英语版

九年义务教育三年制初级中学教科书

GEOMETRY

第三册

课程教材研究所
双语课程教材研究开发中心
组译

几何

人民教育出版社
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英语版初级中学教科书

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说　明

随着改革开放的不断扩大，中国在经济和教育、科学、文化等诸多方面与各国间的交往日益增强，中国人学习英语的热情也日趋高涨。在当今社会，是否熟练掌握英语，已成为衡量一个人的知识结构甚至综合素质的一个重要方面。在这样的形势下，多角度、多渠道提高人们的英语水平，特别是提高基础教育阶段在校学生的英语水平，已经成为社会的迫切需要。

为了适应这种新的形势和需要，从2001年起，作为教育部直属单位的课程教材研究所着手研究开发英语版普通高中教科书（包括数学、物理、化学、生物、历史、地理六门必修课程），已由人民教育出版社出版。随后，又继续开发这套英语版初级中学教科书，将包括初中三个年段的代数、几何、物理、化学、生物、历史、地理和信息技术等。

这套英语版初级中学教科书，根据经全国中小学教材审定委员会2001年审查通过、人民教育出版社出版的《九年义务教育三年制初级中学教科书》编译而成，主要供实行双语教学的学校或班级使用，也可以作为中学生的课外读物，其他有兴趣的读者也可以作为参考书使用，使学科知识的掌握与英语能力的提高形成一种双赢的局面。

为了使这套英语版教科书具有较高的编译质量，课程教材研究所双语课程教材研究开发中心依托所内各学科教材研究开发中心，在国内外特聘学科专家和英语专家联袂翻译，且全部译稿均由中外知名专家共同审校。

我们的宗旨是：以前瞻意识迎接时代挑战，以国际水平奉献中华学子。

人民教育出版社英语版初级中学教科书，愿与广大师生和家长结伴同行，共同打造新世纪的一流英才。

热诚欢迎广大师生和读者将使用中的意见和建议反馈给我们，使这套教材日臻完善。

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汉语版初级中学《几何》

说　明

一、《九年义务教育三年制初级中学教科书·几何》是根据教育部2000年颁发的《九年义务教育全日制初级中学数学教学大纲（试用修订版）》，在原《九年义务教育三年制初级中学教科书·几何》基础上修订的，并经全国中小学教材审定委员会2001年审查通过。这次修订，旨在更加有利于贯彻党和国家的教育方针，更加有利于对青少年进行素质教育，更加有利于初中学生的全面发展，培养学生的创新精神和实践能力。

二、初中几何是初中数学的重要组成部分。通过初中几何的教学，要使学生学会适应日常生活、参加生产和进一步学习所必需的几何基础知识与基本技能，进一步培养运算能力、思维能力和空间观念，能够运用所学知识解决简单的实际问题，培养学生的数学创新意识、良好个性品质以及初步的辩证唯物主义观点。

三、这套《九年义务教育三年制初级中学教科书·几何》分第一、二、三册，共三册。本书是《几何》第三册，供三年制初中三年级使用，每周3课时。

四、在修订中，本书的体例保持了下列特点：

1. 每章都有一段配有插图的引言，可供学生预习用，也可作为教师导入新课的材料。

2. 在课文中适当穿插了“想一想”“读一读”“做一做”等栏目。其中“想一想”是供学生思考的一些问题，“读一读”是供学生阅读的一些短文，“做一做”是供学生课外动手操作的一些实例。这些栏目是为扩大学生知识面、增加趣味性和实践性而设计的，这些都不作为教学要求，只供学生课外参考。

3. 每章后面都安排有“小结与复习”，其中的“学习要求”是对学生学完全章后的要求。

4. 每章最后都配有一套“自我测验题”，供学生自己检查学完这一章后，是否达到本章的基本要求。

5. 本书的练习题分为练习、习题、复习题三类。练习供课内用；习题供课内或课外作业用；复习题供复习每章时选用。其中习题、复习题的题目分为A、B两组，A组属于基本要求范围，B组带有一定的灵活性，仅供学有余力的学生选用。每组习题的第1题，都反映了这一部分知识的基本要求，可以作为预习用，也可作为课后复习用，不要求做出书面答案。

