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Applications of Static Beam Functions in Vibration Analysis of Structures

Ding Zhou

(静力梁函数在结构振动分析中的应用)



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静力梁函数在结构振动 分析中的应用

周 叮 著

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内 容 简 介

本书以著名的结构力学分析方法——李兹法为基础,创造性地提出了以静力梁函数作为基函数,研究梁、板结构的动力学特性,重点分析变截面和变厚度、内部支撑以及边界条件对梁、板结构振动特性的影响。全书共 23 章,第 1 章介绍李兹法的发展史与存在的问题;第 2 章至第 6 章研究各种边界和内部支撑条件下变截面欧拉-伯努利梁和铁摩辛柯梁的振动特性;第 7 章至第 11 章研究各种边界和线支条件下等厚度基尔霍夫薄板的振动特性;第 12 章至第 14 章研究线支和点支等厚度复合材料薄板的振动特性;第 15 章和第 16 章研究变厚度基尔霍夫薄板的振动特性;第 17 章至第 20 章研究等厚度和变厚度米德林中厚板的振动特性;第 21 章和第 22 章研究线支和点支等厚度复合材料厚板的振动特性;第 23 章研究矩形储液罐的流-固耦合振动特性。

本书可供航空航天、机械、土木和力学等方面的科研工作者、工程设计人员、大专院校有关专业教师和研究生使用。

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Preface

The vibration characteristics of thin/thick beams and plates have long been a subject of study because these structural components are widely used in various engineering such as aerospace, marine and building. In practical applications, the complexities such as variable thickness, intermediate supports, elastic boundaries etc. could be encountered, which can greatly influence the dynamic properties of the structural components. Therefore, such a problem deserves to be studied thoroughly.

A monograph written by Leissa summarizes the research work on vibrations of thin plates before the early 1970s, which is a very famous classical book due to the citations more than 6000 times. In general, two different methods, analytical method and numerical method, are commonly used for vibration analysis of beams and plates. The analytical method can only be applied in some special cases while the numerical method has a wider scope of applications. In the numerical method, Ritz approach, comparing with other numerical approaches such as finite element and boundary element, has some distinguishing features, i.e. high accuracy, small computational cost and the ability for parameterization study. It is well known that the efficiency (convergence and accuracy) of the Ritz method greatly depends on the admissible functions (trial functions) selected for the analysis. However, none unified method was developed to construct the admissible functions of structural components, especially for the components with complexities mentioned above. The selection of the admissible functions mainly relies on the experience and preference of the user, which greatly restricts the extending applications of Ritz method.

In the book, a new general method to construct the admissible functions for beams and rectangular plates with complexities is introduced. The so-called complexities mean variable thickness, elastic/rigid point-supports or line-supports, and elastic boundary conditions. The classical Euler-Bernoulli theory is used to describe the slender beams and the classical Kirchhoff theory is used to describe the thin plates. Moreover, the Timoshenko theory is used for the thick beams and the Mindlin theory is used for the moderately thick plates. The static beam functions, which come from the solutions to the beam subjected to a series of static loads, are taken as the admissible functions. Sinusoidal loads, polynomial loads and point loads could be used to construct the admissible functions for different beams and plates.

