时代教育·国外高校优秀教材精选

Addison Wesley

线性代数引论

Introduction to Linear Algebra

（英文版·原书第5版）

李 W·约翰逊(Lee W.Johnson)
(美) R·迪安 里斯(R.Dean Riess) 著
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序

本书是由约翰逊、里斯、阿诺德编著的《线性代数引论》第5版，出版于2002年。

线性代数是学习自然科学、工程和社会科学的学生的一门重要的基础课程，其核心内容包括矩阵理论以及向量空间理论。这些概念和理论不仅为各个专业领域提出相关问题时提供了准确的数学表达语言，而且也为解决问题提供了有力的工具。本书的主要内容有矩阵与线性方程组、二维和三维空间、向量空间 R^n、特征值问题、向量空间和线性变换、行列式、特征值及其应用等。

作为线性代数课程的教材，本书有如下特点：

1. 内容覆盖了我国现行理工科大学线性代数课程的全部内容，与我国现行的线性代数教学大纲和教材体系比较接近，但比国内现有教材的内容更为丰富，且更有深度。

2. 本书的编写采取了模块式的结构，便于使用者取舍。线性代数的核心内容主要由三部分组成，它们是矩阵理论与线性方程组、向量空间的基本概念以及特征值问题，作者将它们分别列入第1章、第3章、第4章。这样，对于学时较少，要求相对较低的专业和读者，可以只选这三章，用30学时左右就可以掌握线性代数的最基本的知识，对于学时充裕、要求较高的专业可以从容地增加其它章节。

3. 在内容处理上，及早引进一些重要的概念，例如在第1章就引入线性组合和线性无关的概念，从而帮助学生很快地从用一般方法解线性方程组过渡到用基、生成系等概念来处理和解决相应的问题。又如在第3章及时建立向量空间 R^n 的概念和从 R^n 到 R^m 的线性变换，那么在第4章一开始简单地引入行列式概念后，就可以比较深入地讨论特征值问题。

4. 向量空间的概念是线性代数学习中的一个难点，为了分散难点，使学生能更平稳地、逐步地接受这个抽象概念，本书作了一系列的铺垫。例如第1章先引入线性无关的概念，然后在第2章讨论二维空间和三维空间，这是两个非常形象，又是学生很熟悉的空间，在这个基础上推广到 R^n 这个 n 维空间，实现了从感性思维到理性思维的第一个飞跃。最后再进入完全抽象的一般线性空间。

5. 线性代数是一门既严谨又抽象的课程，为了使学生既易于入门，又能领会数学抽象的威力，作者通过许多有实际背景的例子，从具体计算入手，自然地、逐步地建立抽象的概念和理论。

6. 21世纪是信息的时代，现代教学手段促进了教学改革的步伐，为进一步提高教学质量提供了有利的教学环境。本书通过例子介绍了在科技工作者中非常流行的数学软件 Matlab 在线性代数中的应用，并且每章结尾都附有专门用 Matlab 做的练习题，这对于培养学生运用现代数学软件学习线性代数和应用代数知识解决实际问题能起到很好的作用。

总之，本书系统新颖、内容丰富、联系实际、语言通畅，且结合数学软件提供了一个现代化的学习环境，相对于国内现行教学大纲，教师便于取舍内容组织教学，不失为一本很好的教科书和教学参考书。

本书可供理工科、经济管理各专业学生作为学习线性代数的教科书或教学参考书，也可供科技人员和自学者参考。

俞正光

清华大学数学科学系

出版说明

随着我国加入WTO，国际间的竞争越来越激烈，而国际间的竞争实际上也就是人才的竞争、教育的竞争。为了加快培养具有国际竞争力的高水平技术人才，加快我国教育改革的步伐，国家教育部近来出台了一系列倡导高校开展双语教学、引进原版教材的政策。以此为契机，机械工业出版社近期推出了一系列国外影印版教材，其内容涉及高等学校公共基础课，以及机、电、信息领域的专业基础课和专业课。

