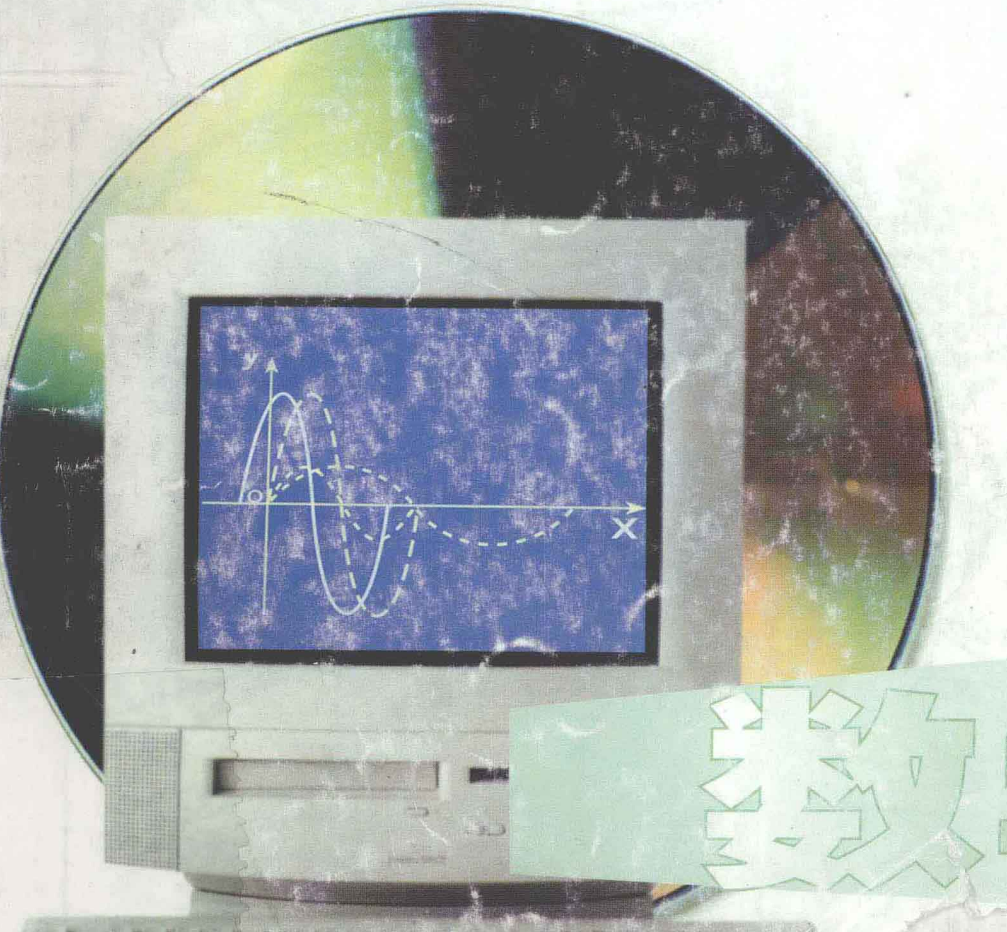
英语版

全日制普通高级中学教科书（试验修订本·必修）

MATHEMATICS

第一册（下）

课程教材研究所
双语课程教材研究开发中心 组译



数学

教育出版社
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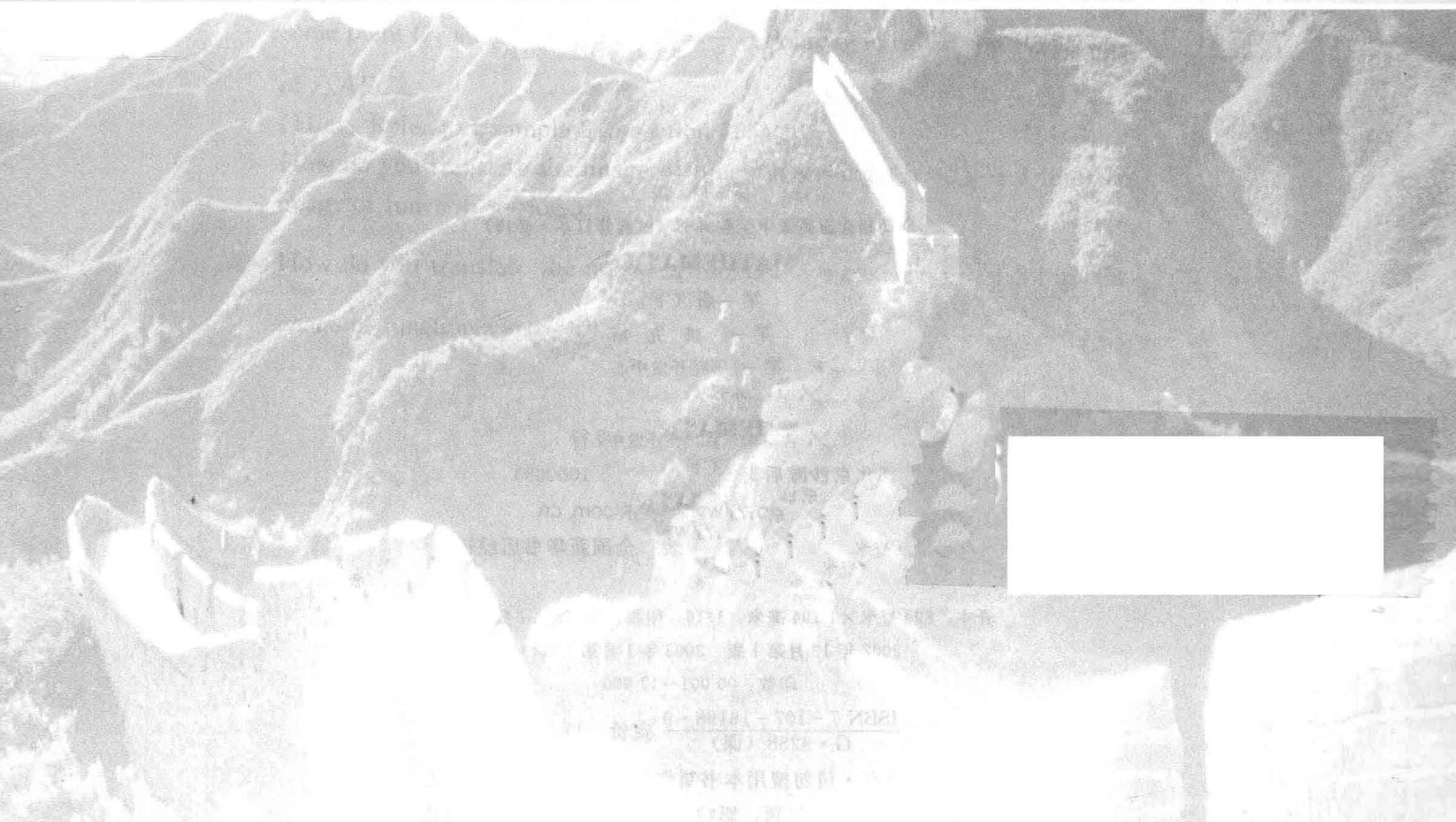

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英语版普通高中教科书

说　明

随着改革开放的不断扩大，中国在经济、文化、教育等诸多方面与各国间的交往日益增强，中国人学习英语的热情也日趋高涨。当今社会，是否熟练掌握英语，已成为衡量一个人的知识结构甚至综合素质的一个重要方面。在这样的形势下，多角度、多渠道提高人们的英语水平，特别是提高基础教育阶段在校高中学生的英语水平，已经成为社会的迫切需要。

为了适应这种新的形势和需要，作为教育部直属单位的课程教材研究所着手研究开发这套英语版普通高中教材，包括数学、物理、化学、生物、历史、地理六门必修课程，由人民教育出版社出版。