This book is made up of twenty-three chapters. The first chapter reviews the

history of the Ritz approach and the existing problems. The second and third chapters study the vibration characteristics of tapered Euler-Bernoulli beams (slender beams) with/without intermediate supports by using the static beam functions under a series of Taylor polynomial loads, respectively. The fourth chapter deals with the vibration characteristics of uniform Timoshenko beams (thick beams) with intermediate supports by using the static beam functions under a series of sinusoidal loads. The fifth chapter studies the vibration characteristics of tapered Timoshenko beams by using the static beam functions under a series of Taylor polynomial loads. The sixth chapter introduces an application of static beam functions in the estimation of vibration characteristics of a spring-mass-beam system. The seventh chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates (thin rectangular plates) by using two sets of static beam functions respectively. One set of static beam functions is under a series of point loads and the other set is under a series of sinusoidal loads. The eighth chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with elastic boundary constraints by using the static beam functions under a series of sinusoidal loads. The ninth chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with intermediate line-supports by using two sets of static beam functions respectively. One set of static beam functions is a combination of vibrating beam functions and simple polynomials and the other set is under a series of sinusoidal loads. The tenth chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with elastic intermediate line-supports and elastic boundary constraints by using the static beam functions under a series of sinusoidal loads. The eleventh chapter studies the vibration characteristics of uniform Kirchhoff rectangular plates with elastic point-supports by using the static beam functions under a series of point loads. The twelfth and thirteenth chapters study the vibration characteristics of symmetrically and asymmetrically laminated rectangular plates with intermediate line-supports by using the static beam functions under a series of sinusoidal loads, respectively. The fourteenth chapter studies the vibration characteristics of composite rectangular plates with point-supports by using the static beam functions under a series of sinusoidal loads. The fifteenth and sixteenth chapters study the vibration characteristics of tapered rectangular plates with/without intermediate line-supports by using the static beam functions under a series of Taylor polynomial loads. The seventeenth and eighteenth chapters study the vibration characteristics of Mindlin rectangular plates (moderately thick rectangular plates) with classical boundary conditions and elastic boundary conditions by using the static beam functions under a series of sinusoidal loads, respectively. The nineteenth chapter studies the vibration characteristics of Mindlin rectangular plates with intermediate line-supports by using

the static beam functions under a series of sinusoidal loads. The twentieth chapter studies the vibration characteristics of tapered Mindlin rectangular plates by using the static beam functions under a series of Taylor polynomial loads. The twenty-first chapter studies the vibration characteristics of thick rectangular plates with intermediate line-supports by using a combination of vibrating beam functions and simple polynomials. The twenty-second chapter studies the vibration characteristics of thick laminated rectangular plates with point-supports by using the static beam functions under a series of sinusoidal loads. The last chapter studies the vibration characteristics of rectangular tanks partially filled with liquid by using the static beam functions under a series of sinusoidal loads.

The accuracy, convergence and numerical robustness for various cases have been investigated in detail, which indicates that the static beam functions can be usefully applied in solving a variety of beam and plate vibration problems. I hope that the present method could be extended to more research topics such as the shell vibrations in future.

Ding Zhou
Nanjing
30 March, 2013

Contents

Preface

Chapter 1	Introduction	1
Chapter 2	Vibration Analysis of Tapered Euler-Bernoulli Beams	4
2.1	Introduction	4
2.2	The Rayleigh-Ritz Method for the Tapered Beams	5
2.3	A New Set of Admissible Functions	7
2.3.1	The coefficients for a truncated beam	8
2.3.2	The coefficients for a sharply ended beam	10
2.3.3	The tapered beam with rigid body motion	11
2.4	Convergency and Comparison Studies	12
2.4.1	Convergency study	13
2.4.2	Optimum expanding point of Taylor series	14
2.5	Numerical Results	15
2.6	Concluding Remarks	19
Chapter 3	Vibration Analysis of Tapered Euler-Bernoulli Beams with Intermediate Supports	21
3.1	Introduction	21
3.2	The Rayleigh-Ritz Method for Tapered Beams with Intermediate Supports	22
3.3	A Set of Static Tapered Beam Functions	25
3.3.1	The truncated beam	26
3.3.2	The sharply ended beam	29
3.3.3	The tapered beam with motions of rigid body	30
3.4	Numerical Examples	31
3.5	Concluding Remarks	35
Chapter 4	Vibration Analysis of Multi-span Timoshenko Beams	37
4.1	Introduction	37
4.2	Eigenfrequency Equation	38
4.3	Static Timoshenko Beam Functions	39
4.4	Convergence and Comparison Studies	45
4.5	Numerical Examples	49
4.6	Concluding Remarks	52