引进国外优秀原版教材，在有条件的学校推动开展英语授课或双语教学，自然也引进了先进的教学思想和教学方法，这对提高我国自编教材的水平，加强学生的英语实际应用能力，使我国的高等教育尽快与国际接轨，必将起到积极的推动作用。

为了做好教材的引进工作，机械工业出版社特别成立了由著名专家组成的国外高校优秀教材审定委员会。这些专家对实施双语教学做了深入细致的调查研究，对引进原版教材提出许多建设性意见，并慎重地对每一本将要引进的原版教材一审再审，精选再精选，确认教材本身的质量水平，以及权威性和先进性，以期所引进的原版教材能适应我国学生的外语水平和学习特点。在引进工作中，审定委员会还结合我国高校教学课程体系的设置和要求，对原版教材的教学思想和方法的先进性、科学性严格把关。同时尽量考虑原版教材的系统性和经济性。

这套教材出版后，我们将根据各高校的双语教学计划，举办原版教材的教师培训，及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议，使我们更好地为教学改革服务。

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PREFACE

Linear algebra is an important component of undergraduate mathematics, particularly for students majoring in the scientific, engineering, and social science disciplines. At the practical level, matrix theory and the related vector-space concepts provide a language and a powerful computational framework for posing and solving important problems. Beyond this, elementary linear algebra is a valuable introduction to mathematical abstraction and logical reasoning because the theoretical development is self-contained, consistent, and accessible to most students.

Therefore, this book stresses both practical computation and theoretical principles and centers on the principal topics of the first four chapters:

matrix theory and systems of linear equations,

elementary vector-space concepts, and

the eigenvalue problem.

This core material can be used for a brief (10-week) course at the late-freshman/sophomore level. There is enough additional material in Chapters 5–7 either for a more advanced or a more leisurely paced course.

FEATURES

Our experience teaching freshman and sophomore linear algebra has led us to carefully choose the features of this text. Our approach is based on the way students learn *and* on the tools they need to be successful in linear algebra as well as in related courses.

We have found that students learn more effectively when the material has a consistent level of difficulty. Therefore, in Chapter 1, we provide early and meaningful coverage of topics such as linear combinations and linear independence. This approach helps the student negotiate what is usually a dramatic jump in level from solving systems of linear equations to working with concepts such as basis and spanning set.

Tools Students Need (When They Need Them)

The following examples illustrate how we provide students with the tools they need for success.

An early introduction to eigenvalues. In Chapter 3, elementary vector-space ideas (subspace, basis, dimension, and so on) are introduced in the familiar setting of R^n. Therefore, it is possible to cover the eigenvalue problem very early and in much greater depth than is usually possible. A brief introduction to determinants is given in Section 4.2 to facilitate the early treatment of eigenvalues.

An early introduction to linear combinations. In Section 1.5, we observe that the matrix-vector product $A\mathbf{x}$ can be expressed as a linear combination of the columns of

A, $A\mathbf{x} = x_1\mathbf{A}_1 + x_2\mathbf{A}_2 + \cdots + x_n\mathbf{A}_n$. This viewpoint leads to a simple and natural development for the theory associated with systems of linear equations. For instance, the equation $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ is expressible as a linear combination of the columns of A. Similarly, a consistent equation $A\mathbf{x} = \mathbf{b}$ has a unique solution if and only if the columns of A are linearly independent. This approach gives some early motivation for the vector-space concepts (introduced in Chapter 3) such as subspace, basis, and dimension. The approach also simplifies ideas such as rank and nullity (which are then naturally given in terms of dimension of appropriate subspaces).

Applications to different fields of study. Some applications are drawn from difference equations and differential equations. Other applications involve interpolation of data and least-squares approximations. In particular, students from a wide variety of disciplines have encountered problems of drawing curves that fit experimental or empirical data. Hence, they can appreciate techniques from linear algebra that can be applied to such problems.