这套英语版高中教材，根据经国家教育部审查通过、人民教育出版社出版的《全日制普通高级中学教科书（试验修订本·必修）》翻译而成，主要供实行双语教学的学校或班级使用，也可以作为高中生的课外读物，其他有兴趣的读者也可以作为参考书使用，使学科知识的掌握与英语能力的提高形成一种双赢的局面。

为了使这套新品种的教材具有较高的编译质量，课程教材研究所双语课程教材研究开发中心依托所内各科教材研究开发中心，在国内外特聘学科专家和英语专家联袂翻译，且全部译稿均由中外知名专家共同审校。

我们的宗旨是：以前瞻意识迎接时代挑战，以国际水平奉献中华学子。

人教版高中英语版教材，愿与广大师生和家长结伴同行，共同打造新世纪的一流英才。

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课程教材研究所
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高中《数学》教科书

说　明

《全日制普通高级中学教科书（试验修订本）·数学》（以下简称《数学》）是根据教育部2000年颁布的《全日制普通高级中学课程计划（试验修订稿）》和《全日制普通高级中学数学教学大纲（试验修订版）》的规定，遵照1999年全国教育工作会议的精神，在两省一市进行试验的《全日制普通高级中学教科书（试验本）·数学》的基础上进行修订的。此次修订的指导思想是：遵循“教育要面向现代化，面向世界，面向未来”的战略思想，贯彻教育必须为社会主义现代化建设服务，必须与生产劳动相结合，培养德、智、体、美全面发展的社会主义事业的建设者和接班人的方针，以全面推进素质教育为宗旨，全面提高普通高中教育质量。

普通高中教育，是与九年义务教育相衔接的高一层次的基础教育。高中教材的编写，旨在进一步提高学生的思想道德品质、文化科学知识、审美情趣和身体心理素质，培养学生的创新精神、实践能力、终身学习的能力和适应社会生活的能力，促进学生的全面发展，为高一级学校和社会输送素质良好的合格的毕业生。

《数学》包括三册，其中第一册、第二册是必修课本，分别在高一、高二学习，每周4课时；第三册是选修课本，在高三学习，它又分为选修Ⅰ和选修Ⅱ两种，每周分别为2课时和4课时。

这套书的第一册又分为上、下两个分册，分别供高一上、下两个学期使用。本书是《数学》第一册（下），内容包括三角函数和平面向量两章。

全套书在体例上有下列特点：

1. 每章均配有章头图和引言，作为全章内容的导入，使学生初步了解学习这一章的必要性。

2. 书中习题共分三类：练习、习题、复习参考题。

练习　以复习相应小节的教学内容为主，供课堂练习用。

习题　每小节后一般配有习题，供课内、外作业选用，少数标有 * 号的题在难度上略有提高，仅供学有余力的学生选用。

复习参考题　每章最后配有复习参考题，分 A、B 两组，A 组题是属于基本要求范围的，供复习全章使用；B 组题带有一定的灵活性，难度上略有提高，仅供学有余力的学生选用。

3. 每章在内容后面均安排有小结与复习，包括内容提要、学习要求和需要注意的问题、参考例题三部分，供复习全章时参考。

4. 每章附有一至两篇不作教学要求的阅读材料，供学生课外阅读，借以扩大知识面、激发学习兴趣、培养应用数学的意识。

本套书由人民教育出版社中学数学室编写，其中《数学》第一册（下）原试验本由饶汉昌、方明一主持编写，参加编写的有蔡上鹤、康合太等，责任编辑为薛彬，审稿为饶汉昌。

《数学》第一册（下）原试验本在编写过程中蒙孔令颐、吴之季、烟学敏、蒋佩锦、戴佳珉等同志提出宝贵意见，在此表示衷心感谢。

参加本次修订的有蔡上鹤、康合太等，责任编辑为薛彬、张劲松，审稿为饶汉昌。

本册教材经教育部中小学教材审定委员会审读，尚待审查。

人民教育出版社中学数学室

2000年11月

CONTENTS

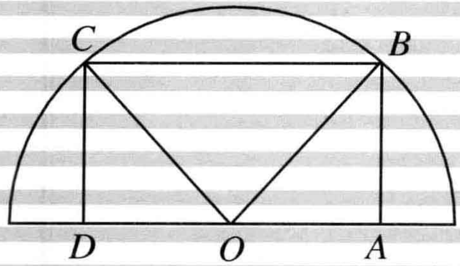
Chapter 4 Trigonometric Functions

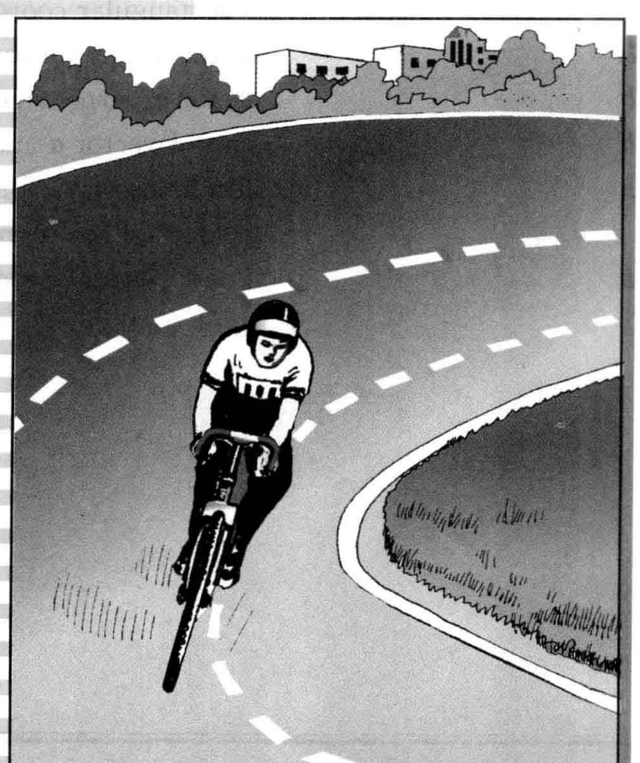
Chapter 5 Planar Vectors

Some Common Symbols in This Book

$\sin x$	sine of x
$\cos x$	cosine of x
$\tan x$	tangent of x
$\cot x$	cotangent of x
$\sin^2 x$	square of $\sin x$
$\arcsin x$	arcsine of x
$\arccos x$	arccosine of x
$\arctan x$	arctangent of x
$\boldsymbol{a}$	vector $\boldsymbol{a}$
$\overrightarrow{AB}$	vector $\overrightarrow{AB}$
$\lvert \boldsymbol{a} \rvert$	modulus of vector $\boldsymbol{a}$ (or length)
$\lvert \overrightarrow{AB} \rvert$	norm of vector $\lvert \overrightarrow{AB} \rvert$ (or length)
$\boldsymbol{0}$	zero vector
$\boldsymbol{e}$	unit vector
$\boldsymbol{i}$, $\boldsymbol{j}$	unit vectors of x-axis, y-axis on the planar rectangular coordinate system
$\boldsymbol{a} /\!/ \boldsymbol{b}$	vector $\boldsymbol{a}$ parallel to vector $\boldsymbol{b}$ (collinear)
$\boldsymbol{a} \perp \boldsymbol{b}$	vector $\boldsymbol{a}$ perpendicular to vector $\boldsymbol{b}$
$\boldsymbol{a}+\boldsymbol{b}$	sum of vector $\boldsymbol{a}$ and vector $\boldsymbol{b}$
$\boldsymbol{a}-\boldsymbol{b}$	difference of vector $\boldsymbol{a}$ and vector $\boldsymbol{b}$
$\lambda \boldsymbol{a}$	product of real number λ and vector $\boldsymbol{a}$
$\boldsymbol{a} \cdot \boldsymbol{b}$	inner (scalar) product of vector $\boldsymbol{a}$ and vector $\boldsymbol{b}$
Rt$\triangle$	right triangle

Chapter 4 Trigonometric Functions





As shown in the figure, a vacant lot which is a semicircle centered at O. Divide an inscribed rectangle $ABCD$ in it as a green field such that AD on the diameter and other two points B and C on half of the circumference. The radius of the semicircle is a. Now how to make A and D symmetrical about O so that the area of rectangle $ABCD$ is maximum?

