

Franz Schwabl

Advanced Quantum Mechanics

4th Edition

高等量子力学

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Franz Schwabl

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Advanced Quantum Mechanics

The true physics is that which will, one day,
achieve the inclusion of man in his wholeness
in a coherent picture of the world.

Pierre Teilhard de Chardin

To my daughter Birgitta

Preface to the Fourth Edition

In this latest edition new material has been added, which includes many additional clarifying remarks and cross references. The design of all figures has been reworked, the layout has been improved and unified to enhance the didactic appeal of the book, however, in the course of these changes I have attempted to keep intact its underlying compact nature. I am grateful to many colleagues for their help with this substantial revision. Again, special thanks go to Uwe Täuber and Roger Hilton for discussions, comments and many constructive suggestions. I should like to thank Dr. Herbert Müller for his generous help in all computer problems. Concerning the graphics, I am very grateful to Mr Wenzel Schürmann for essential support and to Ms Christina Di Stefano and Mr Benjamin Sánchez who undertook the graphical design of the diagrams.

It is my pleasure to thank Dr. Thorsten Schneider and Mrs Jacqueline Lenz of Springer for the excellent co-operation, as well as the le-tex setting team for their careful incorporation of the amendments for this new edition. Finally, I should like to thank all colleagues and students who, over the years, have made suggestions to improve the usefulness of this book.

Munich, June 2008

F. Schwabl

Preface to the First Edition

This textbook deals with advanced topics in the field of quantum mechanics, material which is usually encountered in a second university course on quantum mechanics. The book, which comprises a total of 15 chapters, is divided into three parts: I. Many-Body Systems, II. Relativistic Wave Equations, and III. Relativistic Fields. The text is written in such a way as to attach importance to a rigorous presentation while, at the same time, requiring no prior knowledge, except in the field of basic quantum mechanics. The inclusion of all mathematical steps and full presentation of intermediate calculations ensures ease of understanding. A number of problems are included at the end of each chapter. Sections or parts thereof that can be omitted in a first reading are marked with a star, and subsidiary calculations and remarks not essential for comprehension are given in small print. It is not necessary to have read Part I in order to understand Parts II and III. References to other works in the literature are given whenever it is felt they serve a useful purpose. These are by no means complete and are simply intended to encourage further reading. A list of other textbooks is included at the end of each of the three parts.

In contrast to Quantum Mechanics I, the present book treats relativistic phenomena, and classical and relativistic quantum fields.

Part I introduces the formalism of second quantization and applies this to the most important problems that can be described using simple methods. These include the weakly interacting electron gas and excitations in weakly interacting Bose gases. The basic properties of the correlation and response functions of many-particle systems are also treated here.

The second part deals with the Klein-Gordon and Dirac equations. Important aspects, such as motion in a Coulomb potential are discussed, and particular attention is paid to symmetry properties.

The third part presents Noether's theorem, the quantization of the Klein-Gordon, Dirac, and radiation fields, and the spin-statistics theorem. The final chapter treats interacting fields using the example of quantum electrodynamics: S-matrix theory, Wick's theorem, Feynman rules, a few simple processes such as Mott scattering and electron-electron scattering, and basic aspects of radiative corrections are discussed.

The book is aimed at advanced students of physics and related disciplines, and it is hoped that some sections will also serve to augment the teaching material already available.

This book stems from lectures given regularly by the author at the Technical University Munich. Many colleagues and coworkers assisted in the production and correction of the manuscript: Ms. I. Wefers, Ms. E. Jörg-Müller, Ms. C. Schwierz, A. Vilfan, S. Clar, K. Schenk, M. Hummel, E. Wefers, B. Kaufmann, M. Bulenda, J. Wilhelm, K. Kroy, P. Maier, C. Feuchter, A. Wonhas. The problems were conceived with the help of E. Frey and W. Gasser. Dr. Gasser also read through the entire manuscript and made many valuable suggestions. I am indebted to Dr. A. Lahee for supplying the initial English version of this difficult text, and my special thanks go to Dr. Roginald Hilton for his perceptive revision that has ensured the fidelity of the final rendition.

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Munich, March 1999

F. Schwabl

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Part I

Nonrelativistic Many-Particle Systems

