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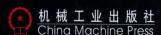
(美) Kenneth H. Rosen 著

英文版・第7版

Kenneth H. Rosen



Discrete
Mathematics
and Its
Applications

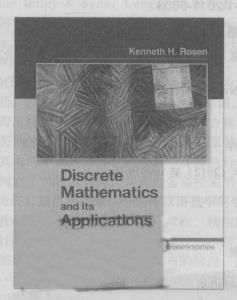


SEVENTH EDITION

# 离散数学及其应用

(英文版・第7版)

I iscrete Mathematics and
Its Applications (Seventh Edition)



(美) Kannath II D.

小 机械工业出版社 China Machine Press Kenneth H. Rosen: Discrete Mathematics and Its Applications, Seventh Edition (ISBN 978-0-07-338309-5). Copyright © 2012 by The McGraw-Hill Companies, Inc.

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近年,在全球信息化大潮的推动下,我国的计算机产业发展迅猛,对专业人才的需求日益 迫切。这对计算机教育界和出版界都既是机遇,也是挑战,而专业教材的建设在教育战略上显 得举足轻重。在我国信息技术发展时间较短的现状下,美国等发达国家在其计算机科学发展的 几十年间积淀和发展的经典教材仍有许多值得借鉴之处。因此,引进一批国外优秀计算机教材 将对我国计算机教育事业的发展起到积极的推动作用,也是与世界接轨、建设真正的世界一流 大学的必由之路。

机械工业出版社华章公司较早意识到"出版要为教育服务"。自1998年开始,我们就将工作重点放在了遴选、移译国外优秀教材上。经过多年的不懈努力,我们与Pearson,McGraw-Hill, Elsevier,MIT, John Wiley & Sons,Cengage等世界著名出版公司建立了良好的合作关系,从他们现有的数百种教材中甄选出Andrew S. Tanenbaum,Bjarne Stroustrup,Brain W. Kernighan,Dennis Ritchie,Jim Gray,Afred V. Aho,John E. Hopcroft,Jeffrey D. Ullman,Abraham Silberschatz,William Stallings,Donald E. Knuth,John L. Hennessy,Larry L. Peterson等大师名家的一批经典作品,以"计算机科学丛书"为总称出版,供读者学习、研究及珍藏。大理石纹理的封面,也正体现了这套丛书的品位和格调。

"计算机科学丛书"的出版工作得到了国内外学者的鼎力襄助,国内的专家不仅提供了中肯的选题指导,还不辞劳苦地担任了翻译和审校的工作,而原书的作者也相当关注其作品在中国的传播,有的还专程为其书的中译本作序。迄今,"计算机科学丛书"已经出版了近两百个品种,这些书籍在读者中树立了良好的口碑,并被许多高校采用为正式教材和参考书籍。其影印版"经典原版书库"作为姊妹篇也被越来越多实施双语教学的学校所采用。

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## Preface

In writing this book, I was guided by my long-standing experience and interest in teaching discrete mathematics. For the student, my purpose was to present material in a precise, readable manner, with the concepts and techniques of discrete mathematics clearly presented and demonstrated. My goal was to show the relevance and practicality of discrete mathematics to students, who are often skeptical. I wanted to give students studying computer science all of the mathematical foundations they need for their future studies. I wanted to give mathematics students an understanding of important mathematical concepts together with a sense of why these concepts are important for applications. And most importantly, I wanted to accomplish these goals without watering down the material.

For the instructor, my purpose was to design a flexible, comprehensive teaching tool using proven pedagogical techniques in mathematics. I wanted to provide instructors with a package of materials that they could use to teach discrete mathematics effectively and efficiently in the most appropriate manner for their particular set of students. I hope that I have achieved these goals.

I have been extremely gratified by the tremendous success of this text. The many improvements in the seventh edition have been made possible by the feedback and suggestions of a large number of instructors and students at many of the more than 600 North American schools, and at any many universities in parts of the world, where this book has been successfully used.

This text is designed for a one- or two-term introductory discrete mathematics course taken by students in a wide variety of majors, including mathematics, computer science, and engineering. College algebra is the only explicit prerequisite, although a certain degree of mathematical maturity is needed to study discrete mathematics in a meaningful way. This book has been designed to meet the needs of almost all types of introductory discrete mathematics courses. It is highly flexible and extremely comprehensive. The book is designed not only to be a successful textbook, but also to serve as valuable resource students can consult throughout their studies and professional life.