Alternative 1: Suppose that $OA=t$ and the area of the rectangle is S. By Shang Gao Theorem, $S=2t\sqrt{a^2-t^2}$. By squaring the two sides, $S^2=4t^2(a^2-t^2)$. This function is complicated. However, let $S^2=y$, $t^2=x$, then y becomes a quadratic function of x, that is $y=-4x^2+4a^2x$. According to known knowledge, y attains its maximum a^4 at $x=\frac{a^2}{2}$.

Since x, y, t and S are all positive numbers, it is easy to see that S is at its maximum when $t=\frac{\sqrt{2}}{2}a$. Thus, points A and D can be found at $\frac{\sqrt{2}}{2}a$ from the left and the right, respectively, of point O.

Alternative 2: Look at the angle. Suppose $\angle AOB=\theta$, then $AB=a\sin\theta$, $OA=a\cos\theta$, and hence

$$S=a\sin\theta\cdot 2a\cos\theta=a^2\cdot 2\sin\theta\cos\theta.$$

This is a function of θ. This chapter will tell you what θ makes and when S is at its greatest. Alternative 2 is much simpler than Alternative 1.

In this chapter, we shall systematically observe trigonometric functions of arbitrary angle via sets and functions, get basic transformations of trigonometric relations and some transforming methods and then, on this basis, understand images and properties of trigonometric functions. In addition, we shall also know the methods of calculating the values of a trigonometric function. All these will be very important in future study and have wide application in science and technology.

I Trigonometric Functions of an Arbitrary Angle

4.1 Concept of Angle and Its Extension

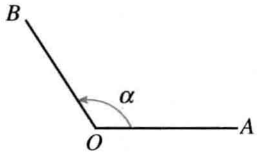


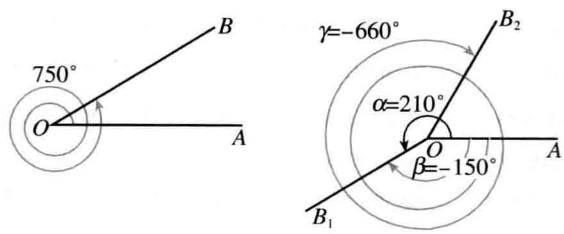
Fig. 4-1

As known, an angle can be seen as the figure formed by rotating a ray about its end in one direction or another. In Fig. 4-1, the angle α is formed by rotating the ray OA about its end O in an anti-clockwise direction to OB. Point O is the vertex of the angle. The rays OA and OB form, respectively, the initial edge (or side) and the terminal edge (or side).

An angle is defined as **positive** if it is formed in an anti-clockwise direction; **negative** otherwise. In Fig. 4-1, angle α is positive. An angle formed by the hour hand or minute hand, moving on a clock, is always negative. For short, 'angle α', or '$\angle\alpha$', is simply written as 'α' to avoid confusion.

Angles we had studied in the past are only in the range of $0°\sim 360°$. However, other angles are often met in everyday life. For example, in gymnastics, 'turn round 720°' (i. e., 'turn round 2 circular courses'), 'turn round 1080°' (i. e., 'turn round 3 circular courses') are possible. In other words, angles are not limited in the range of $0°\sim 360°$. For another example, in Fig. 4-2 (1), the angle is positive, it is 750°; in Fig. 4-2 (2), the positive angle $\alpha = 210°$, the negative angles $\beta = -150°$, $\gamma = -660°$.

If a ray is without rotation, the angle formed in this way is called **zero angle**. In another statement, zero angle is with its terminal edge the same as its initial edge. If α is the zero angle, then $\alpha = 0°$.



