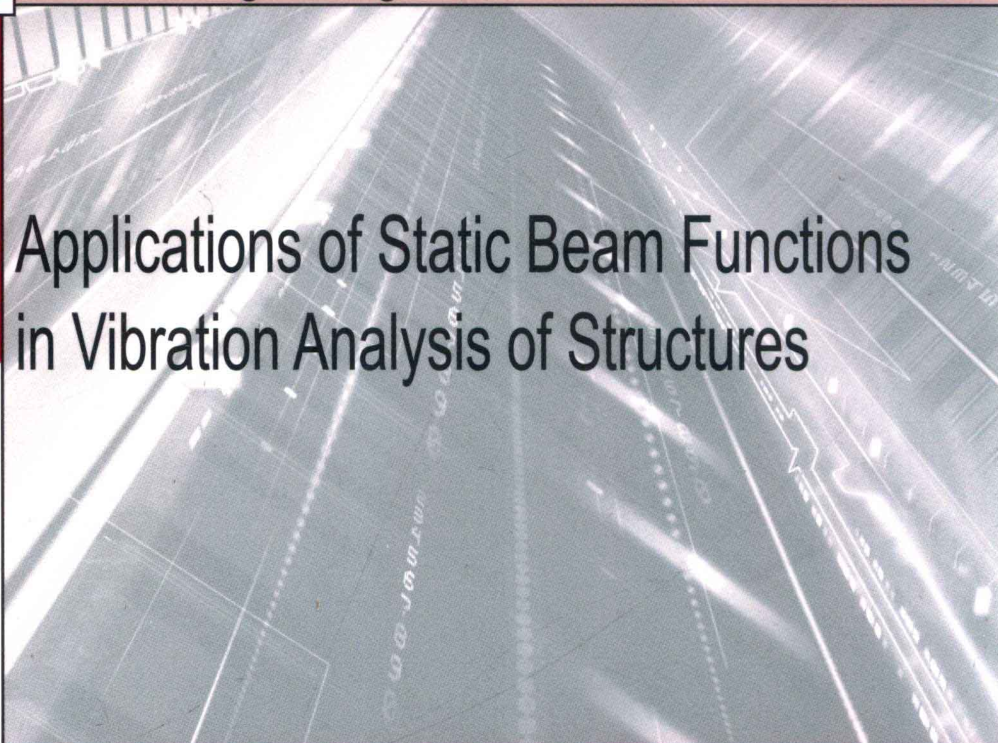
Structural Engineering Frontier Series



Applications of Static Beam Functions in Vibration Analysis of Structures

Ding Zhou

（静力梁函数在结构振动分析中的应用）

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结构工程前沿丛书

静力梁函数在结构振动分析中的应用

周　叮　著

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内 容 简 介

本书以著名的结构力学分析方法——李兹法为基础，创造性地提出了以静力梁函数作为基函数，研究梁、板结构的动力学特性，重点分析变截面和变厚度、内部支撑以及边界条件对梁、板结构振动特性的影响. 全书共23章，第1章介绍李兹法的发展史与存在的问题；第2章至第6章研究各种边界和内部支撑条件下变截面欧拉-伯努利梁和铁摩辛柯梁的振动特性；第7章至第11章研究各种边界和线支条件下等厚度基尔霍夫薄板的振动特性；第12章至第14章研究线支和点支等厚度复合材料薄板的振动特性；第15章和第16章研究变厚度基尔霍夫薄板的振动特性；第17章至第20章研究等厚度和变厚度米德林中厚板的振动特性；第21章和第22章研究线支和点支等厚度复合材料厚板的振动特性；第23章研究矩形储液罐的流-固耦合振动特性.

本书可供航空航天、机械、土木和力学等方面的科研工作者、工程设计人员、大专院校有关专业教师和研究生使用.

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Preface

The vibration characteristics of thin/thick beams and plates have long been a subject of study because these structural components are widely used in various engineering such as aerospace, marine and building. In practical applications, the complexities such as variable thickness, intermediate supports, elastic boundaries etc. could be encountered, which can greatly influence the dynamic properties of the structural components. Therefore, such a problem deserves to be studied thoroughly.

A monograph written by Leissa summarizes the research work on vibrations of thin plates before the early 1970s, which is a very famous classical book due to the citations more than 6000 times. In general, two different methods, analytical method and numerical method, are commonly used for vibration analysis of beams and plates. The analytical method can only be applied in some special cases while the numerical method has a wider scope of applications. In the numerical method, Ritz approach, comparing with other numerical approaches such as finite element and boundary element, has some distinguishing features, i.e. high accuracy, small computational cost and the ability for parameterization study. It is well known that the efficiency (convergence and accuracy) of the Ritz method greatly depends on the admissible functions (trial functions) selected for the analysis. However, none unified method was developed to construct the admissible functions of structural components, especially for the components with complexities mentioned above. The selection of the admissible functions mainly relies on the experience and preference of the user, which greatly restricts the extending applications of Ritz method.