五、教科书原试用本由吕学礼、饶汉昌、蔡上鹤任主编，李慧君任副主编，参加编写的有蔡上鹤、陈汶、李慧君、许缦阁。责任编辑为李慧君。丁石孙、丁尔升、梅向明、张玺恩、张孝达任顾问。

参加本次修订的有饶汉昌、蔡上鹤、颜其鹏、张劲松，责任编辑为颜其鹏。

本书在编写和修订过程中，吸收了全国各地许多教师和教研人员的意见和建议，在此向他们表示衷心感谢。

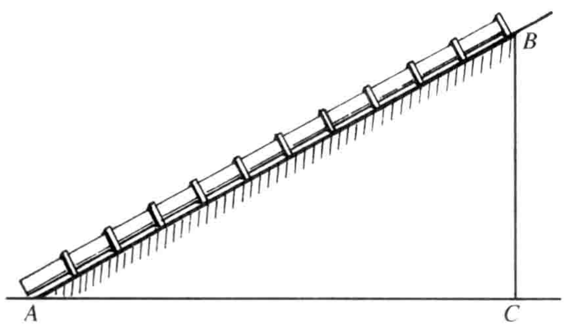
人民教育出版社中学数学室

2001年12月

CONTENTS

Chapter 6

Solving Right Triangles



A pumping station, with a water pipe laid along a slope, is to be built. It can be seen from the above figure that the size of $\angle A$ formed by the slope and the horizontal plane can be measured with a clinometer and the length of water pipe AB can also be measured directly. They are two known numbers.

When the water pipe is laid up to place B, suppose the height from B to the horizontal plane is BC. Because point C is inaccessible, the length of BC cannot be measured directly. How can we work out BC with what we know?

The above problem can be summed up that in Rt$\triangle ABC$, when $\angle A$ and hypotenuse are given, how to calculate side BC opposite to $\angle A$. After learning this chapter, the problem can be easily solved.

I Acute Trigonometric Functions

6.1 Sine and Cosine

Let's start with a set square to study the problem on the previous page. In all set squares with no equal sides (Figure 6-1(1)), the length of the opposite right-angle side of $\angle 30^\circ$ (indicated with BC, and $\angle C$ is a right angle) is half that of hypotenuse (indicated with AB), that is

$$\frac{\text{opposite side of } \angle A}{\text{hypotenuse}}=\frac{BC}{AB}=\frac{1}{2}.$$

That's to say, when $\angle A=30^\circ$, the ratio of the opposite side of $\angle A$ to the hypotenuse always equals $\frac{1}{2}$, no matter how big a triangle is. According to this ratio, the length of side BC opposite to $\angle A$ can be calculated if hypotenuse AB is given.

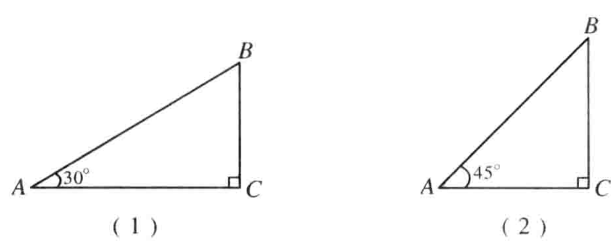


Figure 6-1

Similarly, in all isosceles set squares (Figure 6-1(2)), by Pythagorean Theorem, we have

$$\frac{\text{opposite side of } \angle A}{\text{hypotenuse}}=\frac{BC}{AB}=\frac{BC}{\sqrt{BC^2+BC^2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}.$$

That is to say, when $\angle A=45^\circ$, the ratio of the opposite side of $\angle A$ to the hypotenuse equals $\frac{\sqrt{2}}{2}$. According to this ratio, the length of side BC opposite to $\angle A$ can be calculated if hypotenuse AB is given.

