 我不知道谁将成为“赢家”，是自然界的丰富性还是人类想象力的无限性。

本书是 1996 年至 2002 年我在麻省理工学院讲授量子多体物理课程期间成型的。读者是对现代理论物理感兴趣的研究生。本书第一部分（第 2 章—第 5 章）介绍传统的多体物理，其中包括路径积分、线性响应、摩擦的量子理论、相互作用玻色子和费米子的平均场理论、对称性破缺和长程有序、重整化群、正交突变、费米液体理论和非线性 σ 模型。第二部分（第 6 章—第 10 章）涵盖了现代多体物理的很多主题，其中包括分数量子霍尔理论、分数统计、流代数和玻色化、量子规范理论、拓扑序和量子序、弦网凝聚、衍生的规范玻色子和费米子、量子自旋液体的平均场理论和二维或三维精确可解模型。

本书所采用的方法大多是基于量子场论和路径积分。在我们讨论的许多问题中，低能有效理论起着核心作用。即使在第一部分，我尝试用更为现代方法来处理一些老问题。我也试着强调一些传统凝聚态物理中的更现代的论题。第二部分包括了最近的一些工作。其中大约有一半来自最近几年的研究工作，部分内容取自我的研究论文和综述文章（当然一些研究论文则源自于本书的部分章节）。

本书在写作上不追求以简洁和紧凑的数学公式表述内容，而是强调物理图像以及想法和观点的发展，计算和结果的阐述都是为了揭示物理图像。本书不是要清除那些丑陋的假设，而是试图揭露它们。本书还尝试揭示了一些利用常见方法得到不正确的结果，目的是为了强调（而不是隐藏）这些方法的局限性。

本书没有罗列众多不同的系统和不同的现象，只考虑了几个简单的系统。通过这些简单系统，广泛讨论了凝聚态理论的物理思想、概念和方法的。书中的小字部分是评论或者更高级的议题，初次阅读可以忽略。

本书的另一个特点是试图质疑和揭示多体物理和，更普遍地，理论物理中的一些基本思想和图像。例如："什么是费米子?"，"什么是规范玻色子?"，相变和对称性破缺的思想，"一种序总是用一个序参量来描述吗?" 等。这里，我认为并非什么都是理所当然的。我希望这些讨论能够鼓励读者探究漂亮的数学公式背后包含的物理思想，使读者意识到某一些物理概念的丑陋性和随意性。

由于数学的形式体系变得越来越漂亮了，人们很容易被这种数学形式体系所羁绊，变成形式体系的"奴隶"。当我们把一切都看作是粒子的集合时，我们曾经成为牛顿定律的"奴隶"。当量子理论被发现之后[1]，我们成为量子场论的"奴隶"。目前，我们希望用量子场论解释一切，我们的教育不鼓励我们去超越量子场论。

然而，为了使物理学取得革命性进展，我们不能允许我们想象力为形式体系所困束缚。我们也不能允许形式体系限定我们想象力的边界。数学形式体系只不过是描述和传达我们的想象的一个工具或一种语言。有时，当你有了一个新的思想或者新的想法，你或许会发现你什么都说不出来。无论你说什么都是错的，这是因为能够描述这种新思想或新想法的合适的数学或恰当的语言还没有被发明出来。事实上，真正新的物理思想通常需要一个新的数学形式体系来描述它们。这让我想起了一个关于一个部落的故事。这个部落只有四个字用来计数：一个，两个，三个和许多许多。试想一下，一个部落成员有了关于两个苹果加两个苹果、三个苹果加三个苹果的一种想法。他将很难把他的理论解释给其他成员。这应该就是当你有了一个真正意义上的新思想时你的感觉。虽然本书命名为"多体系统的量

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1 经典粒子的概念在量子理论中被打破了，见 2.2 节的讨论。

子场论”，我希望阅读本书后，读者将会看到量子场论不是万能的。自然界的丰富性不会被量子场论所束缚。

我要感谢 Margaret O'Meara，她校对了本书的许多章节。我还要感谢 Anthony Zee，Michael Levin，Bas Overbosch，Ying Ran，Tiago Ribeiro 和 Fei-Lin Wang，他们给本书提出了很多意见和建议。最后，但并非最不重要，我要感谢技术编辑 Julie Harris 博士，她为本书的编辑和润色付出了很大的努力。

文小刚
Lexington，MA
2003 年 10 月

目 录

1

INTRODUCTION

1.1 More is different

- **The collective excitations of a many-body system can be viewed as particles. However, the properties of those particles can be very different from the properties of the particles that form the many-body system.**
- **Guessing is better than deriving.**
- **Limits of classical computing.**
- **Our vacuum is just a special material.**

A quantitative change can lead to a qualitative change. This philosophy is demonstrated over and over again in systems that contain many particles (or many degrees of freedom), such as solids and liquids. The physical principles that govern a system of a few particles can be very different from the physical principles that govern the collective motion of many-body systems. New physical concepts (such as the concepts of fermions and gauge bosons) and new physical laws and principles (such as the law of electromagnetism) can arise from the correlations of many particles (see Chapter 10).