#### Goals of a Discrete Mathematics Course

A discrete mathematics course has more than one purpose. Students should learn a particular set of mathematical facts and how to apply them; more importantly, such a course should teach students how to think logically and mathematically. To achieve these goals, this text stresses mathematical reasoning and the different ways problems are solved. Five important themes are interwoven in this text: mathematical reasoning, combinatorial analysis, discrete structures, algorithmic thinking, and applications and modeling. A successful discrete mathematics course should carefully blend and balance all five themes.

1 Mathematical Reasoning: Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. This text starts with a discussion of mathematical logic, which serves as the foundation for the subsequent discussions of methods of proof. Both the science and the art of constructing proofs are addressed. The technique of mathematical induction is stressed through many different types of examples of such proofs and a careful explanation of why mathematical induction is a valid proof technique.

- 3. Discrete Structures: A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, permutations, relations, graphs, trees, and finite-state machines.
- 4. Algorithmic Thinking: Certain classes of problems are solved by the specification of an algorithm. After an algorithm has been described, a computer program can be constructed implementing it. The mathematical portions of this activity, which include the specification of the algorithm, the verification that it works properly, and the analysis of the computer memory and time required to perform it, are all covered in this text. Algorithms are described using both English and an easily understood form of pseudocode.
- 5 Applications and Modeling: Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science and data networking in this text, as well as applications to such diverse areas as chemistry, biology, linguistics, geography, business, and the Internet. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises.

#### Changes in the Seventh Edition

Although the sixth edition has been an extremely effective text, many instructors, including longtime users, have requested changes designed to make this book more effective. I have devoted a significant amount of time and energy to satisfy their requests and I have worked hard to find my own ways to make the book more effective and more compelling to students.

The seventh edition is a major revision, with changes based on input from more than 40 formal reviewers, feedback from students and instructors, and author insights. The result is a new edition that offers an improved organization of topics making the book a more effective teaching tool. Substantial enhancements to the material devoted to logic, algorithms, number theory, and graph theory make this book more flexible and comprehensive. Numerous changes in the seventh edition have been designed to help students more easily learn the material. Additional explanations and examples have been added to clarify material where students often have difficulty. New exercises, both routine and challenging, have been added. Highly relevant applications, including many related to the Internet, to computer science, and to mathematical biology, have been added. The companion website has benefited from extensive development activity and now provides tools students can use to master key concepts and explore the world of discrete mathematics, and many new tools under development will be released in the year following publication of this book.

I hope that instructors will closely examine this new edition to discover how it might meet their needs. Although it is impractical to list all the changes in this edition, a brief list that highlights some key changes, listed by the benefits they provide, may be useful.

#### More Flexible Organization

- Applications of propositional logic are found in a new dedicated section, which briefly introduces logic circuits.
- Recurrence relations are now covered in Chapter 2.
- Expanded coverage of countability is now found in a dedicated section in Chapter 2.

- - Separate chapters now provide expanded coverage of algorithms (Chapter 3) and number theory and cryptography (Chapter 4).
  - More second and third level heads have been used to break sections into smaller coherent

#### Tools for Easier Learning

- Difficult discussions and proofs have been marked with the famous Bourbaki "dangerous bend" symbol in the margin.
- New marginal notes make connections, add interesting notes, and provide advice to students.
- More details and added explanations, in both proofs and exposition, make it easier for students to read the book.
- Many new exercises, both routine and challenging, have been added, while many existing exercises have been improved.

#### Enhanced Coverage of Logic, Sets, and Proof Jones W. and Date (See 1997)

- The satisfiability problem is addressed in greater depth, with Sudoku modeled in terms of satisfiability.
- Hilbert's Grand Hotel is used to help explain uncountability.
- Proofs throughout the book have been made more accessible by adding steps and reasons behind these steps.
- A template for proofs by mathematical induction has been added.
- The step that applies the inductive hypothesis in mathematical induction proof is now explicitly noted.

### Algorithms mor mann a based sended of the evision revision as a major revision, with charges based on more revision.

- The pseudocode used in the book has been updated.
- Explicit coverage of algorithmic paradigms, including brute force, greedy algorithms, and dynamic programing, is now provided.
- Useful rules for big-O estimates of logarithms, powers, and exponential functions have been added.