In the book, a new general method to construct the admissible functions for beams and rectangular plates with complexities is introduced. The so-called complexities mean variable thickness, elastic/rigid point-supports or line-supports, and elastic boundary conditions. The classical Euler-Bernoulli theory is used to describe the slender beams and the classical Kirchhoff theory is used to describe the thin plates. Moreover, the Timoshenko theory is used for the thick beams and the Mindlin theory is used for the moderately thick plates. The static beam functions, which come from the solutions to the beam subjected to a series of static loads, are taken as the admissible functions. Sinusoidal loads, polynomial loads and point loads could be used to construct the admissible functions for different beams and plates.

This book is made up of twenty-three chapters. The first chapter reviews the history of the Ritz approach and the existing problems. The second and third chapters study the vibration characteristics of tapered Euler-Bernoulli beams (slender beams) with/without intermediate supports by using the static beam functions under a series of Taylor polynomial loads, respectively. The fourth chapter deals with the vibration characteristics of uniform Timoshenko beams (thick beams) with intermediate supports by using the static beam functions under a series of sinusoidal loads. The fifth chapter studies the vibration characteristics of tapered Timoshenko beams by using the static beam functions under a series of Taylor polynomial loads. The sixth chapter introduces an application of static beam functions in the estimation of vibration characteristics of a spring-mass-beam system. The seventh chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates (thin rectangular plates) by using two sets of static beam functions respectively. One set of static beam functions is under a series of point loads and the other set is under a series of sinusoidal loads. The eighth chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with elastic boundary constraints by using the static beam functions under a series of sinusoidal loads. The ninth chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with intermediate line-supports by using two sets of static beam functions respectively. One set of static beam functions is a combination of vibrating beam functions and simple polynomials and the other set is under a series of sinusoidal loads. The tenth chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with elastic intermediate line-supports and elastic boundary constraints by using the static beam functions under a series of sinusoidal loads. The eleventh chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with elastic point-supports by using the static beam functions under a series of point loads. The twelfth and thirteenth chapters study the vibration characteristics of symmetrically and asymmetrically laminated rectangular plates with intermediate line-supports by using the static beam functions under a series of sinusoidal loads, respectively. The fourteenth chapter studies the vibration characteristics of composite rectangular plates with point-supports by using the static beam functions under a series of sinusoidal loads. The fifteenth and sixteenth chapters study the vibration characteristics of tapered rectangular plates with/without intermediate line-supports by using the static beam functions under a series of Taylor polynomial loads. The seventeenth and eighteenth chapters study the vibration characteristics of Mindlin rectangular plates (moderately thick rectangular plates) with classical boundary conditions and elastic boundary conditions by using the static beam functions under a series of sinusoidal loads, respectively. The nineteenth chapter studies the vibration characteristics of Mindlin rectangular plates with intermediate line-supports by using the static beam functions under a series of sinusoidal loads. The twentieth chapter studies the vibration characteristics of tapered Mindlin rectangular plates by using the static beam functions under a series of Taylor polynomial loads. The twenty-first chapter studies the vibration characteristics of thick rectangular plates with intermediate line-supports by using a combination of vibrating beam functions and simple polynomials. The twenty-second chapter studies the vibration characteristics of thick laminated rectangular plates with point-supports by using the static beam functions under a series of sinusoidal loads. The last chapter studies the vibration characteristics of rectangular tanks partially filled with liquid by using the static beam functions under a series of sinusoidal loads.

The accuracy, convergence and numerical robustness for various cases have been investigated in detail, which indicates that the static beam functions can be usefully applied in solving a variety of beam and plate vibration problems. I hope that the present method could be extended to more research topics such as the shell vibrations in future.

Ding Zhou
Nanjing
30 March, 2013

Contents

Chapter 1

Introduction

It is well known that beams and rectangular plates are some of the most commonly used structural elements in aerospace, civil and marine engineering. Their dynamic behaviors are of practical importance to design engineers. In the practical applications, beams and rectangular plates are either uniform or non-uniform. Sometimes, internal point-supports or line-supports may be placed to reduce the magnitude of displacements and stresses of the structures or to satisfy special architectural and functional requirements. In general, the Euler-Bernoulli beam theory (Gorman, 1975) for slender beams and the classical plate theory (Timoshenko and Woinowsky-Krieger, 1959) for thin plates are sufficient to give satisfactory accuracy. However, the Timoshenko beam theory (Timoshenko, 1921) for thick beams and the Mindlin plate theory (Mindlin, 1951) for moderately thick plates should be applied if one wants to obtain more reasonable results because of the fact that the effects of transverse shear deformation and rotary inertia have been taken into account in these cases.