(1) (2)

Fig. 4-2

As such an extension, angles can be positive, negative and zero.

In what follows, angles are often discussed in the rectangular coordinate system. The vertex of an angle is taken to be the origin. The initial edge of an angle is the same as the non-negative x-axis. An angle whose terminal edge (excluding the origin) is in the i-th quadrant is called an i-th quadrant angle ($i=1, 2, 3, 4$). For example, in Fig. 4-3 (1), angles $30°$, $390°$, $-330°$ are all 1st quadrant angles; in Fig. 4-3 (2), angles $300°$, $-60°$ are all 4th quadrant angles; angle $585°$ is a 3rd quadrant angle. If the terminal edge of an angle is on a coordinate axis, then it does not belong to any quadrant.

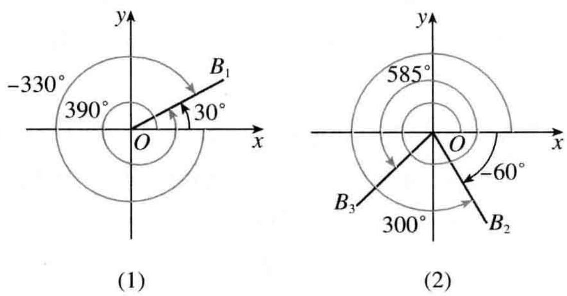


Fig. 4-3

In Fig. 4-3 (1), the terminal edges of angles $390°$, $-330°$ are the same of $30°$ and both of them can be represented by an angle in the range from $0°$ to $360°$[1] plus k times circular course, or k perigons ($k \in \mathbf{Z}$), i. e.,

$$390° = 30° + 360° \text{ (here } k=1\text{)},$$

$$-330° = 30° - 360° \text{ (here } k=-1\text{)}.$$

[1] In this book, we define that $0° \sim 360°$ includes $0°$, but does not include $360°$.

Suppose $S = \{\beta \mid \beta = 30° + k \cdot 360°, k \in \mathbf{Z}\}$, then angles $390°$, $-330°$ are all an element of S. Angle $30°$ is an element of S as well (here $k=0$). It is easy to see that all the angles whose terminal edges are the same as $30°$(included) are elements of S. Conversely, any element of S has its terminal edge as the same as that of $30°$. In general, we have:

All the angles whose terminal edges are the same as the terminal of an angle α (included) form the set

$$S = \{\beta \mid \beta = \alpha + k \cdot 360°, k \in \mathbf{Z}\},$$

i. e., any angle whose terminal edge is the same of α can always be represented by α plus integral number of perigons.

Example 1 Find the angles in the range $0°$ to $360°$ whose terminal edges are the same as that of one of the following angles and justify the quadrants they are in.

(1) $-120°$; (2) $640°$; (3) $-950°12'$.

Solution (1) $-120°=240°-360°$.

Then the angle with the same terminal edge as angle $-120°$ is angle $240°$. It is a 3rd quadrant angle.

(2) $640°=280°+360°$.

Then the angle with the same terminal edge as angle $640°$ is angle $280°$. It is a 4th quadrant angle.

(3) $-950°12'=129°48'-3\times 360°$.

Then the angle with the same terminal edge as angle $-950°12'$ is angle $129°48'$. It is a 2nd quadrant angle.

Example 2 Provide the set of all the angles whose terminal edges are on y-axis (represented by an angle in the range from $0°$ to $360°$).

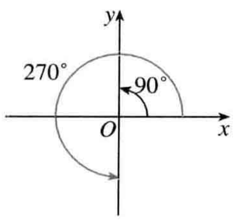


Fig. 4-4

Solution There are two angles from $0°$ to $360°$ with their terminal edges on y-axis, i. e., angle $90°$ and angle $270°$ (Fig. 4-4). Then, the set of all the angles whose terminals are the same as angle 90°'s is

$$\begin{aligned} S_1 &= \{\beta \mid \beta = 90° + k \cdot 360°, k \in \mathbf{Z}\} \\ &= \{\beta \mid \beta = 90° + 2k \cdot 180°, k \in \mathbf{Z}\}, \end{aligned}$$