#### Number Theory and Cryptography and washing to be 1 belies used was worked

- Expanded coverage allows instructors to include just a little or a lot of number theory in their courses.
- The relationship between the mod function and congruences has been explained more
- The sieve of Eratosthenes is now introduced earlier in the book.
- Linear congruences and modular inverses are now covered in more detail.
- Applications of number theory, including check digits and hash functions, are covered in great depth.
- A new section on cryptography integrates previous coverage, and the notion of a cryptosystem has been introduced.
- Cryptographic protocols, including digital signatures and key sharing, are now covered.

#### Graph Theory

- A structured introduction to graph theory applications has been added.
- More coverage has been devoted to the notion of social networks.
- Applications to the biological sciences and motivating applications for graph isomorphism and planarity have been added.
- Matchings in bipartite graphs are now covered, including Hall's theorem and its proof.
- Coverage of vertex connectivity, edge connectivity, and *n*-connectedness has been added, providing more insight into the connectedness of graphs.

#### **Enrichment Material**

- Many biographies have been expanded and updated, and new biographies of Bellman, Bézout Bienyamé, Cardano, Catalan, Cocks, Cook, Dirac, Hall, Hilbert, Ore, and Tao have been added.
- Historical information has been added throughout the text.
- Numerous updates for latest discoveries have been made.

#### Expanded Media

- Extensive effort has been devoted to producing valuable web resources for this book.
- Extra examples in key parts of the text have been provided on companion website.
- Interactive algorithms have been developed, with tools for using them to explore topics and for classroom use.
- A new online ancillary, *The Virtual Discrete Mathematics Tutor*, available in fall 2012, will help students overcome problems learning discrete mathematics.
- A new homework delivery system, available in fall 2012, will provide automated homework for both numerical and conceptual exercises.
- Student assessment modules are available for key concepts.
- Powerpoint transparencies for instructor use have been developed.
- A supplement *Exploring Discrete Mathematics* has been developed, providing extensive support for using Maple<sup>TM</sup> or Mathematica<sup>TM</sup> in conjunction with the book.
- An extensive collection of external web links is provided.

#### Features of the Book

ACCESSIBILITY This text has proved to be easily read and understood by beginning students. There are no mathematical prerequisites beyond college algebra for almost all the content of the text. Students needing extra help will find tools on the companion website for bringing their mathematical maturity up to the level of the text. The few places in the book where calculus is referred to are explicitly noted. Most students should easily understand the pseudocode used in the text to express algorithms, regardless of whether they have formally studied programming languages. There is no formal computer science prerequisite.

Each chapter begins at an easily understood and accessible level. Once basic mathematical concepts have been carefully developed, more difficult material and applications to other areas of study are presented.

FLEXIBILITY This text has been carefully designed for flexible use. The dependence of chapters on previous material has been minimized. Each chapter is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructors can easily pace their lectures using these blocks.

The writing style in this book is direct and pragmatic. Precise mathematical language is used without excessive formalism and abstraction. Care has been taken to balance the mix of notation and words in mathematical statements.

MATHEMATICAL RIGOR AND PRECISION All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematics. Proofs are motivated and developed slowly; their steps are all carefully justified. The axioms used in proofs and the basic properties that follow from them are explicitly described in an appendix, giving students a clear idea of what they can assume in a proof. Recursive definitions are explained and used extensively.

WORKED EXAMPLES Over 800 examples are used to illustrate concepts, relate different topics, and introduce applications. In most examples, a question is first posed, then its solution is presented with the appropriate amount of detail.

APPLICATIONS The applications included in this text demonstrate the utility of discrete mathematics in the solution of real-world problems. This text includes applications to a wide variety of areas, including computer science, data networking, psychology, chemistry, engineering, linguistics, biology, business, and the Internet.

ALGORITHMS Results in discrete mathematics are often expressed in terms of algorithms; hence, key algorithms are introduced in each chapter of the book. These algorithms are expressed in words and in an easily understood form of structured pseudocode, which is described and specified in Appendix 3. The computational complexity of the algorithms in the text is also analyzed at an elementary level.

