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Computational Geometry

Algorithms and Applications

Third Edition

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Preface

Computational geometry emerged from the field of algorithms design and analysis in the late 1970s. It has grown into a recognized discipline with its own journals, conferences, and a large community of active researchers. The success of the field as a research discipline can on the one hand be explained from the beauty of the problems studied and the solutions obtained, and, on the other hand, by the many application domains—computer graphics, geographic information systems (GIS), robotics, and others—in which geometric algorithms play a fundamental role.

For many geometric problems the early algorithmic solutions were either slow or difficult to understand and implement. In recent years a number of new algorithmic techniques have been developed that improved and simplified many of the previous approaches. In this textbook we have tried to make these modern algorithmic solutions accessible to a large audience. The book has been written as a textbook for a course in computational geometry, but it can also be used for self-study.

Structure of the book. Each of the sixteen chapters (except the introductory chapter) starts with a problem arising in one of the application domains. This problem is then transformed into a purely geometric one, which is solved using techniques from computational geometry. The geometric problem and the concepts and techniques needed to solve it are the real topic of each chapter. The choice of the applications was guided by the topics in computational geometry we wanted to cover; they are not meant to provide a good coverage of the application domains. The purpose of the applications is to motivate the reader; the goal of the chapters is not to provide ready-to-use solutions for them. Having said this, we believe that knowledge of computational geometry is important to solve geometric problems in application areas efficiently. We hope that our book will not only raise the interest of people from the algorithms community, but also from people in the application areas.

For most geometric problems treated we give just one solution, even when a number of different solutions exist. In general we have chosen the solution that is easiest to understand and implement. This is not necessarily the most efficient solution. We also took care that the book contains a good mixture of techniques like divide-and-conquer, plane sweep, and randomized algorithms. We decided not to treat all sorts of variations to the problems; we felt it is more important to introduce all main topics in computational geometry than to give more detailed information about a smaller number of topics.

Several chapters contain one or more sections marked with a star. They contain improvements of the solution, extensions, or explain the relation between various problems. They are not essential for understanding the remainder of the book.

Every chapter concludes with a section that is entitled *Notes and Comments*. These sections indicate where the results described in the chapter originated, mention other solutions, generalizations, and improvements, and provide references. They can be skipped, but do contain useful material for those who want to know more about the topic of the chapter.

At the end of each chapter a number of exercises is provided. These range from easy tests to check whether the reader understands the material to more elaborate questions that extend the material covered. Difficult exercises and exercises about starred sections are indicated with a star.

A course outline. Even though the chapters in this book are largely independent, they should preferably not be treated in an arbitrary order. For instance, Chapter 2 introduces plane sweep algorithms, and it is best to read this chapter before any of the other chapters that use this technique. Similarly, Chapter 4 should be read before any other chapter that uses randomized algorithms.

For a first course on computational geometry, we advise treating Chapters 1–10 in the given order. They cover the concepts and techniques that, according to us, should be present in any course on computational geometry. When more material can be covered, a selection can be made from the remaining chapters.

Prerequisites. The book can be used as a textbook for a high-level undergraduate course or a low-level graduate course, depending on the rest of the curriculum. Readers are assumed to have a basic knowledge of the design and analysis of algorithms and data structures: they should be familiar with big-Oh notations and simple algorithmic techniques like sorting, binary search, and balanced search trees. No knowledge of the application domains is required, and hardly any knowledge of geometry. The analysis of the randomized algorithms uses some very elementary probability theory.

Implementations. The algorithms in this book are presented in a pseudo-code that, although rather high-level, is detailed enough to make it relatively easy to implement them. In particular we have tried to indicate how to handle degenerate cases, which are often a source of frustration when it comes to implementing.

We believe that it is very useful to implement one or more of the algorithms; it will give a feeling for the complexity of the algorithms in practice. Each chapter can be seen as a programming project. Depending on the amount of time available one can either just implement the plain geometric algorithms, or implement the application as well.