Computer awareness. The increased accessibility of computers (especially personal computers) is beginning to affect linear algebra courses in much the same way as it has calculus courses. Accordingly, this text has somewhat of a numerical flavor, and (when it is appropriate) we comment on various aspects of solving linear algebra problems in a computer environment.

A Comfort in the Storm

We have attempted to provide the type of student support that will encourage success in linear algebra—one of the most important undergraduate mathematics courses that students take.

A gradual increase in the level of difficulty. In a typical linear algebra course, the students find the techniques of Gaussian elimination and matrix operations fairly easy. Then, the ensuing material relating to vector spaces is suddenly much harder. We do three things to lessen this abrupt midterm jump in difficulty:

1. We introduce linear independence early in Section 1.7.
2. We include a new Chapter 2, "Vectors in 2-Space and 3-Space."
3. We first study vector space concepts such as subspace, basis, and dimension in Chapter 3, in the familiar geometrical setting of R^n.

Clarity of exposition. For many students, linear algebra is the most rigorous and abstract mathematical course they have taken since high-school geometry. We have tried to write the text so that it is accessible, but also so that it reveals something of the power of mathematical abstraction. To this end, the topics have been organized so that they flow logically and naturally from the concrete and computational to the more abstract. Numerous examples, many presented in extreme detail, have been included in order to illustrate the concepts. The sections are divided into subsections with boldface headings. This device allows the reader to develop a mental outline of the material and to see how the pieces fit together.

Extensive exercise sets. We have provided a large number of exercises, ranging from routine drill exercises to interesting applications and exercises of a theoretical nature. The more difficult theoretical exercises have fairly substantial hints. The computational

exercises are written using workable numbers that do not obscure the point with a mass of cumbersome arithmetic details.

Trustworthy answer key. Except for the theoretical exercises, solutions to the odd-numbered exercises are given at the back of the text. We have expended considerable effort to ensure that these solutions are correct.

Spiraling exercises. Many sections contain a few exercises that hint at ideas that will be developed later. Such exercises help to get the student involved in thinking about extensions of the material that has just been covered. Thus the student can anticipate a bit of the shape of things to come. This feature helps to lend unity and cohesion to the material.

Historical notes. We have a number of historical notes. These assist the student in gaining a historical and mathematical perspective of the ideas and concepts of linear algebra.

Supplementary exercises. We include, at the end of each chapter, a set of supplementary exercises. These exercises, some of which are true–false questions, are designed to test the student's understanding of important concepts. They often require the student to use ideas from several different sections.

Integration of MATLAB. We have included a collection of MATLAB projects at the end of each chapter. For the student who is interested in computation, these projects provide hands-on experience with MATLAB.

A short MATLAB appendix. Many students are not familiar with MATLAB. Therefore, we include a *very* brief appendix that is sufficient to get the student comfortable with using MATLAB for problems that typically arise in linear algebra.

The vector form for the general solution. To provide an additional early introduction to linear combinations and spanning sets, in Section 1.5 we introduce the idea of the vector form for the general solution of $A\mathbf{x} = \mathbf{b}$.

SUPPLEMENTS

SOLUTIONS MANUALS

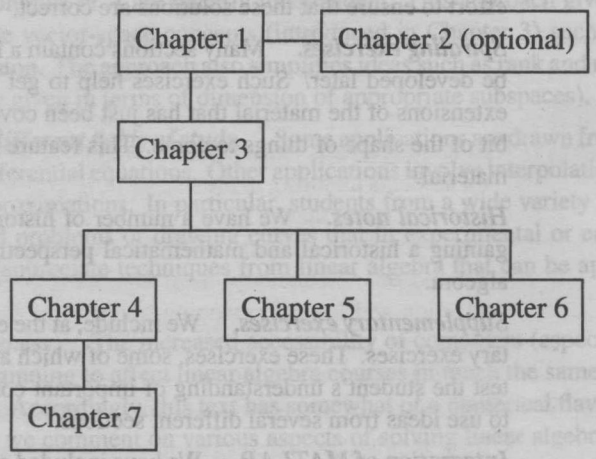
An Instructor's Solutions Manual and a Student's Solutions Manual are available. The odd-numbered computational exercises have answers at the back of the book. The student's solutions manual (ISBN 0-201-65860-7) includes detailed solutions for these exercises. The instructor's solutions manual (ISBN 0-201-75814-8) contains solutions to all the exercises.