HISTORICAL INFORMATION The background of many topics is succinctly described in the text. Brief biographies of 83 mathematicians and computer scientists are included as footnotes. These biographies include information about the lives, careers, and accomplishments of these important contributors to discrete mathematics and images, when available, are displayed.

In addition, numerous historical footnotes are included that supplement the historical information in the main body of the text. Efforts have been made to keep the book up-to-date by reflecting the latest discoveries.

A list of key terms and results follows each chapter. The KEY TERMS AND RESULTS key terms include only the most important that students should learn, and not every term defined in the chapter.

EXERCISES There are over 4000 exercises in the text, with many different types of questions posed. There is an ample supply of straightforward exercises that develop basic skills, a large number of intermediate exercises, and many challenging exercises. Exercises are stated clearly and unambiguously, and all are carefully graded for level of difficulty. Exercise sets contain special discussions that develop new concepts not covered in the text, enabling students to discover new ideas through their own work.

Exercises that are somewhat more difficult than average are marked with a single star \*; those that are much more challenging are marked with two stars \*\*. Exercises whose solutions require calculus are explicitly noted. Exercises that develop results used in the text are clearly identified with the right pointing hand symbol s. Answers or outlined solutions to all oddnumbered exercises are provided at the back of the text. The solutions include proofs in which most of the steps are clearly spelled out.

REVIEW QUESTIONS A set of review questions is provided at the end of each chapter.

These questions are designed to help students focus their study on the most important concepts and techniques of that chapter. To answer these questions students need to write long answers, rather than just perform calculations or give short replies.

SUPPLEMENTARY EXERCISE SETS Each chapter is followed by a rich and varied set of supplementary exercises. These exercises are generally more difficult than those in the exercise sets following the sections. The supplementary exercises reinforce the concepts of the chapter and integrate different topics more effectively.

**COMPUTER PROJECTS** Each chapter is followed by a set of computer projects. The approximately 150 computer projects tie together what students may have learned in computing and in discrete mathematics. Computer projects that are more difficult than average, from both a mathematical and a programming point of view, are marked with a star, and those that are extremely challenging are marked with two stars.

COMPUTATIONS AND EXPLORATIONS A set of computations and explorations is included at the conclusion of each chapter. These exercises (approximately 120 in total) are designed to be completed using existing software tools, such as programs that students or instructors have written or mathematical computation packages such as Maple<sup>TM</sup> or Mathematica<sup>TM</sup>. Many of these exercises give students the opportunity to uncover new facts and ideas through computation. (Some of these exercises are discussed in the *Exploring Discrete Mathematics* companion workbooks available online.)

WRITING PROJECTS Each chapter is followed by a set of writing projects. To do these projects students need to consult the mathematical literature. Some of these projects are historical in nature and may involve looking up original sources. Others are designed to serve as gateways to new topics and ideas. All are designed to expose students to ideas not covered in depth in the text. These projects tie mathematical concepts together with the writing process and help expose students to possible areas for future study. (Suggested references for these projects can be found online or in the printed *Student's Solutions Guide*.)

APPENDIXES There are three appendixes to the text. The first introduces axioms for real numbers and the positive integers, and illustrates how facts are proved directly from these axioms. The second covers exponential and logarithmic functions, reviewing some basic material used heavily in the course. The third specifies the pseudocode used to describe algorithms in this text.

SUGGESTED READINGS A list of suggested readings for the overall book and for each chapter is provided after the appendices. These suggested readings include books at or below the level of this text, more difficult books, expository articles, and articles in which discoveries in discrete mathematics were originally published. Some of these publications are classics, published many years ago, while others have been published in the last few years.

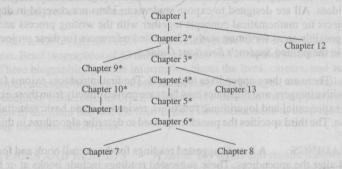
#### How to Use This Book method and the second of the second o

This text has been carefully written and constructed to support discrete mathematics courses at several levels and with differing foci. The following table identifies the core and optional sections. An introductory one-term course in discrete mathematics at the sophomore level can be based on the core sections of the text, with other sections covered at the discretion of the

instructor. A two-term introductory course can include all the optional mathematics sections in addition to the core sections. A course with a strong computer science emphasis can be taught by covering some or all of the optional computer science sections. Instructors can find sample syllabi for a wide range of discrete mathematics courses and teaching suggestions for using each section of the text can be found in the *Instructor's Resource Guide* available on the website for this book.