To implement a geometric algorithm a number of basic data types—points, lines, polygons, and so on—and basic routines that operate on them are needed. Implementing these basic routines in a robust manner is not easy, and takes a lot

of time. Although it is good to do this at least once, it is useful to have a software library available that contains the basic data types and routines. Pointers to such libraries can be found on our Web site.

Web site. This book is accompanied by a Web site, which contains a list of errata collected for each edition of the book, all figures and the pseudo code for all algorithms, as well as some other resources. The address is

`http://www.cs.uu.nl/geobook/`

You can also use the address given on our Web site to send us errors you have found, or any other comments you have about the book.

About the third edition. This third edition contains two major additions: In Chapter 7, on Voronoi diagrams, we now also discuss Voronoi diagrams of line segments and farthest-point Voronoi diagrams. In Chapter 12, we have included an extra section on binary space partition trees for low-density scenes, as an introduction to realistic input models. In addition, a large number of small and some larger errors have been corrected (see the list of errata for the second edition on the Web site). We have also updated the notes and comments of every chapter to include references to recent results and recent literature. We have tried not to change the numbering of sections and exercises, so that it should be possible for students in a course to still use the second edition.

Acknowledgments. Writing a textbook is a long process, even with four authors. Many people contributed to the original first edition by providing useful advice on what to put in the book and what not, by reading chapters and suggesting changes, and by finding and correcting errors. Many more provided feedback and found errors in the first two editions. We would like to thank all of them, in particular Pankaj Agarwal, Helmut Alt, Marshall Bern, Jit Bose, Hazel Everett, Gerald Farin, Steve Fortune, Geert-Jan Giezeman, Mordecai Golin, Dan Halperin, Richard Karp, Matthew Katz, Klara Kedem, Nelson Max, Joseph S. B. Mitchell, René van Oostrum, Günter Rote, Henry Shapiro, Sven Skyum, Jack Snoeyink, Gert Vegter, Peter Widmayer, Chee Yap, and Günther Ziegler. We also would like to thank Springer-Verlag for their advice and support during the creation of this book, its new editions, and the translations into other languages (at the time of writing, Japanese, Chinese, and Polish).

Finally we would like to acknowledge the support of the Netherlands' Organization for Scientific Research (N.W.O.) and the Korea Research Foundation (KRF).

January 2008

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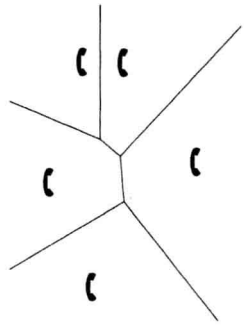
Contents

1 Computational Geometry

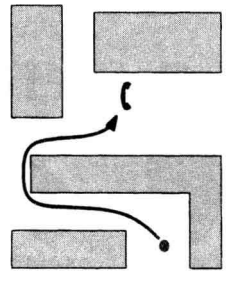
Introduction

Imagine you are walking on the campus of a university and suddenly you realize you have to make an urgent phone call. There are many public phones on campus and of course you want to go to the nearest one. But which one is the nearest? It would be helpful to have a map on which you could look up the nearest public phone, wherever on campus you are. The map should show a subdivision of the campus into regions, and for each region indicate the nearest public phone. What would these regions look like? And how could we compute them?



Even though this is not such a terribly important issue, it describes the basics of a fundamental geometric concept, which plays a role in many applications. The subdivision of the campus is a so-called *Voronoi diagram*, and it will be studied in Chapter 7 in this book. It can be used to model trading areas of different cities, to guide robots, and even to describe and simulate the growth of crystals. Computing a geometric structure like a Voronoi diagram requires geometric algorithms. Such algorithms form the topic of this book.

A second example. Assume you located the closest public phone. With a campus map in hand you will probably have little problem in getting to the phone along a reasonably short path, without hitting walls and other objects. But programming a robot to perform the same task is a lot more difficult. Again, the heart of the problem is geometric: given a collection of geometric obstacles, we have to find a short connection between two points, avoiding collisions with the obstacles. Solving this so-called *motion planning* problem is of crucial importance in robotics. Chapters 13 and 15 deal with geometric algorithms required for motion planning.



