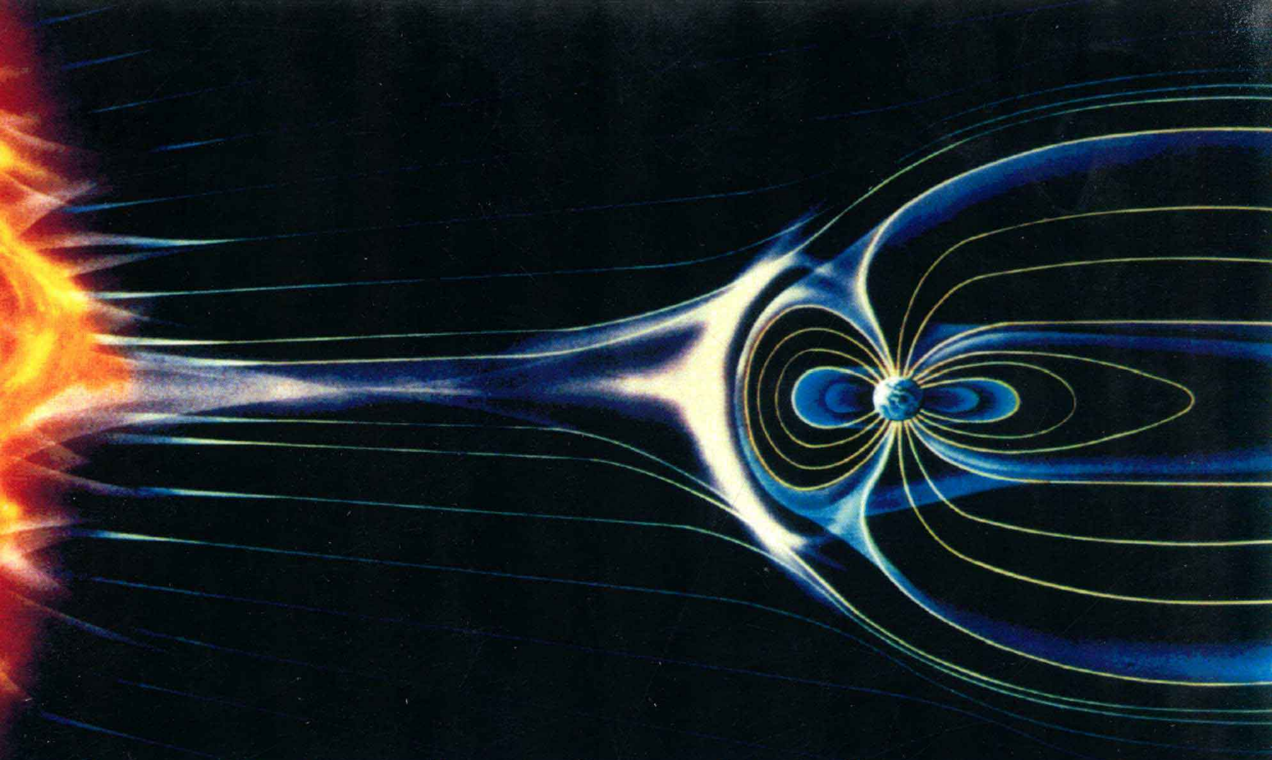

# 电磁场与电磁波（第二版）

## （英文版）

# