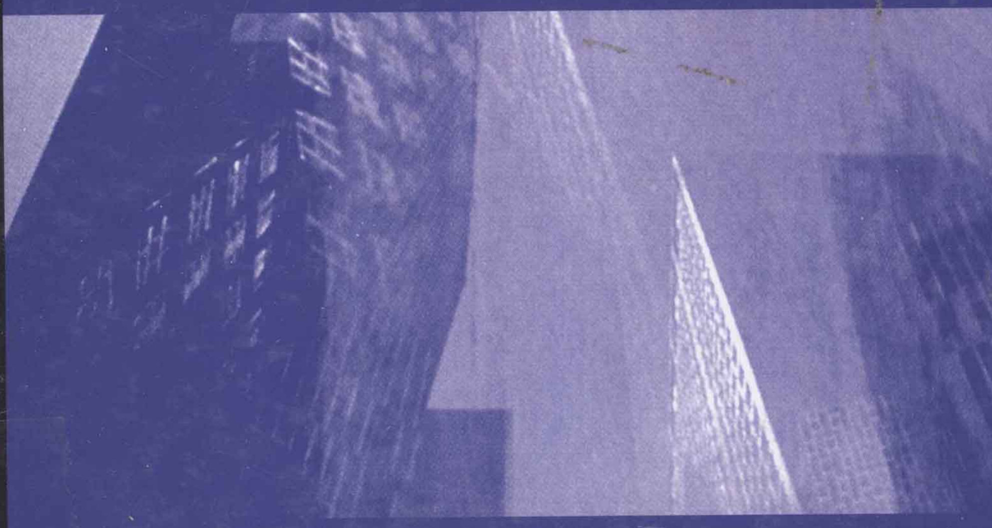
ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

Zheng-Ming Huang
Ye-Xin Zhou

Strength of Fibrous Composites



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Preface

Laminated composites made of continuous fibers and metal, ceramic, or polymer matrices have been used for structural applications for more than half a century. Many modern industries such as aerospace engineering or wind power energy engineering would not have advanced to their current levels if composites had not been used. Among all of the superior characteristics of composites in comparison with other more traditional, isotropic structural materials, three are the most well known. They are high-specific stiffness (stiffness to mass ratio), high-specific strength and the ability to tailor desired properties by choosing suitable fiber and matrix materials as well as the fiber architecture geometry.

Determination of the composite mechanical properties has attracted the attention of scientists, researchers and engineers. From an application point of view, it would be best if all of the mechanical properties of the composites can be estimated by using their constituent fiber and matrix properties and the fiber architecture parameters, i.e., by using a micromechanical approach. For the composite stiffness, this is feasible. There are many micromechanical models for efficiently estimating the effective elastic properties of laminated composites, which have been the focus of most of the available mechanics of composite materials textbooks and monographs. A very challenging problem, however, is to estimate the composite strength as well as other inelastic behaviors micromechanically. In the current literature, there is a lack of a book systematically addressing this problem. Almost all of the monographs dealing with laminate strength follow a phenomenological philosophy. Namely, the laminate strength is estimated based on the information of lamina strengths, which must be measured on composites themselves. However, predicting laminate strength micromechanically is very important, as one of the most critical issues in designing a composite structure is to know its load carrying capacity *in priori*. Only when this capacity has been explicitly related to the constituent properties and geometric parameters, can an optimal design choosing proper constituent materials, fiber content and architecture, and laminate layups for the structure before fabrication, be achieved.

Would it be possible to dream that any mechanical property, including the ultimate load carrying capacity of a composite made using any continuous fiber architecture subjected to arbitrary loads, would be simply available without any experiment on it but be based only on an established database containing the required constituent properties? Will this become a reality? More than a decade ago, the first author of this book established a unified micromechanical theory, the bridging model, to describe the constitutive relationship of a composite up to the point of failure. The unique feature of this theory is that the internal stresses in the constituent fiber and matrix materials of the composite under any arbitrary load conditions, including a temperature variation, can be evaluated using rigorous and explicit equations. By assuming that a composite failure is caused by either the fiber or the matrix failure, a micromechanical strength theory for the composite is established. The last decade has seen sound development of the bridging model as well as its applications to the analysis of mechanical properties, especially strengths of various fibrous composites. The assessment by the World Wide Failure Exercises (WWFE-I and WWFE-II, also known as "Failure Olympics" in the composite community) has confirmed the efficiency and accuracy of this model.

This book systematically deals with the bridging model development as well as applications to strength prediction of unidirectional (UD) laminas and multidirectional laminates. The model can be derived in terms of an Eshelby's tensor. Presented in Chapter 1 is the classical Eshelby's problem as well as other pre-requirements in mechanics and mathematics to understand the bridging model theory and applications. Chapter 2 addresses a general elastic-plastic constitutive theory, the Prandtl-Reuss theory, for isotropic materials. This theory is used to describe the matrix behavior in a composite. Chapter 3 is the key to this book, where the bridging model development is shown in detail. An interesting outcome is that by making use of a bridging matrix, any micromechanical model for predicting effective elastic moduli of a UD composite can be formulated into a unified expression. In Chapter 4, the strength of UD composites is dealt with. Closed-form formulae for strengths of a UD lamina under uniaxial loads are derived. Modified maximum normal stress failure criteria for both multiaxial tension and multiaxial compression of a constituent are set forth. Strengths at elevated temperatures or subjected to fatigue loads are analyzed. Application of the bridging model to predict the strength of multidirectional laminates subjected to various load conditions is a main focus of this book, and is addressed in Chapter 5. Either the classical or a pseudo 3D laminate theory is incorporated with the bridging model to determine the internal stresses in the fibers and matrix of the laminate subjected to 2D or 3D load conditions. Fatal and nonfatal failures are classified. In additional to a variety of strength prediction examples, the WWFE-I and WWFE-II problems are analyzed with detailed discussions. The chapter ends with the highlight of the simulation procedure for inelastic and strength properties of woven, braided and knitted fabric reinforced composite laminates. The analyzing formulae have been programmed into a computer routine in the FORTRAN language, which is shown in Chapter 6. Supplymentary materials to this book containing the original code of the computer routine can be found from http://extra.springer.com. Input data for running the routine to resolve several illustrated examples and to analyze the WWFE-I and WWFE-II problems are included in the supplymentary materials.

The book is intended for senior and postgraduate students in engineering. It can be regarded as an extension to Strength/Mechanics of Materials textbooks. Researchers and engineers who are working with composite materials will also find this book useful. Any comment on the book can be sent to huangzm@tongji.edu.cn or huangzm@email.com. The authors would like to express their heartiest gratitude for any comments, in advance.

Zheng-Ming Huang
Ye-Xin Zhou
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Contents

1

Background

1.1 Scope of This Book

It has been recognized that technological development depends on advances in the field of materials. Whatever the field may be, the final limitations will rest on the available materials. In some industries, conventional monolithic materials are currently operating at or near their limits and do not offer the potential for meeting the demands of further technical advancement (Lerch & Saltsman, 1993). In this regard, composites represent nothing less than a giant step in the ever-lasting endeavor to achieve optimisation of materials.

Composite materials are made on a macroscopic scale from two or more distinct phases of constituent materials. They are developed to achieve unique mechanical properties and other superior performance characteristics that would be impossible with any of the constituent materials alone. As most practical synthetic composites are essentially constructed from two-phase composite components, we thus only need to focus on those composites having two distinct constituent materials, a continuous phase and a reinforcement phase. The continuous phase is commonly referred to as a matrix, which may be metal, ceramic or polymer. The geometric form of the reinforcement phase can be powders, particles, short fibers, whiskers or continuous fibers. Only continuous fiber reinforced composites are considered in this book. Thus, the fiber reinforced or, simply, the fibrous composites referred to throughout this book are considered as those made from continuous fiber reinforcement. However, the continuous fibers can be arranged in an arbitrary form, such as uni-/multi-directional, woven, braided or knitted preforms.

Modern composites made using continuous fiber preforms and various types of matrices have generated a revolution in high-performance structures in a number of industries such as aerospace, shipbuilding, sports equipment, automobile construction, energy, and so on. Advanced fibrous composites offer significantly high stiffness and strength to weight ratios, compared to conventional monolithic materials such as metallic materials. This is mainly because a material in very thin fiber form has a much higher mechanical performance than in its bulk form (Griffith, 1920; Gordon, 1976). Another advantage of fibrous composites is that people can freely select different constituent materials, their contents and their arrangement for an optimum performance.

A fundamental issue in making use of a composite, the same as in the use of any other material, is to understand thoroughly its mechanical properties, especially its ultimate load-carrying capacity. Metals, polymers and ceramics are essentially isotropic and homogenous, and have predictable properties. Hence, material selection, component design and manufacturing are fairly straightforward. On the other hand, composites essentially display anisotropic behaviors and their mechanical responses are different if loaded in different directions. Not surprisingly, the use of composites presents a whole new array of challenges for a designer. The designer must deal with anisotropic materials in his component design and understand how the properties of raw constituent materials, together with the specifics of potential manufacturing methods (possible reinforcement form and geometry, and relative proportions of fiber and matrix) will influence the properties of the final product.

The purpose of this book is to provide a comprehensive methodology to determine the mechanical behaviors, particularly the ultimate load-carrying capacity of fibrous composites from the knowledge of their constituent properties, volume fractions of the constituent materials, geometric arrangement of the reinforcing phase in the matrix, the laminate stacking sequence, etc. The composite forms considered include unidirectional laminae and multidirectional laminates.

1.2 Linear Elasticity

In order to investigate the mechanical behaviors of a material, especially for practical applications, stress and deformation analysis is necessary. The mechanics of elasticity can be considered as the theoretical basis for estimating the elastic stress and deformation of any solid structure or structural material under the action of any general loading (Zhang, 2003). Basic assumptions and concepts of linear elasticity will be briefly summarized here.

Two types of notations are used to designate rectangular coordinates of a point in a material geometry. One is (x, y, z)-notation and another is (x_1, x_2, x_3)-notation. Usually, they refer to two different right-hand coordinate systems, the origins of which may or may not coincide. As long as they refer to the same coordinate system, it is always true that $x_1=x$, $x_2=y$, and $x_3=z$.

When the material under consideration is subjected to some excitation, such as an external load, the initial point P: (x_1, x_2, x_3), will deform to a new point P': $(x_1+u_1, x_2+u_2, x_3+u_3)$, as shown in Fig. 1.1, where u_1, u_2, and u_3 are the displacement components of the point P along x_1-, x_2- and x_3-directions, respectively. One of the fundamental assumptions for linear elasticity is that all of the three displacement components, u_1, u_2, and u_3, are infinitesimal. From these displacement components, we get the infinitesimal strains of the point P as (Timoshenko & Goodier, 1970)

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad i, j = 1, 2, 3 \tag{1.1}$$

Using these strain components, we can define a second-order strain tensor (matrix) $[\varepsilon_{ij}]$, which has a dimension of 3×3. It is seen from Eq. (1.1) that the strain tensor is symmetric. Only six of them are independent. Thus, instead of the strain tensor, we can use a contracted strain vector, $\{\varepsilon_i\}$, to represent the strain state of the point P where

$$\{\varepsilon_i\}^{\mathrm{T}}=\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6\}=\{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{23}, 2\varepsilon_{13}, 2\varepsilon_{12}\} \tag{1.2}$$

The superscript "T" in Eq. (1.2) denotes a transposition. Note that there is a factor "2" before the shear strains, ε_{23}, ε_{13}, and ε_{12}.

As the material has been subjected to the external load, stresses are generated. Let $[\sigma_{ij}]$ denote a 3×3 stress tensor at the point P. By using an infinitesimal volume element containing P and by applying equilibrium conditions, it can be shown (Timoshenko & Goodier, 1970) that the stress tensor is always symmetric. We can thus also use a contracted vector, $\{\sigma_i\}$, to represent the stress state of the point P where

$$\{\sigma_i\}^{\mathrm{T}}= \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\}=\{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\} \tag{1.3}$$

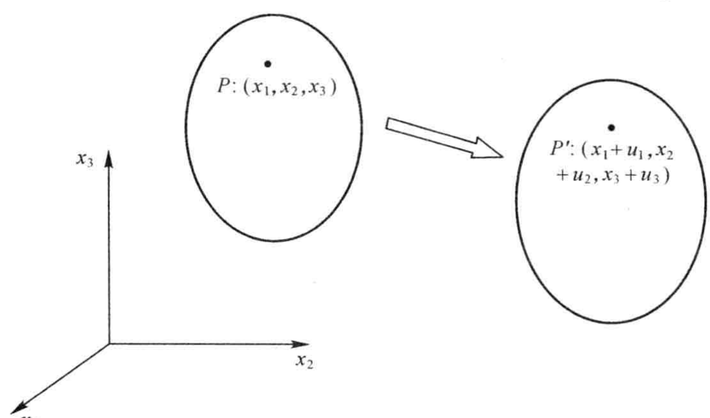

Fig. 1.1 Deformation of a material point

At any point P of the material, the elastic strain $\{\varepsilon_i\}$ is related to the stress $\{\sigma_j\}$ by Hooke's law,

$$\{\varepsilon_i\}=[S_{ij}]\{\sigma_j\} \tag{1.4.1}$$

or

$$\{\sigma_i\}=[C_{ij}]\{\varepsilon_j\} \tag{1.4.2}$$

where the 6×6 matrices $[S_{ij}]$ and $[C_{ij}]$ are named as compliance and stiffness matrices of the material, respectively. Each can be obtained from inverting the other, i.e.,

$$[S_{ij}]= [C_{ij}]^{-1} \tag{1.5.1}$$

$$[C_{ij}]= [S_{ij}]^{-1} \tag{1.5.2}$$

When the deformation of the material is in an elastic range (i.e., all of the displacement components, u_1, u_2, and u_3, vanish if the external load is reduced to zero), there is a strain energy function W such that

$$W=\frac{1}{2}\sigma_{ij}\varepsilon_{ij}=\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\sigma_{ij}\varepsilon_{ij} \tag{1.6}$$

Here and in the following, a summation convention is applied to any repeated subscripts, such as i and j in Eq. (1.6), in their variation range. Therefore,

$$\sigma_{ij}=\frac{\partial W}{\partial \varepsilon_{ij}}, \text{ or } \varepsilon_{ij}=\frac{\partial W}{\partial \sigma_{ij}} \tag{1.7}$$

The function W is always positive for any non-zero stress or strain tensor. This means that the compliance and stiffness matrices, $[S_{ij}]$ and $[C_{ij}]$, are always positive definite. From Eq. (1.7), we can further conclude that the matrices $[S_{ij}]$ and $[C_{ij}]$ are always symmetric because, after substituting Eq. (1.4.1) or Eq. (1.4.2) into Eq. (1.6), the resulting function W is a quadratic equation and the coefficient matrix of a quadratic can always be made to be symmetric. Hence, there are at most 21 independent elastic constants for any material. If, however, the material has some symmetric planes, i.e., the planes with respect to which the material properties are the same, the number of the independent constants can be reduced further (Timoshenko & Goodier, 1970). In engineering practice, there are three kinds of materials that are most commonly encountered. They are isotropic, transversely isotropic and orthotropic materials.