Condensed matter physics is a branch of physics which studies systems of many particles in the 'condensed' (i.e. solid or liquid) states. The starting-point of current condensed matter theory is the Schrödinger equation that governs the motion of a number of particles (such as electrons and nuclei). The Schrödinger equation is mathematically complete. In principle, we can obtain all of the properties of any many-body system by solving the corresponding Schrödinger equation.

However, in practice, the required computing power is immense. In the 1980s, a workstation with 32 Mbyte RAM could solve a system of eleven interacting electrons. After twenty years the computing power has increased by 100-fold, which allows us to solve a system with merely two more electrons. The computing power required to solve a typical system of 10^{23} interacting electrons is beyond the imagination of the human brain. A classical computer made by all of the atoms in our universe would not be powerful enough to handle the problem.[5] Such an impossible computer could only solve the Schrödinger equation for merely about 100

[5] It would not even have enough memory to store a single state vector of such a system.

particles.[6] We see that an generic interacting many-body system is an extremely complex system. Practically, it is impossible to deduce all of its exact properties from the Schrödinger equation. So, even if the Schrödinger equation is the correct theory for condensed matter systems, it may not always be helpful for obtaining physical properties of an interacting many-body system.

Even if we do get the exact solution of a generic interacting many-body system, very often the result is so complicated that it is almost impossible to understand it in full detail. To appreciate the complexity of the result, let us consider a tiny interacting system of 200 electrons. The energy eigenvalues of the system are distributed in a range of about 200 eV. The system has at least 2^{200} energy levels. The level spacing is about $200\,\text{eV}/2^{200} = 10^{-60}\,\text{eV}$. Had we spent a time equal to the age of the universe in measuring the energy, then, due to the energy–time uncertainty relation, we could only achieve an energy resolution of order 10^{-33} eV. We see that the exact result of the interacting many-body system can be so complicated that it is impossible to check its validity experimentally in full detail.[7] To really understand a system, we need to understand the connection and the relationship between different phenomena of a system. Very often, the Schrödinger equation does not directly provide such an understanding.

As we cannot generally directly use the Schrödinger equation to understand an interacting system, we have to start from the beginning when we are faced with a many-body system. We have to treat the many-body system as a black box, just as we treat our mysterious and unknown universe. We have to guess a low-energy effective theory that directly connects different experimental observations, instead of deducing it from the Schrödinger equation. We cannot assume that the theory that describes the low-energy excitations bears any resemblance to the theory that describes the underlying electrons and nuclei.

This line of thinking is very similar to that of high-energy physics. Indeed, the study of strongly-correlated many-body systems and the study of high-energy physics share deep-rooted similarities. In both cases, one tries to find theories that connect one observed experimental fact to another. (Actually, connecting one observed experimental fact to another is almost the definition of a physical theory.) One major difference is that in high-energy physics we only have one 'material' (our vacuum) to study, while in condensed matter physics there are many different materials which may contain new phenomena not present in our vacuum (such as fractional statistics, non-abelian statistics, and gauge theories with all kinds of gauge groups).

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[6] This raises a very interesting question—how does nature do its computation? How does nature figure out the state of 10^{23} particles one second later? It appears that the mathematics that we use is too inefficient. Nature does not do computations this way.

[7] As we cannot check the validity of the result obtained from the Schrödinger equation in full detail, our belief that the Schrödinger equation determines all of the properties of a many-body system is just a *faith*.

1.2 'Elementary' particles and physics laws are emergent phenomena

- **Emergence—the first principle of many-body systems.**
- **Origin of 'elementary' particles.**
- **Origin of the 'beauty' of physics laws. (Why nature behaves reasonably.)**

Historically, in our quest to understand nature, we have been misled by a fundamental (and incorrect) assumption that the vacuum is empty. We have (incorrectly) assumed that matter placed in a vacuum can always be divided into smaller parts. We have been dividing matter into smaller and smaller parts, trying to discover the smallest 'elementary' particles—the fundamental building block of our universe. We have been believing that the physics laws that govern the 'elementary' particles must be simple. The rich phenomena in nature come from these simple physics laws.

However, many-body systems present a very different picture. At high energies (or high temperatures) and short distances, the properties of the many-body system are controlled by the interaction between the atoms/molecules that form the system. The interaction can be very complicated and specific. As we lower the temperature, depending on the form of the interaction between atoms, a crystal structure or a superfluid state is formed. In a crystal or a superfluid, the only low-energy excitations are collective motions of the atoms. Those excitations are the sound waves. In quantum theory, all of the waves correspond to particles, and the particle that corresponds to a sound wave is called a phonon.[8] Therefore, at low temperatures, a new 'world' governed by a new kind of particle—phonons—emerges. The world of phonons is a simple and 'beautiful' world, which is very different from the original system of atoms/molecules.