A third example. Assume you don't have one map but two: one with a description of the various buildings, including the public phones, and one indicating the roads on the campus. To plan a motion to the public phone we have to *overlay* these maps, that is, we have to combine the information in the two maps. Overlaying maps is one of the basic operations of geographic information systems. It involves locating the position of objects from one map in the other, computing the intersection of various features, and so on. Chapter 2 deals with this problem.

These are just three examples of geometric problems requiring carefully designed geometric algorithms for their solution. In the 1970s the field of computational geometry emerged, dealing with such geometric problems. It can be defined as the systematic study of algorithms and data structures for geometric objects, with a focus on exact algorithms that are asymptotically fast. Many researchers were attracted by the challenges posed by the geometric problems. The road from problem formulation to efficient and elegant solutions has often been long, with many difficult and sub-optimal intermediate results. Today there is a rich collection of geometric algorithms that are efficient, and relatively easy to understand and implement.

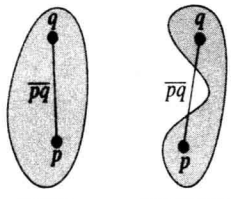
This book describes the most important notions, techniques, algorithms, and data structures from computational geometry in a way that we hope will be attractive to readers who are interested in applying results from computational geometry. Each chapter is motivated with a real computational problem that requires geometric algorithms for its solution. To show the wide applicability of computational geometry, the problems were taken from various application areas: robotics, computer graphics, CAD/CAM, and geographic information systems.

You should not expect ready-to-implement software solutions for major problems in the application areas. Every chapter deals with a single concept in computational geometry; the applications only serve to introduce and motivate the concepts. They also illustrate the process of modeling an engineering problem and finding an exact solution.

1.1 An Example: Convex Hulls

Good solutions to algorithmic problems of a geometric nature are mostly based on two ingredients. One is a thorough understanding of the geometric properties of the problem, the other is a proper application of algorithmic techniques and data structures. If you don't understand the geometry of the problem, all the algorithms of the world won't help you to solve it efficiently. On the other hand, even if you perfectly understand the geometry of the problem, it is hard to solve it effectively if you don't know the right algorithmic techniques. This book will give you a thorough understanding of the most important geometric concepts and algorithmic techniques.

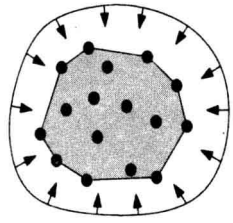
To illustrate the issues that arise in developing a geometric algorithm, this section deals with one of the first problems that was studied in computational geometry: the computation of planar convex hulls. We'll skip the motivation for this problem here; if you are interested you can read the introduction to Chapter 11, where we study convex hulls in 3-dimensional space.



convex not convex

A subset S of the plane is called *convex* if and only if for any pair of points $p,q \in S$ the line segment $\overline{pq}$ is completely contained in S. The *convex hull* $\mathcal{CH}(S)$ of a set S is the smallest convex set that contains S. To be more precise, it is the intersection of all convex sets that contain S.

We will study the problem of computing the convex hull of a finite set P of n points in the plane. We can visualize what the convex hull looks like by a thought experiment. Imagine that the points are nails sticking out of the plane, take an elastic rubber band, hold it around the nails, and let it go. It will snap around the nails, minimizing its length. The area enclosed by the rubber band is the convex hull of P. This leads to an alternative definition of the convex hull of a finite set P of points in the plane: it is the unique convex polygon whose vertices are points from P and that contains all points of P. Of course we should prove rigorously that this is well defined—that is, that the polygon is unique—and that the definition is equivalent to the one given earlier, but let's skip that in this introductory chapter.