and the set of all the angles whose terminals are the same as angle 270°'s is

$$\begin{aligned} S_2 &= \{\beta \mid \beta = 270° + k \cdot 360°, k \in \mathbf{Z}\} \\ &= \{\beta \mid \beta = 90° + 180° + 2k \cdot 180°, k \in \mathbf{Z}\} \\ &= \{\beta \mid \beta = 90° + (2k+1)180°, k \in \mathbf{Z}\}, \end{aligned}$$

and hence, the set of angles whose terminal edges are on y-axis is

$$\begin{aligned} S &= S_1 \cup S_2 \\ &= \{\beta \mid \beta = 90° + 2k \cdot 180°, k \in \mathbf{Z}\} \\ &\quad \cup \{\beta \mid \beta = 90° + (2k+1)180°, k \in \mathbf{Z}\} \\ &= \{\beta \mid \beta = 90° + \text{even times } 180°\} \\ &\quad \cup \{\beta \mid \beta = 90° + \text{odd times } 180°\} \\ &= \{\beta \mid \beta = 90° + \text{integer times } 180°\} \\ &= \{\beta \mid \beta = 90° + n \cdot 180°, n \in \mathbf{Z}\}. \end{aligned}$$

Example 3 Work out the sets of angles which have their terminal edges the same as one of the following angles, and select all the elements β such that $-360° \leqslant \beta < 720°$:

(1) $60°$; (2) $-21°$; (3) $363°14'$.

Solution (1) $S=\{\beta \mid \beta=60°+k \cdot 360°, k\in \mathbf{Z}\}$.

The elements of S satisfying $-360° \leqslant \beta < 720°$ are

$$60°-1\times360°=-300°,$$
$$60°+0\times360°=60°,$$
$$60°+1\times360°=420°.$$

(2) $-21°$ is not an angle from $0°$ to $360°$, but the set of angles with the same terminal as $-21°$ can still be constructed in the above way, i. e. ,

$$S=\{\beta \mid \beta=-21°+k\cdot360°,\ k\in\mathbf{Z}\}.$$

The elements of S satisfying $-360°\leqslant\beta<720°$ are

$$-21°+0\times360°=-21°,$$
$$-21°+1\times360°=339°,$$
$$-21°+2\times360°=699°.$$

(3) $S=\{\beta \mid \beta=363°14'+k\cdot360°,\ k\in\mathbf{Z}\}.$

The elements of S satisfying $-360°\leqslant\beta<720°$ are

$$363°14'-2\times360°=-356°46',$$
$$363°14'-1\times360°=3°14',$$
$$363°14'+0\times360°=363°14'.$$

Training Exercises

1. (Oral) What quadrant is an acute angle in? Is a first quadrant angle always acute? Answer the two questions again for respectively a right angle and an obtuse angle.
2. (Oral) Today is Wednesday. What day is the day after $7k$ ($k\in\mathbf{Z}$) days? What day is the day before $7k$ ($k\in\mathbf{Z}$) days? What day is the day after 100 days?
3. Given an angle whose vertex is the origin of the coordinate system and whose initial edge is the non-negative semi-axis of x-axis, construct the following angles and show what quadrants they are in:
   (1) $420°$; (2) $-75°$; (3) $855°$; (4) $-510°$.
4. Find the angles which have the same terminal edge of each of the following angles in the range from $0°$ to $360°$ and show what quadrants they are in:
   (1) $-54°18'$; (2) $395°8'$; (3) $-1\ 190°30'$; (4) $1\ 563°$.
5. Work out the sets of angles which have the same terminal edge of each of the following angles and select all those β satisfying $-720°\leqslant\beta<360°$:
   (1) $45°$; (2) $-30°$; (3) $1\ 303°18'$; (4) $-225°$.