New Technology Resource Manual. This manual was designed to assist in the teaching of the MATLAB, Maple, and Mathematica programs in the context of linear algebra. This manual is available from Addison-Wesley (ISBN 0-201-75812-1) or via [our website,] http://www.aw.com/jra.

ORGANIZATION

To provide greater flexibility, Chapters 4, 5, and 6 are essentially independent. These chapters can be taken in any order once Chapters 1 and 3 are covered. Chapter 7 is a mélange of topics related to the eigenvalue problem: quadratic forms, differential

equations, *QR* factorizations, Householder transformations, generalized eigenvectors, and so on. The sections in Chapter 7 can be covered in various orders. A schematic diagram illustrating the chapter dependencies is given below. Note that Chapter 2, "Vectors in 2-Space and 3-Space," can be omitted with no loss of continuity.



We especially note that Chapter 6 (Determinants) can be covered before Chapter 4 (Eigenvalues). However, Chapter 4 contains a brief introduction to determinants that should prove sufficient to users who do not wish to cover Chapter 6.

A very short but useful course at the beginning level can be built around the following sections:

Section 1.1–1.3, 1.5–1.7, 1.9

Sections 3.1–3.6

Sections 4.1–4.2, 4.4–4.5

A syllabus that integrates abstract vector spaces. Chapter 3 introduces elementary vector-space ideas in the familiar setting of R^n. We designed Chapter 3 in this way so that it is possible to cover the eigenvalue problem much earlier and in greater depth than is generally possible. Many instructors, however, prefer an integrated approach to vector spaces, one that combines R^n and abstract vector spaces. The following syllabus, similar to ones used successfully at several universities, allows for a course that integrates abstract vector spaces into Chapter 3. This syllabus also allows for a detailed treatment of determinants:

Sections 1.1–1.3, 1.5–1.7, 1.9

Sections 3.1–3.3, 5.1–5.3, 3.4–3.5, 5.4–5.5

Sections 4.1–4.3, 6.4–6.5, 4.4–4.7

Augmenting the core sections. As time and interest permit, the core of Sections 1.1–1.3, 1.5–1.7, 1.9, 3.1–3.6, 4.1–4.2, and 4.4–4.5 can be augmented by including various combinations of the following sections:

(a) *Data fitting and approximation:* 1.8, 3.8–3.9, 7.5–7.6.

(b) *Eigenvalue applications:* 4.8, 7.1–7.2.

(c) *More depth in vector space theory:* 3.7, Chapter 5.

(d) *More depth in eigenvalue theory:* 4.6–4.7, 7.3–7.4, 7.7–7.8.

(e) *Determinant theory:* Chapter 6.

To allow the possibility of getting quickly to eigenvalues, Chapter 4 contains a brief introduction to determinants. If the time is available and if it is desirable, Chapter 6 (Determinants) can be taken after Chapter 3. In such a course, Section 4.1 can be covered quickly and Sections 4.2–4.3 can be skipped.

Finally, in the interest of developing the student's mathematical sophistication, we have provided proofs for almost every theorem. However, some of the more technical proofs (such as the demonstration that $\det(AB) = \det(A)\det(B)$) are deferred to the end of the sections. As always, constraints of time and class maturity will dictate which proofs should be omitted.

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Blacksburg, Virginia

L.W.J.
R.D.R.
J.T.A.