How do we compute the convex hull? Before we can answer this question we must ask another question: what does it mean to compute the convex hull? As we have seen, the convex hull of P is a convex polygon. A natural way to represent a polygon is by listing its vertices in clockwise order, starting with an arbitrary one. So the problem we want to solve is this: given a set $P = \{p_1, p_2, \ldots, p_n\}$ of points in the plane, compute a list that contains those points from P that are the vertices of $\mathcal{CH}(P)$, listed in clockwise order.

input = set of points:
$p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9$

output = representation of the convex hull:
p_4, p_5, p_8, p_2, p_9

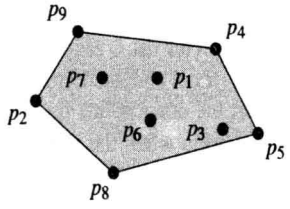
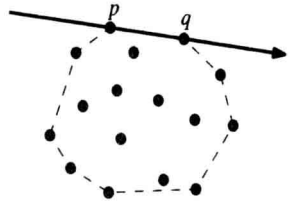


Figure 1.1
Computing a convex hull

The first definition of convex hulls is of little help when we want to design an algorithm to compute the convex hull. It talks about the intersection of all convex sets containing P, of which there are infinitely many. The observation that $\mathcal{CH}(P)$ is a convex polygon is more useful. Let's see what the edges of $\mathcal{CH}(P)$ are. Both endpoints p and q of such an edge are points of P, and if we direct the line through p and q such that $\mathcal{CH}(P)$ lies to the right, then all the points of P must lie to the right of this line. The reverse is also true: if all points of $P \setminus \{p, q\}$ lie to the right of the directed line through p and q, then $\overline{pq}$ is an edge of $\mathcal{CH}(P)$.



Now that we understand the geometry of the problem a little bit better we can develop an algorithm. We will describe it in a style of pseudocode we will use throughout this book.

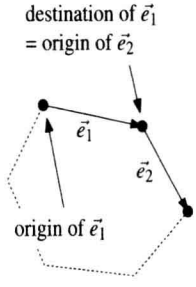
Algorithm SLOWCONVEXHULL(P)
Input. A set P of points in the plane.
Output. A list $\mathcal{L}$ containing the vertices of $\mathcal{CH}(P)$ in clockwise order.
1. $E \leftarrow \emptyset$.
2. **for** all ordered pairs $(p, q) \in P \times P$ with p not equal to q
3. **do** *valid* $\leftarrow$ **true**

4. **for** all points $r \in P$ not equal to p or q
5. **do if** r lies to the left of the directed line from p to q
6. **then** *valid* $\leftarrow$ **false**.
7. **if** *valid* **then** Add the directed edge $\overrightarrow{pq}$ to E.
8. From the set E of edges construct a list $\mathcal{L}$ of vertices of $\mathcal{CH}(P)$, sorted in clockwise order.

Two steps in the algorithm are perhaps not entirely clear.

The first one is line 5: how do we test whether a point lies to the left or to the right of a directed line? This is one of the primitive operations required in most geometric algorithms. Throughout this book we assume that such operations are available. It is clear that they can be performed in constant time so the actual implementation will not affect the asymptotic running time in order of magnitude. This is not to say that such primitive operations are unimportant or trivial. They are not easy to implement correctly and their implementation will affect the actual running time of the algorithm. Fortunately, software libraries containing such primitive operations are nowadays available. We conclude that we don't have to worry about the test in line 5; we may assume that we have a function available performing the test for us in constant time.

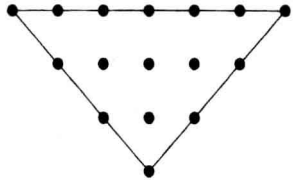
The other step of the algorithm that requires some explanation is the last one. In the loop of lines 2–7 we determine the set E of convex hull edges. From E we can construct the list $\mathcal{L}$ as follows. The edges in E are directed, so we can speak about the origin and the destination of an edge. Because the edges are directed such that the other points lie to their right, the destination of an edge comes after its origin when the vertices are listed in clockwise order. Now remove an arbitrary edge $\vec{e}_1$ from E. Put the origin of $\vec{e}_1$ as the first point into $\mathcal{L}$, and the destination as the second point. Find the edge $\vec{e}_2$ in E whose origin is the destination of $\vec{e}_1$, remove it from E, and append its destination to $\mathcal{L}$. Next, find the edge $\vec{e}_3$ whose origin is the destination of $\vec{e}_2$, remove it from E, and append its destination to $\mathcal{L}$. We continue in this manner until there is only one edge left in E. Then we are done; the destination of the remaining edge is necessarily the origin of $\vec{e}_1$, which is already the first point in $\mathcal{L}$. A simple implementation of this procedure takes $O(n^2)$ time. This can easily be improved to $O(n \log n)$, but the time required for the rest of the algorithm dominates the total running time anyway.



Analyzing the time complexity of SLOWCONVEXHULL is easy. We check $n^2 - n$ pairs of points. For each pair we look at the $n-2$ other points to see whether they all lie on the right side. This will take $O(n^3)$ time in total. The final step takes $O(n^2)$ time, so the total running time is $O(n^3)$. An algorithm with a cubic running time is too slow to be of practical use for anything but the smallest input sets. The problem is that we did not use any clever algorithmic design techniques; we just translated the geometric insight into an algorithm in a brute-force manner. But before we try to do better, it is useful to make several observations about this algorithm.

We have been a bit careless when deriving the criterion of when a pair p,q defines an edge of $\mathcal{CH}(P)$. A point r does not always lie to the right or to the

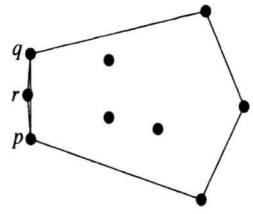
left of the line through p and q, it can also happen that it lies *on* this line. What should we do then? This is what we call a *degenerate case*, or a *degeneracy* for short. We prefer to ignore such situations when we first think about a problem, so that we don't get confused when we try to figure out the geometric properties of a problem. However, these situations do arise in practice. For instance, if we create the points by clicking on a screen with a mouse, all points will have small integer coordinates, and it is quite likely that we will create three points on a line.

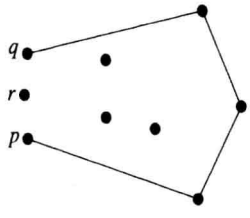


To make the algorithm correct in the presence of degeneracies we must reformulate the criterion above as follows: a directed edge $\overrightarrow{pq}$ is an edge of $\mathcal{CH}(P)$ if and only if all other points $r \in P$ lie either strictly to the right of the directed line through p and q, or they lie on the open line segment $\overline{pq}$. (We assume that there are no coinciding points in P.) So we have to replace line 5 of the algorithm by this more complicated test.

We have been ignoring another important issue that can influence the correctness of the result of our algorithm. We implicitly assumed that we can somehow test exactly whether a point lies to the right or to the left of a given line. This is not necessarily true: if the points are given in floating point coordinates and the computations are done using floating point arithmetic, then there will be rounding errors that may distort the outcome of tests.

Imagine that there are three points p, q, and r, that are nearly collinear, and that all other points lie far to the right of them. Our algorithm tests the pairs (p,q), (r,q), and (p,r). Since these points are nearly collinear, it is possible that the rounding errors lead us to decide that r lies to the right of the line from p to q, that p lies to the right of the line from r to q, and that q lies to the right of the line from p to r. Of course this is geometrically impossible—but the floating point arithmetic doesn't know that! In this case the algorithm will accept all three edges. Even worse, all three tests could give the opposite answer, in which case the algorithm rejects all three edges, leading to a gap in the boundary of the convex hull. And this leads to a serious problem when we try to construct the sorted list of convex hull vertices in the last step of our algorithm. This step assumes that there is exactly one edge starting in every convex hull vertex, and exactly one edge ending there. Due to the rounding errors there can suddenly be two, or no, edges starting in vertex p. This can cause the program implementing our simple algorithm to crash, since the last step has not been designed to deal with such inconsistent data.





Although we have proven the algorithm to be correct and to handle all special cases, it is not *robust*: small errors in the computations can make it fail in completely unexpected ways. The problem is that we have proven the correctness assuming that we can compute exactly with real numbers.

We have designed our first geometric algorithm. It computes the convex hull of a set of points in the plane. However, it is quite slow—its running time is $O(n^3)$—, it deals with degenerate cases in an awkward way, and it is not robust. We should try to do better.