Exercises 4.1

1. Find the angles which have the same terminal edge of each of the following angles in the range from $0°$ to $360°$ and show what quadrants they are in:
   (1) $-265°$; (2) $1\,185°14'$; (3) $-1\,000°$; (4) $-843°10'$;
   (5) $-15°$; (6) $3\,900°$; (7) $560°24'$; (8) $2\,903°15'$.
2. Work out the set of all the angles whose terminal edges are on the x-axis (expressed by an angle from $0°$ to $360°$).
3. Work out the sets of angles which have the same terminal edge of each of the following angles and select all those γ satisfying $-360° \leqslant \gamma < 360°$:
   (1) $60°$; (2) $-75°$; (3) $-824°30'$; (4) $475°$;
   (5) $90°$; (6) $270°$; (7) $180°$; (8) $0°$.
4. Work out the sets of first, second, third and fourth quadrant angles separately.
5. Choose the correct answer:
   (1) If α is an acute angle, then 2α is a ( )
   (A) first quadrant angle.
   (B) second quadrant angle.
   (C) positive angle less than $180°$.
   (D) positive angle not greater than right angle.
   (2) If α is an obtuse angle, then $\frac{\alpha}{2}$ is a ( )
   (A) first quadrant angle.
   (B) second quadrant angle.
   (C) first and second quadrant angle.
   (D) positive angle not less than right angle.

4.2 Radian System

We had learnt to the measure angles in junior high school geometry that $1°$ (1 degree) of an angle was defined to be $\frac{1}{360}$ of the round angle (a perigon). Such a unit system for measuring angles by taking degree as a unit is called a **degree system.** In what follows, we will discuss another unit system—— radian system for measuring angles that we often encounter in mathematics and other sciences. Its unit symbol is rad, short for as radian.

The central angle opposite to an arc of length equal to the radius is called an **angle of 1 radian**, i. e. , such a central angle equals 1 rad in the radian system. In Fig. 4-5, the length of arc $\overset{\frown}{AB}$ equals radius r and hence the central angle $\angle AOB$ opposite to $\overset{\frown}{AB}$ is an angle of

1 radian. In Fig. 4-6, since the length of arc $\overset{\frown}{AC}$ opposite to central angle $\angle AOC$ is $l=2r$, the radian number of $\angle AOC$ is

$$\frac{l}{r}=\frac{2r}{r}=2.$$

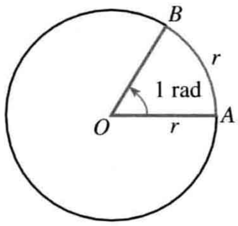


Fig. 4-5

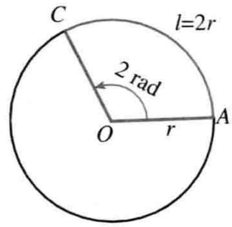


Fig. 4-6

When the central angle is a perigon, the arc opposite to it has its length $l=2\pi r$, hence the radian number of a perigon is

$$\frac{l}{r}=\frac{2\pi r}{r}=2\pi.$$

On this basis, for any angle from $0°$ to $360°$, its radian number $x=\frac{l}{r}$ must satisfy the inequality $0\leqslant x<2\pi$. Just like angles, the concept of arcs is the same, the radian number of a positive angle is always positive.

If angle α is negative, then its radian number is negative. The radian number of angle 0 is 0. For example, a negative central angle with its opposite arc length $l=4\pi r$ has its radian number

$$-\frac{l}{r}=-\frac{4\pi r}{r}=-4\pi.$$

In general, we have that **the radian number of a positive angle is positive, the radian number of a negative angle is negative, the radian number of angle 0 is 0; the absolute value of the radian number of angle α is**

$$|\alpha|=\frac{l}{r},$$

where l is the arc length opposite to α as a central angle, r is the radius of the circle.

Such a unit system with the radian as the unit of measuring angles is called the **radian system.**

In degree and radian systems for measuring angle 0, the quantity is the same (always is 0) while the units are different. However, for angles other than 0, both quantity and units are different. In what follows, discuss the relationship between the degree and radian systems.