CONTENTS

MATRICES AND SYSTEMS OF LINEAR EQUATIONS

1

OVERVIEW

In this chapter we discuss systems of linear equations and methods (such as Gauss-Jordan elimination) for solving these systems. We introduce matrices as a convenient language for describing systems and the Gauss-Jordan solution method.

We next introduce the operations of addition and multiplication for matrices and show how these operations enable us to express a linear system in matrix-vector terms as

$$A\mathbf{x} = \mathbf{b}.$$

Representing the matrix A in column form as $A = [\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n]$, we then show that the equation $A\mathbf{x} = \mathbf{b}$ is equivalent to

$$x_1\mathbf{A}_1 + x_2\mathbf{A}_2 + \cdots + x_n\mathbf{A}_n = \mathbf{b}.$$

The equation above leads naturally to the concepts of linear combination and linear independence. In turn, those ideas allow us to address questions of existence and uniqueness for solutions of $A\mathbf{x} = \mathbf{b}$ and to introduce the idea of an inverse matrix.

CORE SECTIONS

1.1 Introduction to Matrices and Systems of Linear Equations
1.2 Echelon Form and Gauss-Jordan Elimination
1.3 Consistent Systems of Linear Equations
1.5 Matrix Operations
1.6 Algebraic Properties of Matrix Operations
1.7 Linear Independence and Nonsingular Matrices
1.9 Matrix Inverses and Their Properties

INTRODUCTION TO MATRICES AND SYSTEMS OF LINEAR EQUATIONS

In the real world, problems are seldom so simple that they depend on a single input variable. For example, a manufacturer's profit clearly depends on the cost of materials, but it also depends on other input variables such as labor costs, transportation costs, and plant overhead. A realistic expression for profit would involve all these variables. Using mathematical language, we say that profit is a *function of several variables*.

In linear algebra we study the simplest functions of several variables, the ones that are *linear*. We begin our study by considering linear equations. By way of illustration, the equation

$$x_1 + 2x_2 + x_3 = 1$$

is an example of a linear equation, and $x_1 = 2, x_2 = 1, x_3 = -3$ is one solution for the equation. In general a ***linear equation*** in n unknowns is an equation that can be put in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b. \tag{1}$$

In (1), the coefficients $a_1, a_2, \ldots, a_n$ and the constant b are known, and $x_1, x_2, \ldots, x_n$ denote the unknowns. A ***solution*** to Eq. (1) is any sequence $s_1, s_2, \ldots, s_n$ of numbers such that the substitution $x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$ satisfies the equation.

Equation (1) is called linear because each term has degree one in the variables $x_1, x_2, \ldots, x_n$. (Also, see Exercise 37.)

EXAMPLE 1 Determine which of the following equations are linear.

(i) $x_1 + 2x_1x_2 + 3x_2 = 4$

(ii) $x_1^{1/2} + 3x_2 = 4$

(iii) $2x_1^{-1} + \sin x_2 = 0$

(iv) $3x_1 - x_2 = x_3 + 1$

Solution Only Eq. (iv) is linear. The terms x_1x_2, $x_1^{1/2}$, x_1^{-1}, and $\sin x_2$ are all nonlinear. ■

Linear Systems

Our objective is to obtain simultaneous solutions to a system (that is, a set) of one or more linear equations. Here are three examples of systems of linear equations.

(a)
$$\begin{aligned} x_1 + x_2 &= 3 \\ x_1 - x_2 &= 1 \end{aligned}$$

(b)
$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= -11 \\ -x_1 + 3x_2 + 5x_3 &= 15 \end{aligned}$$

(c)
$$\begin{aligned} 3x_1 - 2x_2 &= 1 \\ 6x_1 - 4x_2 &= 6 \end{aligned}$$

In terms of solutions, it is easy to check that $x_1 = 2, x_2 = 1$ is one solution to system (a). Indeed, it can be shown that this is the *only* solution to the system.