Exact analytical solutions for vibrations of beams and plates have been given only to special cases such as uniform beams and simply supported uniform rectangular plates. Besides this, various numerical methods such as finite element method (Zienkiewicz and Taylor, 1991), finite difference method (Gumeniuk, 1956), finite strip method (Cheung and Tham, 1997), Rayleigh-Ritz method (Young, 1950; Warburton and Edney, 1984; Bhat, 1985; Liew et al., 1998) as well as differential quadrature method (Bert and Malik, 1996) etc. have been developed to investigate those cases for which exact solutions are not available. Among these methods, the Rayleigh-Ritz method can give high accuracy with low computational effort if proper admissible functions (i.e., trial functions or basis functions) are selected. Using vibrating beam functions as the admissible functions to analyze vibrations of thin rectangular plates in the Rayleigh-Ritz method may be traced back to the beginning of the 1950s (Young, 1950). In the beginning of the 1980s, the vibration of moderately thick rectangular plates (Dawe and Roufaeil, 1980) was analyzed by using the vibrating Timoshenko beam functions as the admissible functions in the Rayleigh-Ritz method. Moreover, the vibrating beam functions of uniform beams have also been extensively used to study the vibration of non-uniform beams (Chehil and Jategaonkar, 1987; Jategaonkar and Chehil, 1989) and plates (Chopra and Durvasula, 1971; Malhotra et al., 1987) as well as elastically restrained rectangular plates (Warburton and Edney, 1984; Saha and Kar, 1996).

The effectiveness of vibrating beam functions in solving the vibration of uniform beams and rectangular plates is generally acknowledged. However, it has been found that for beams or plates with varying thickness, the results with satisfactory accuracy can be obtained only when the truncation factor (the ratio of smaller end thickness to the larger end thickness) is larger than 0.3 (Cheung and Zhou, 1999c; Zhou and Cheung, 2000c). For a small truncation factor (e.g. lower than 0.3), the convergence rate of the eigenfrequencies quickly decreases with the decrease of the truncation factor and the accuracy becomes worse. The reason is that the effect of the thickness variation of the beams and plates on the admissible functions is not considered when the vibrating beam functions of the uniform beams are used for the analysis. The same problems also exist in other commonly used admissible functions. Furthermore, as a global method, the Rayleigh-Ritz method can give rather accurate eigenfrequencies of the beam and plate structures if the suitable admissible functions can be selected. However, the conventional admissible functions (having infinite orders of continuous derivatives) used in the Rayleigh-Ritz method are commonly inefficient in the local stress/strain analysis of structures with complexity. A typical case is that the effect of the reactions of point-supports or line-supports on the stress distributions of beams and plates can not be reasonably described because the reactions result in the abrupt jumps of shear force distributions at these supports (Cheung and Zhou, 2000c). In practical engineering, it is important to provide the accurate description of dynamic stress distributions within the structures since the beams and plates may be damaged at these internal supports due to the concentrate support forces.

In this book, sets of static beam functions are developed as the admissible functions to analyze the free and forced vibrations of uniform/non-uniform beams or plates with/without internal point-supports or line-supports in the Rayleigh-Ritz method. The method is not only suitable for the vibration analysis of Euler-Bernoulli beams and thin rectangular plates (Cheung and Zhou, 1999a, 1999b, 2000c; Zhou and Cheung, 1999) but also suitable for the vibration analysis of Timoshenko beams and Mindlin rectangular plates (Zhou and Cheung, 2000b; Cheung and Zhou, 2000b). Moreover, the method has been extended to study vibrations of composite rectangular plates (Zhou et al. 2000; Cheung and Zhou, 1999b). It is seen that unlike those already existing admissible functions, the present admissible functions are closely connected with the thickness variation of the beams and plates. Therefore, lower computational effort and more accurate eigenfrequencies can be obtained using these static beam functions to analyze vibrations of beams and rectangular plates with variable thickness. Moreover, in the present study, the effect of reactions of the point-supports and line-supports on stress/train distributions of the structures has been considered in the development of the admissible functions. For example, the jumps of shear forces at internal supports may be accurately described by using these static beam functions. As a result, more accurate dynamic response and stress/strain analysis can be obtained.