Then, when acute angle A has other fixed value, will the ratio of the opposite side of $\angle A$ to the hypotenuse be constant, too?

As we say acute angle A has a fixed value, it means that we have a lot of right triangles such as $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$, …, they have one equal acute angle. If we overlap A_1, A_2, A_3, … and mark them A, and make right-angle sides AC_1, AC_2, AC_3, … fall on a same straight line and hypotenuses AB_1, AB_2, AB_3 … on the other same straight line (Figure 6-2), it is easy to know,

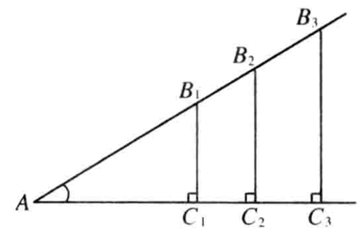


Figure 6-2

$$B_1C_1 \parallel B_2C_2 \parallel B_3C_3 \parallel \cdots,$$

$$\therefore \quad \triangle AB_1C_1 \backsim \triangle AB_2C_2 \backsim \triangle AB_3C_3 \backsim \cdots.$$

$$\therefore \quad \frac{B_1C_1}{AB_1}=\frac{B_2C_2}{AB_2}=\frac{B_3C_3}{AB_3}=\cdots.$$

Therefore, among these right triangles, the ratio of the opposite side of $\angle A$ to the hypotenuse is a constant.

Training Exercise

In $\triangle ABC$, $\angle C$ is a right angle. If $\angle A=60^\circ$, what is the ratio of the opposite side of $\angle A$ to the hypotenuse?

As shown in Figure 6-3, in $\triangle ABC$, $\angle C$ is a right angle, the ratio of the length of the opposite side of acute angle A to the length of hypotenuse is called the **sine** of $\angle A$, represented as $\sin A$❶, so

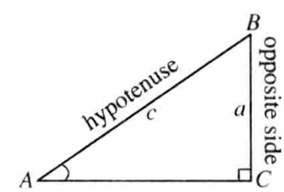


Figure 6-3

$$\sin A=\frac{\textbf{opposite side of } \angle A}{\textbf{hypotenuse}}.$$

If we denote side BC opposite to $\angle A$ as a, side BC opposite to $\angle C$ as c, then

$$\sin A=\frac{a}{c}.$$

For example, when $\angle A=30^\circ$, we have

❶ Notice: this is a complete sign and it should not be written as $\sin \cdot A$. Angle "$\angle$" is customarily left out, the first letter "s" should be a small letter. The same rule applies to $\cos A$, $\tan A$, $\cot A$, and so on.

$$\sin A=\sin 30°=\frac{1}{2};$$

When $\angle A=45°$, we have

$$\sin A=\sin 45°=\frac{\sqrt{2}}{2}.$$

Since the hypotenuse of a right triangle is greater than its right-angle side, we can obtain from Figure 6-3

$$0<\frac{a}{c}<1.$$

$\therefore\quad 0<\sin A<1$ ($\angle A$ being an acute angle).

Example 1 Find the values of $\sin A$ and $\sin B$ in Rt$\triangle ABC$ as shown in Figure 6-4.

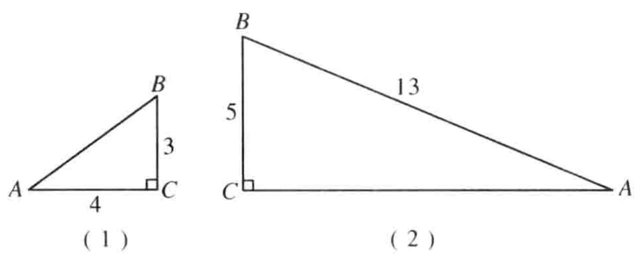


Figure 6-4

Analysis: To calculate $\sin A$, the ratio of the opposite side of $\angle A$ to the hypotenuse is needed; to find $\sin B$, the ratio of the opposite side of $\angle B$ to the hypotenuse is also needed.