Let us explain what we mean by 'the world of phonons is simple and beautiful'. For simplicity, we will concentrate on a superfluid. Although the interaction between atoms in a gas can be complicated and specific, the properties of emergent phonons at low energies are simple and universal. For example, all of the phonons have an energy-independent velocity, regardless of the form of the interactions between the atoms. The phonons pass through each other with little interaction despite the strong interactions between the atoms. In addition to the phonons, the superfluid also has another excitation called rotons. The rotons can interact with each other by exchanging phonons, which leads to a dipolar interaction with a force proportional to $1/r^4$. We see that not only are the phonons emergent, but even the physics laws which govern the low-energy world of the phonons and rotons are emergent. The emergent physics laws (such as the law of the dipolar interaction and the law of non-interacting phonons) are simple and beautiful.

[8] A crystal has three kinds of phonons, while a superfluid has only one kind of phonon.

I regard the law of $1/r^4$ dipolar interaction to be beautiful because it is not $1/r^3$, or $1/r^{4.13}$, or one of billions of other choices. It is precisely $1/r^4$, and so it is fascinating to understand why it has to be $1/r^4$. Similarly, the $1/r^2$ Coulomb law is also beautiful and fascinating. We will explain the emergence of the law of dipolar interaction in superfluids in the first half of this book and the emergence of Coulomb's law in the second half of this book.

If our universe itself was a superfluid and the particles that form the superfluid were yet to be discovered, then we would only know about low-energy phonons. It would be very tempting to regard the phonon as an elementary particle and the $1/r^4$ dipolar interaction between the rotons as a fundamental law of nature. It is hard to imagine that those phonons and the law of the $1/r^4$ dipolar interaction come from the particles that are governed by a very different set of laws.

We see that in many-body systems the laws that govern the emergent low-energy collective excitations are simple, and those collective excitations behave like particles. If we want to draw a connection between a many-body system and our vacuum, then we should connect the low-energy collective excitations in the many-body system to the 'elementary' particles (such as the photon and the electron) in the vacuum. But, in the many-body system, the collective excitations are not elementary. When we examine them at short length scales, a complicated non-universal atomic/molecular system is revealed. Thus, in many-body systems we have collective excitations (also called quasiparticles) at low energies, and those collective excitations very often do not become the building blocks of the model at high energies and short distances. The theory at the atomic scale is usually complicated, specific, and unreasonable. The simplicity and the beauty of the physics laws that govern the collective excitations do not come from the simplicity of the atomic/molecular model, but from the fact that those laws have to allow the collective excitations to survive at low energies. A generic interaction between collective excitations may give those excitations a large energy gap, and those excitations will be unobservable at low energies. The interactions (or physics laws) that allow gapless (or almost gapless) collective excitations to exist must be very special—and 'beautiful'.

If we believe that our vacuum can be viewed as a special many-body material, then we have to conclude that there are no 'elementary' particles. All of the so-called 'elementary' particles in our vacuum are actually low-energy collective excitations and they may not be the building blocks of the fundamental theory. The fundamental theory and its building blocks at high energies[9] and short distances are governed by a different set of physical laws. According to the point of view of emergence, those laws may be specific, non-universal, and complicated.

[9] Here, by high energies we mean the energies of the order of the Planck scale $M_P = 1.2 \times 10^{19}$ GeV.

The beautiful world and reasonable physical laws at low energies and long distances emerge as a result of a 'natural selection': the physical laws that govern the low-energy excitations should allow those excitations to exist at low energies. In a sense, the 'natural selection' explains why our world is reasonable.

Someone who knows both condensed matter physics and high-energy physics may object to the above picture because our vacuum appears to be very different from the solids and liquids that we know of. For example, our vacuum contains Dirac fermions (such as electrons and quarks) and gauge bosons (such as light), while solids and liquids seemingly do not contain these excitations. It appears that light and electrons are fundamental and cannot be emergent. So, to apply the picture of emergence in many-body systems to elementary particles, we have to address the following question: can gauge bosons and Dirac fermions emerge from a many-body system? Or, more interestingly, can gauge bosons and Dirac fermions emerge from a many-boson system?

The fundamental issue here is where do fermions and gauge bosons come from? What is the origin of light and fermions? Can light and fermions be an emergent phenomenon? We know that massless (or gapless) particles are very rare in nature. If they exist, then they must exist for a reason. But what is the reason behind the existence of the massless photons and nearly massless fermions (such as electrons)? (The electron mass is smaller than the natural scale—the Planck mass—by a factor of 10^{22} and can be regarded as zero for our purpose.) Can many-body systems provide an answer to the above questions?

In the next few sections we will discuss some basic notions in many-body systems. In particular, we will discuss the notion that leads to gapless excitations and the notion that leads to emergent gauge bosons and fermions from local bosonic models. We will see that massless photons and massless fermions can be emergent phenomena.

1.3 Corner-stones of condensed matter physics

- **Landau's symmetry-breaking theory (plus the renormalization group theory) and Landau's Fermi liquid theory form the foundation of traditional condensed matter physics.**

The traditional many-body theory is based on two corner-stones, namely Landau's Fermi liquid theory and Landau's symmetry-breaking theory (Landau, 1937; Ginzburg and Landau, 1950). The Fermi liquid theory is a perturbation theory around a particular type of ground state—the states obtained by filling single-particle energy levels. It describes metals, semiconductors, magnets, superconductors, and superfluids. Landau's symmetry-breaking theory points out that