Chapter 5	Vibration Analysis of Tapered Timoshenko Beams	53
5.1	Introduction	53
5.2	Eigenfrequency Equation of Tapered Beam	54
5.3	The Static Timoshenko Beam Functions (STBF)	56
5.3.1	Truncated beam	58
5.3.2	Sharply ended beam	59
5.4	Convergence and Comparison Study	60
5.5	Numerical Results	64
5.6	Conclusions	69
Chapter 6	Estimation of Dynamic Characteristics of a Spring-Mass-Beam System	70
6.1	Introduction	70
6.2	Governing Differential Equations	71
6.3	Galerkin Solutions	73
6.4	Basic Characteristics of Solutions	75
6.5	Static Beam Functions	75
6.6	Determination of Factors	77
6.7	An Example	80
6.8	Characteristics of Solutions	81
6.9	Conclusions	84
Chapter 7	Vibration Analysis of Kirchhoff Rectangular Plates	85
Part I	Using Static Beam Functions under Point Loads	85
7.1	Introduction	85
7.2	Sets of Static Beam Functions under Point Loads	85
7.3	Rayleigh-Ritz Solution for Rectangular Plates	88
7.4	Numerical Results	89
7.5	Concluding Remarks	91
Part II	Using Static Beam Functions under Sinusoidal Loads	92
7.1	Introduction	92
7.2	The Set of Static Beam Functions	92
7.3	The Rayleigh-Ritz Approach	94
7.4	Numerical Results	96
7.5	Concluding Remarks	98
Chapter 8	Vibration Analysis of Kirchhoff Rectangular Plates with Elastic Edge Constraints	99
8.1	Introduction	99

8.2	The Set of Static Beam Functions	100
8.3	The Rayleigh-Ritz Solution	101
8.4	Numerical Examples	104
8.5	Discussion and Conclusions	106
Chapter 9 Vibration Analysis of Kirchhoff Rectangular Plates with Intermediate Line-supports		107
Part I Using a Combination of Vibrating Beam Functions and Polynomials		107
9.1	Introduction	107
9.2	Mathematical Model	109
9.3	Numerical Examples	113
9.4	Concluding Remarks	120
Part II Using the Static Beam Functions for Beam with Point-supports ..		121
9.1	Introduction	121
9.2	A New Set of Admissible Functions	122
9.3	Eigenfrequency Equation	125
9.4	Some Numerical Results	127
9.5	Conclusions	133
Chapter 10 Vibration Analysis of Kirchhoff Rectangular Plates with Elastic Intermediate Line-supports and Edge Constraints		134
10.1	Introduction	134
10.2	A Set of Static Beam Functions	135
10.3	Formulation of Eigenvalue Equation	140
10.4	Numerical Examples	143
10.5	Conclusions	152
Chapter 11 Vibration Analysis of Kirchhoff Rectangular Plates with Elastic Point-supports		154
11.1	Introduction	154
11.2	Sets of Static Beam Functions under Point Loads	155
11.3	Eigenvalue Problem with Rayleigh-Ritz Method	156
11.4	Numerical Results	158
11.5	Conclusion	163
Chapter 12 Vibration Analysis of Symmetrically Laminated Rectangular Plates with Intermediate Line-supports ..		164
12.1	Introduction	164
12.2	A Set of Static Beam Functions	165
12.3	Eigenfrequency Equation	168

12.4	Numerical Results	171
12.4.1	Accuracy and convergency study	171
12.4.2	Numerical examples	173
12.5	Concluding remarks	177
Chapter 13	Vibration Analysis of Asymmetrically Laminated Rectangular Plates with Internal Line-supports	179
13.1	Introduction	179
13.2	Energy Functional	180
13.3	Rayleigh-Ritz Solution	183
13.4	Trial Functions	185
13.5	Convergence and Comparison Study	187
13.6	Numerical Results	191
13.7	Conclusion	193
Chapter 14	Vibration Analysis of Composite Rectangular Plates with Point-supports	194
14.1	Introduction	194
14.2	Static Beam Functions	195
14.2.1	The static beam functions under sine series loads	195
14.2.2	The static beam functions under a point-load	197
14.3	Eigenfrequency Equation	197
14.4	Admissible Functions	200
14.5	Comparison and Convergence	202
14.5.1	Isotropic square plates with point-supports	202
14.5.2	Laminated square composite plates	204
14.6	Numerical Results	204
14.7	Conclusions	207
Chapter 15	Vibration Analysis of Tapered Kirchhoff Rectangular Plates	208
15.1	Introduction	208
15.2	The development of a set of tapered beam functions	210
15.3	The Rayleigh-Ritz method	216
15.4	Numerical examples	219
15.5	Concluding remarks	224
	Appendix	225
Chapter 16	Vibration Analysis of Tapered Kirchhoff Rectangular Plates with Intermediate Line-supports	226
16.1	Introduction	226
16.2	The Rayleigh-Ritz Method for Tapered Rectangular Plates	227