Solution: (1) $\because$ Hypotenuse $AB=\sqrt{AC^2+BC^2}=5$,

$\therefore\quad \sin A=\frac{3}{5}$, and $\sin B=\frac{4}{5}$.

(2) $\sin A=\frac{5}{13}$.

$\because\quad AC=\sqrt{AB^2-BC^2}=\sqrt{144}=12$,

$\therefore\quad \sin B=\frac{12}{13}$.

Similar to the case of sine, we can prove that when acute angle A has any fixed value, the ratio of the adjacent side of $\angle A$ to the hypotenuse is a constant, too.

In $\triangle ABC$, as shown in Figure 6-5, $\angle C$ is a right angle, we call the ratio of the adjacent side of $\angle A$ to the hypotenuse **cosine** of $\angle A$, represented as $\cos A$, that is

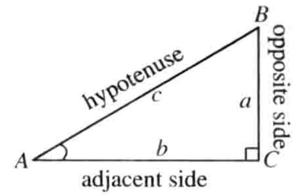


Figure 6-5

$$\cos A=\frac{\textbf{adjacent side of } \angle A}{\textbf{hypotenuse}}.$$

If we denote the adjacent side of $\angle A$ (the opposite side of $\angle B$) as b, then

$$\cos A=\frac{b}{c}.$$

$\because$ $0<b<c$,

$\therefore$ $0<\cos A<1$ ($\angle A$ being an acute angle).

From Figure 6-1 and other known knowledge, we have:

> (1) $\sin 30^\circ=\frac{1}{2}$, $\sin 45^\circ=\frac{\sqrt{2}}{2}$, $\sin 60^\circ=\frac{\sqrt{3}}{2}$;
>
> (2) $\cos 30^\circ=\frac{\sqrt{3}}{2}$, $\cos 45^\circ=\frac{\sqrt{2}}{2}$, $\cos 60^\circ=\frac{1}{2}$.

Example 2 Find the values of the following expressions:

(1) $\sin 30^\circ+\cos 30^\circ$;

(2) $\sqrt{2}\sin 45^\circ-\frac{1}{2}\cos 60^\circ$.

Solution: (1) $\sin 30^\circ+\cos 30^\circ$

$$=\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2};$$

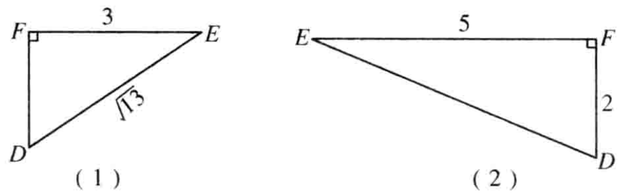
(2) $\sqrt{2}\sin 45^\circ-\frac{1}{2}\cos 60^\circ$

$$=\sqrt{2}\times\frac{\sqrt{2}}{2}-\frac{1}{2}\times\frac{1}{2}$$

$$=\frac{3}{4}.$$

Training Exercises

1. Find the values of sin D and sin E in the figure.



(Exercise 1)

2. (1) Find the values of cos A and cos B in Figure 6-4;
   (2) Compare the values of cos A and cos B with those of sin A and sin B (refer to the solutions in example 1), and write out the expressions of sin A and cos B, and also those of sin B and cos A.
3. (1) Find the values of cos D and cos E in exercise 1;
   (2) Compare the values of cos D and cos E with those of sin D and sin E obtained from exercise 1, and write out the expressions of sin D and cos E, and also those of sin E and cos D.
4. Find the values of the following expressions:
   (1) $\sin 45^\circ + \cos 45^\circ$; (2) $\sin 30^\circ \cdot \cos 60^\circ$;
   (3) $0.5 - \sin 60^\circ$; (4) $\dfrac{\sin 30^\circ}{\cos 30^\circ}$.