1.2.1 Isotropic Material

If the material is symmetric with respect to every direction, it is said to be isotropic. Most metals, ceramics and polymers are isotropic materials. In general, matrix materials used in composite fabrication are essentially taken as isotropic. For this kind of material, there are only two independent elastic constants. They are usually given in terms of engineering moduli, i.e., Young's modulus, E, and Poisson's ratio, ν. Young's modulus is defined as the slope of a uniaxial stress-strain curve of the material at an initial stage, whereas the Poisson's ratio is defined as the negative of the ratio of the transverse strain over the longitudinal strain when a testing load is applied in the longitudinal direction. The compliance matrix, $[S_{ij}]$, of an isotropic material takes the form

$$\left[S_{ij}\right]=\begin{bmatrix}\left[S_{ij}\right]_\sigma & 0\\ 0 & \left[S_{ij}\right]_\tau\end{bmatrix} \tag{1.8}$$

where $[S_{ij}]_\sigma$ and $[S_{ij}]_\tau$ are the sub-matrices of the compliance relating normal stresses with elongation strains and shear stresses with shear strains, respectively, and are given by

$$[S_{ij}]_\sigma=\begin{bmatrix}\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E}\\ & \frac{1}{E} & -\frac{\nu}{E}\\ \text{symmetry} & & \frac{1}{E}\end{bmatrix} \tag{1.9}$$

$$[S_{ij}]_\tau=\begin{bmatrix}\frac{1}{G} & 0 & 0\\ & \frac{1}{G} & 0\\ \text{symmetry} & & \frac{1}{G}\end{bmatrix} \tag{1.10}$$

In Eq. (1.10), G is the shear modulus defined as

$$G=0.5E/(1+\nu) \tag{1.11}$$

1.2.2 Transversely Isotropic Material

A material is said to be transversely isotropic if its elastic properties are kept unchanged with respect to an arbitrary rotation around a given axis. For convenience of illustration, this axis is called the symmetric axis (or direction). Such a kind of material is of special importance in the study of fibrous composites, since a unidirectional (abbreviated to "UD") composite, the most important fibrous composite, is generally considered as transversely isotropic. When fibers are uniformly arranged in the matrix in such a way that the axes of the fibers are parallel to each other, the material is said to be an unidirectionally fiber-reinforced composite. A UD composite is also called a UD lamina. Fig. 1.2 shows a high-contrast micrograph of the transverse plane section of such a boron fiber-aluminium matrix composite. The dark dots represent the cross-sections of the boron fibers and the white area designates the continuous aluminium matrix. From the figure, it can be easily concluded that the material properties are best considered as invariant (symmetric) with respect to any rotation about the fiber axis, since both the boron and the aluminium are isotropic. A further conclusion is that the resulting composite is still transversely isotropic even if the fiber material is transversely isotropic but has a symmetric direction along the fiber axis. This is important because a number of commonly used fibers such as graphite, carbon and aramid (Kevlar) are transversely isotropic.

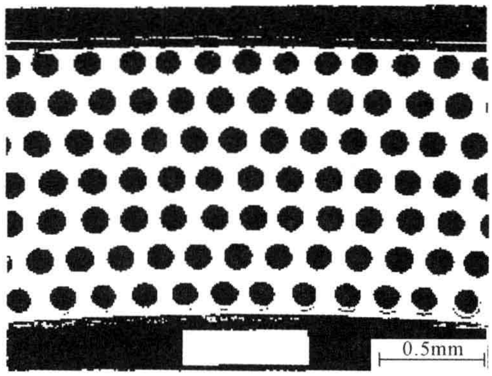

Fig. 1.2 A micrograph of the cross-sectional plane of a UD composite. The black dots are fibers and the white continuum is the matrix

For a transversely isotropic material, let its symmetric axis (the fiber axis in a UD composite) be x_1. The compliance matrix of the material is the same as that given by Eq. (1.8) but with different sub-matrices, which read

$$[S_{ij}]_\sigma = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{12}}{E_{11}} \\ & \frac{1}{E_{22}} & -\frac{\nu_{23}}{E_{22}} \\ \text{symmetry} & & \frac{1}{E_{22}} \end{bmatrix} \tag{1.12}$$

$$[S_{ij}]_\tau = \begin{bmatrix} \frac{1}{G_{23}} & 0 & 0 \\ & \frac{1}{G_{12}} & 0 \\ \text{symmetry} & & \frac{1}{G_{12}} \end{bmatrix} \tag{1.13}$$