16.3	A Set of Static Beam Functions	230
16.3.1	The truncated beam	232
16.3.2	The sharp ended beam	235
16.3.3	The tapered beam with rigid body motions	236
16.4	Numerical Examples	237
16.5	Conclusions	246
Chapter 17	Vibration Analysis of Mindlin Rectangular Plates	247
17.1	Introduction	247
17.2	A Set of Static Timoshenko Beam Functions	248
17.3	Eigenfrequency Equation of Mindlin Plate	252
17.4	Comparison and Convergency Studies	256
17.5	The Parametric Study	260
17.6	Conclusions	264
Chapter 18	Vibration Analysis of Mindlin Rectangular Plates with Elastically Restrained Edges	265
18.1	Introduction	265
18.2	Rayleigh-Ritz Formulae for Mindlin Rectangular Plates	266
18.3	A Set of Static Timoshenko Beam Functions	270
18.4	Comparison and Convergency Studies	273
18.5	Numerical Results	277
18.6	Conclusions	282
Chapter 19	Vibration Analysis of Mindlin Rectangular Plates with Intermediate Line-supports	283
19.1	Introduction	283
19.2	Rayleigh-Ritz Solution of Mindlin Plate	284
19.3	Static Timoshenko Beam Functions	288
19.4	Convergence and Comparison Study	293
19.5	Numerical Results	296
19.6	Conclusions	298
Chapter 20	Vibrations Analysis of Tapered Mindlin Plates	300
20.1	Introduction	300
20.2	The Eigenfrequency Equation of Tapered Plates	301
20.3	Two Sets of Static Timoshenko Beam Functions (STBF)	305
20.3.1	Truncated beam	307
20.3.2	Sharp-ended beam	307
20.4	Convergence and Comparison Studies	308
20.5	Numerical Results	311
20.6	Concluding Remarks	314

Chapter 21	Vibration Analysis of Thick Rectangular Plates with Internal Line-supports	315
21.1	Introduction	315
21.2	Trial Functions	316
21.3	Numerical Examples	318
21.3.1	Preliminary assessment: simply supported laminated plates	318
21.3.2	Continuous rectangular plates	319
21.4	Conclusions	324
Chapter 22	Vibration Analysis of Layered Thick Rectangular Plates with Internal Point-supports	326
22.1	Introduction	326
22.2	Two Sets of Static Beam Functions	327
22.2.1	Static beam functions under a series of sinusoidal loads	328
22.2.2	Static beam functions under a series of point-loads	329
22.3	Finite Layer Formulation	329
22.4	Basic Functions	331
22.5	Numerical Studies	333
22.5.1	Convergence and comparison	333
22.5.2	Numerical examples	336
22.6	Concluding Remarks	338
	Appendix A	339
	Appendix B	340
Chapter 23	Vibration Analysis of Rectangular Tanks Partially Filled with Liquid	344
23.1	Introduction	344
23.2	Basic Equations	345
23.3	Solution of Velocity Potential	347
23.4	Rayleigh-Ritz-Galerkin Method	350
23.4.1	Rayleigh quotient	350
23.4.2	Eigenfrequency equation	352
23.5	Admissible Functions	357
23.6	Numerical Results	360
23.6.1	Convergence and comparison study	360
23.6.2	Parametric effect study	363
23.7	Conclusions	368
	References	369

Chapter 1

Introduction

It is well known that beams and rectangular plates are some of the most commonly used structural elements in aerospace, civil and marine engineering. Their dynamic behaviors are of practical importance to design engineers. In the practical applications, beams and rectangular plates are either uniform or non-uniform. Sometimes, internal point-supports or line-supports may be placed to reduce the magnitude of displacements and stresses of the structures or to satisfy special architectural and functional requirements. In general, the Euler-Bernoulli beam theory (Gorman, 1975) for slender beams and the classical plate theory (Timoshenko and Woinowsky-Krieger, 1959) for thin plates are sufficient to give satisfactory accuracy. However, the Timoshenko beam theory (Timoshenko, 1921) for thick beams and the Mindlin plate theory (Mindlin, 1951) for moderately thick plates should be applied if one wants to obtain more reasonable results because of the fact that the effects of transverse shear deformation and rotary inertia have been taken into account in these cases.