We have learned the sine and cosine of an acute angle, we are going to study their relationships. We know:

$$\sin 30^\circ = \frac{1}{2},\ \sin 45^\circ = \frac{\sqrt{2}}{2},\ \text{and } \sin 60^\circ = \frac{\sqrt{3}}{2};$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2},\ \cos 45^\circ = \frac{\sqrt{2}}{2},\ \text{and } \cos 60^\circ = \frac{1}{2}.$$

From the above, we can see:

$$\sin 30^\circ = \cos 60^\circ,$$
$$\sin 45^\circ = \cos 45^\circ,$$
$$\sin 60^\circ = \cos 30^\circ.$$

This is to say that the values of sine of the three special angles of 30°, 45° and 60° are respectively equal to the values of cosine of their complementary angles of 60°, 45° and 30° (in the other way, the values of cosine of the three special angles of 30°, 45° and 60° are respectively equal to the values of sine of their complementary angles of 60°, 45° and 30°).

Then, is the value of sine of any acute angle equal to the value of cosine of its complementary angle ? Figure 6-5 shows

$$\sin A=\frac{\text{opposite side of } \angle A}{\text{hypotenuse}}=\frac{a}{c},$$

$$\cos B=\frac{\text{adjacent side of } \angle B}{\text{hypotenuse}}=\frac{\text{opposite side of } \angle A}{\text{hypotenuse}}=\frac{a}{c},$$

$\therefore$ $$\sin A=\cos B.$$

By the same reason $\cos A=\sin B$.

We notice in Figure 6-5 that $\angle A+\angle B=90°$, i. e. $\angle B=90°-\angle A$. So we can also write the above formula as

$$\sin A=\cos(90°-A),$$
$$\cos A=\sin(90°-A).$$

This is to say, **the sine of any acute angle is equal to the cosine of its complementary angle, and the cosine of any acute angle is equal to the sine of its complementary angle.**

Example 3 (1) Given $\sin A=\frac{1}{2}$ and $\angle B=90°-\angle A$, find the value of $\cos B$;

(2) Given $\sin 35°=0.573\ 6$, find the value of $\cos 55°$;

(3) Given $\cos 47°6'=0.680\ 7$, find the value of $\sin 42°54'$.

Solution: (1) $\cos B=\cos(90°-A)=\sin A=\frac{1}{2}$;

(2) $\cos 55°=\cos(90°-35°)=\sin 35°=0.573\ 6$;

(3) $\sin 42°54'=\sin(90°-47°6')$

$=\cos 47°6'=0.680\ 7$.

Training Exercises

1. Given two acute angles of $\angle A$ and $\angle B$.

(1) Write $\cos(90°-A)$ in the sine form of $\angle A$;

(2) Write $\sin(90°-B)$ in the cosine form of $\angle A$.

2. (1) Given $\cos A=\frac{\sqrt{2}}{2}$ and $\angle B=90^\circ-\angle A$, calculate $\sin B$;

   (2) Given $\sin 67^\circ18'=0.922\,5$, calculate $\cos 22^\circ42'$;

   (3) Given $\cos 4^\circ24'=0.997\,1$, calculate $\sin 85^\circ36'$.

3. In $\triangle ABC$, $\angle C$ is a right angle, the opposite sides of $\angle A$, $\angle B$ and $\angle C$ are a, b and c respectively. Please use the following conditions to find the values of sine and cosine of $\angle A$ first, then those of $\angle B$:

   (1) $a=2$, and $b=1$; (2) $a=3$, and $c=4$;

   (3) $b=2$, and $c=\sqrt{29}$; (4) $a=4\sqrt{5}$, and $b=8$.