Chapter	Core	Optional CS	Optional Math
1	1.1–1.8 (as needed)	Lacobordo on America	Walla taa
2	2.1–2.4, 2.6 (as needed)		2.5
3		3.1–3.3 (as needed)	20712A7
4	4.1–4.4 (as needed)	4.5, 4.6	P TOWNEY
5	5.1–5.3	5.4, 5.5	en them.
6	6.1–6.3	6.6	6.4, 6.5
7	7.1	7.4	7.2, 7.3
8	8.1, 8.5	8.3	8.2, 8.4, 8.6
9	9.1, 9.3, 9.5	9.2	9.4, 9.6
10	10.1–10.5	ONLING OUR SHERICHERS A	10.6-10.8
11	11.1	11.2, 11.3	11.4, 11.5
12	TO THE ENDLEMENT	12.1-12.4	P LIN D. J.
13	A CHAPLET, I HOBE EXCLUSES OF	13.1–13.5	discoste

Instructors using this book can adjust the level of difficulty of their course by choosing either to cover or to omit the more challenging examples at the end of sections, as well as the more challenging exercises. The chapter dependency chart shown here displays the strong dependencies. A star indicates that only relevant sections of the chapter are needed for study of a later chapter. Weak dependencies have been ignored. More details can be found in the Instructor Resource Guide.



#### Ancillaries

STUDENT'S SOLUTIONS GUIDE This student manual, available separately, contains full solutions to all odd-numbered problems in the exercise sets. These solutions explain why a particular method is used and why it works. For some exercises, one or two other possible approaches are described to show that a problem can be solved in several different ways. Suggested references for the writing projects found at the end of each chapter are also included in this volume. Also included are a guide to writing proofs and an extensive description of common

mistakes students make in discrete mathematics, plus sample tests and a sample crib sheet for each chapter designed to help students prepare for exams.

(ISBN-10: 0-07-735350-1) (ISBN-13: 978-0-07-735350-6)

INSTRUCTOR'S RESOURCE GUIDE This manual, available on the website and in printed form by request for instructors, contains full solutions to even-numbered exercises in the text. Suggestions on how to teach the material in each chapter of the book are provided, including the points to stress in each section and how to put the material into perspective. It also offers sample tests for each chapter and a test bank containing over 1500 exam questions to choose from. Answers to all sample tests and test bank questions are included. Finally, several sample syllabi are presented for courses with differing emphases and student ability levels.

(ISBN-10: 0-07-735349-8)

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I would like to thank the many instructors and students at a variety of schools who have used this book and provided me with their valuable feedback and helpful suggestions. Their input has made this a much better book than it would have been otherwise. I especially want to thank Jerrold Grossman, Jean-Claude Evard, and Georgia Mederer for their technical reviews of the seventh edition and their "eagle eyes," which have helped ensure the accuracy of this book. I also appreciate the help provided by all those who have submitted comments via the website.

I thank the reviewers of this seventh and the six previous editions. These reviewers have provided much helpful criticism and encouragement to me. I hope this edition lives up to their high expectations.

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Kenneth H. Rosen

# About the Author

enneth H. Rosen has had a long career as a Distinguished Member of the Technical Staff at AT&T Laboratories in Monmouth County, New Jersey. He currently holds the position of Visiting Research Professor at Monmouth University, where he teaches graduate courses in computer science.

Dr. Rosen received his B.S. in Mathematics from the University of Michigan, Ann Arbor (1972), and his Ph.D. in Mathematics from M.I.T. (1976), where he wrote his thesis in the area of number theory under the direction of Harold Stark. Before joining Bell Laboratories in 1982, he held positions at the University of Colorado, Boulder; The Ohio State University, Columbus; and the University of Maine, Orono, where he was an associate professor of mathematics. While working at AT&T Labs, he taught at Monmouth University, teaching courses in discrete mathematics, coding theory, and data security. He currently teaches courses in algorithm design and in computer security and cryptography.