Exact analytical solutions for vibrations of beams and plates have been given only to special cases such as uniform beams and simply supported uniform rectangular plates. Besides this, various numerical methods such as finite element method (Zienkiewicz and Taylor, 1991), finite difference method (Gumeniuk, 1956), finite strip method (Cheung and Tham, 1997), Rayleigh-Ritz method (Young, 1950; Warburton and Edney, 1984; Bhat, 1985; Liew et al., 1998) as well as differential quadrature method (Bert and Malik, 1996) etc. have been developed to investigate those cases for which exact solutions are not available. Among these methods, the Rayleigh-Ritz method can give high accuracy with low computational effort if proper admissible functions (i.e., trial functions or basis functions) are selected. Using vibrating beam functions as the admissible functions to analyze vibrations of thin rectangular plates in the Rayleigh-Ritz method may be traced back to the beginning of the 1950s (Young, 1950). In the beginning of the 1980s, the vibration of moderately thick rectangular plates (Dawe and Roufaeil, 1980) was analyzed by using the vibrating Timoshenko beam functions as the admissible functions in the Rayleigh-Ritz method. Moreover, the vibrating beam functions of uniform beams

have also been extensively used to study the vibration of non-uniform beams (Chehil and Jategaonkar, 1987; Jategaonkar and Chehil, 1989) and plates (Chopra and Durvasula, 1971; Malhotra et al., 1987) as well as elastically restrained rectangular plates (Warburton and Edney, 1984; Saha and Kar, 1996).

The effectiveness of vibrating beam functions in solving the vibration of uniform beams and rectangular plates is generally acknowledged. However, it has been found that for beams or plates with varying thickness, the results with satisfactory accuracy can be obtained only when the truncation factor (the ratio of smaller end thickness to the larger end thickness) is larger than 0.3 (Cheung and Zhou, 1999c; Zhou and Cheung, 2000c). For a small truncation factor (e.g. lower than 0.3), the convergence rate of the eigenfrequencies quickly decreases with the decrease of the truncation factor and the accuracy becomes worse. The reason is that the effect of the thickness variation of the beams and plates on the admissible functions is not considered when the vibrating beam functions of the uniform beams are used for the analysis. The same problems also exist in other commonly used admissible functions. Furthermore, as a global method, the Rayleigh-Ritz method can give rather accurate eigenfrequencies of the beam and plate structures if the suitable admissible functions can be selected. However, the conventional admissible functions (having infinite orders of continuous derivatives) used in the Rayleigh-Ritz method are commonly inefficient in the local stress/strain analysis of structures with complexity. A typical case is that the effect of the reactions of point-supports or line-supports on the stress distributions of beams and plates can not be reasonably described because the reactions result in the abrupt jumps of shear force distributions at these supports (Cheung and Zhou, 2000c). In practical engineering, it is important to provide the accurate description of dynamic stress distributions within the structures since the beams and plates may be damaged at these internal supports due to the concentrate support forces.

In this book, sets of static beam functions are developed as the admissible functions to analyze the free and forced vibrations of uniform/non-uniform beams or plates with/without internal point-supports or line-supports in the Rayleigh-Ritz method. The method is not only suitable for the vibration analysis of Euler-Bernoulli beams and thin rectangular plates (Cheung and Zhou, 1999a, 1999b, 2000c; Zhou and Cheung, 1999) but also suitable for the vibration analysis of Timoshenko beams and Mindlin rectangular plates (Zhou and Cheung, 2000b; Cheung and Zhou, 2000b). Moreover, the method has been extended to study vibrations of composite rectangular plates (Zhou et al. 2000; Cheung and Zhou, 1999b). It is seen that unlike those already existing admissible functions, the present admissible functions are closely connected with the thickness variation of the beams and plates. Therefore,

lower computational effort and more accurate eigenfrequencies can be obtained using these static beam functions to analyze vibrations of beams and rectangular plates with variable thickness. Moreover, in the present study, the effect of reactions of the point-supports and line-supports on stress/strain distributions of the structures has been considered in the development of the admissible functions. For example, the jumps of shear forces at internal supports may be accurately described by using these static beam functions. As a result, more accurate dynamic response and stress/strain analysis can be obtained.