Dr. Rosen has published numerous articles in professional journals in number theory and in mathematical modeling. He is the author of the widely used Elementary Number Theory and Its Applications, published by Pearson, currently in its sixth edition, which has been translated into Chinese. He is also the author of Discrete Mathematics and Its Applications, published by McGraw-Hill, currently in its seventh edition. Discrete Mathematics and Its Applications has sold more than 350,000 copies in North America during its lifetime, and hundreds of thousands of copies throughout the rest of the world. This book has also been translated into Spanish, French, Greek, Chinese, Vietnamese, and Korean. He is also co-author of UNIX: The Complete Reference; UNIX System V Release 4: An Introduction; and Best UNIX Tips Ever, all published by Osborne McGraw-Hill. These books have sold more than 150,000 copies, with translations into Chinese, German, Spanish, and Italian. Dr. Rosen is also the editor of the Handbook of Discrete and Combinatorial Mathematics, published by CRC Press, and he is the advisory editor of the CRC series of books in discrete mathematics, consisting of more than 55 volumes on different aspects of discrete mathematics, most of which are introduced in this book. Dr. Rosen serves as an Associate Editor for the journal Discrete Mathematics, where he works with submitted papers in several areas of discrete mathematics, including graph theory, enumeration, and number theory. He is also interested in integrating mathematical software into the educational and professional environments, and worked on several projects with Waterloo Maple Inc.'s Maple TM software in both these areas. Dr. Rosen has also worked with several publishing companies on their homework delivery platforms.

At Bell Laboratories and AT&T Laboratories, Dr. Rosen worked on a wide range of projects, including operations research studies, product line planning for computers and data communications equipment, and technology assessment. He helped plan AT&T's products and services in the area of multimedia, including video communications, speech recognition, speech synthesis, and image networking. He evaluated new technology for use by AT&T and did standards work in the area of image networking. He also invented many new services, and holds more than 55 patents. One of his more interesting projects involved helping evaluate technology for the AT&T attraction that was part of EPCOT Center.

# The Companion Website

he extensive companion website accompanying this text has been substantially enhanced for the seventh edition This website is accessible at www.mhhe.com/rosen. The homepage shows the Information Center, and contains login links for the site's Student Site and Instructor Site. Key features of each area are described below:

#### THE INFORMATION CENTER

The Information Center contains basic information about the book including the expanded table of contents (including subsection heads), the preface, descriptions of the ancillaries, and a sample chapter. It also provides a link that can be used to submit errata reports and other feedback about the book.

The Student site contains a wealth of resources available for student use, including the following, tied into the text wherever the special icons displayed below are found in the text:



**Extra Examples** You can find a large number of additional examples on the site, covering all chapters of the book. These examples are concentrated in areas where students often ask for additional material. Although most of these examples amplify the basic concepts, more-challenging examples can also be found here.



Interactive Demonstration Applets These applets enable you to interactively explore how important algorithms work, and are tied directly to material in the text with linkages to examples and exercises. Additional resources are provided on how to use and apply these applets.



Self Assessments These interactive guides help you assess your understanding of 14 key concepts, providing a question bank where each question includes a brief tutorial followed by a multiple-choice question. If you select an incorrect answer, advice is provided to help you understand your error. Using these Self Assessments, you should be able to diagnose your problems and find appropriate help.



Web Resources Guide This guide provides annotated links to hundreds of external websites containing relevant material such as historical and biographical information, puzzles and problems, discussions, applets, programs, and more. These links are keyed to the text by page molimocha, including video communications, speech radmun

Additional resources in the Student site include:

- bra system to do a wide range of computations in discrete mathematics. Each chapter provides a description of relevant functions in the computer algebra system and how they are used, programs to carry out computations in discrete mathematics, examples, and exercises that can be worked using this computer algebra system. Two versions, Exploring Discrete Mathematics with Maple TM and Exploring Discrete Mathematics with Mathematica TM will be available.
  - \* Applications of Discrete Mathematics This ancillary contains 24 chapters—each with its own set of exercises—presenting a wide variety of interesting and important applications

- A Guide to Proof-Writing This guide provides additional help for writing proofs, a skill that many students find difficult to master. By reading this guide at the beginning of the course and periodically thereafter when proof writing is required, you will be rewarded as your proof-writing ability grows. (Also available in the Student's Solutions Guide.)
- Common Mistakes in Discrete Mathematics This guide includes a detailed list of common misconceptions that students of discrete mathematics often have and the kinds of errors they tend to make. You are encouraged to review this list from time to time to help avoid these common traps. (Also available in the Student's Solutions Guide.)
- Advice on Writing Projects This guide offers helpful hints and suggestions for the Writing Projects in the text, including an extensive bibliography of helpful books and articles for research; discussion of various resources available in print and online; tips on doing library research; and suggestions on how to write well. (Also available in the Student's Solutions Guide.)
- The Virtual Discrete Mathematics Tutor This extensive ancillary provides students with valuable assistance as they make the transition from lower-level courses to discrete mathematics. The errors students have made when studying discrete mathematics using this text has been analyzed to design this resource. Students will be able to get many of their questions answered and can overcome many obstacles via this ancillaries. The Virtual Discrete Mathematics Tutor is expected to be available in the fall of 2012.

#### **INSTRUCTOR SITE**

This part of the website provides access to all of the resources on the Student Site, as well as these resources for instructors:

- Suggested Syllabi Detailed course outlines are shown, offering suggestions for courses with different emphases and different student backgrounds and ability levels.
- Teaching Suggestions This guide contains detailed teaching suggestions for instructors, including chapter overviews for the entire text, detailed remarks on each section, and comments on the exercise sets.
- Printable Tests Printable tests are offered in TeX and Word format for every chapter, and can be customized by instructors.
- PowerPoints Lecture Slides and PowerPoint Figures and Tables An extensive collection
  of PowerPoint slides for all chapters of the text are provided for instructor use. In addition,
  images of all figures and tables from the text are provided as PowerPoint slides.
- Homework Delivery System An extensive homework delivery system, under development for availability in fall 2012, will provide questions tied directly to the text, so that students will be able to do assignments on-line. Moreover, they will be able to use this system in a tutorial mode. This system will be able to automatically grade assignments, and deliver free-form student input to instructors for their own analysis. Course management capabilities will be provided that will allow instructors to create assignments, automatically assign and grade homework, quiz, and test questions from a bank of questions tied directly to the text, create and edit their own questions, manage course announcements and due dates, and track student progress.

### To the Student

hat is discrete mathematics? Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here discrete means consisting of distinct or unconnected elements.) The kinds of problems solved using discrete mathematics include:

- How many ways are there to choose a valid password on a computer system?
- What is the probability of winning a lottery?
- Is there a link between two computers in a network?
- How can I identify spam e-mail messages?
- How can I encrypt a message so that no unintended recipient can read it?
- What is the shortest path between two cities using a transportation system?
- How can a list of integers be sorted so that the integers are in increasing order?
  - How many steps are required to do such a sorting?
  - How can it be proved that a sorting algorithm correctly sorts a list?
  - How can a circuit that adds two integers be designed?
  - How many valid Internet addresses are there?

You will learn the discrete structures and techniques needed to solve problems such as these.

More generally, discrete mathematics is used whenever objects are counted, when relationships between finite (or countable) sets are studied, and when processes involving a finite number of steps are analyzed. A key reason for the growth in the importance of discrete mathematics is that information is stored and manipulated by computing machines in a discrete fashion.

WHY STUDY DISCRETE MATHEMATICS? There are several important reasons for studying discrete mathematics. First, through this course you can develop your mathematical maturity: that is, your ability to understand and create mathematical arguments. You will not get very far in your studies in the mathematical sciences without these skills.

Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Discrete mathematics provides the mathematical foundations for many computer science courses including data structures, algorithms, database theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Students find these courses much more difficult when they have not had the appropriate mathematical foundations from discrete math. One student has sent me an e-mail message saying that she used the contents of this book in every computer science course she took!

Math courses based on the material studied in discrete mathematics include logic, set theory, number theory, linear algebra, abstract algebra, combinatorics, graph theory, and probability theory (the discrete part of the subject).

Also, discrete mathematics contains the necessary mathematical background for solving problems in operations research (including many discrete optimization techniques), chemistry, engineering, biology, and so on. In the text, we will study applications to some of these areas.

Many students find their introductory discrete mathematics course to be significantly more challenging than courses they have previously taken. One reason for this is that one of the primary goals of this course is to teach mathematical reasoning and problem solving, rather than a discrete set of skills. The exercises in this book are designed to reflect this goal. Although there are plenty of exercises in this text similar to those addressed in the examples, a large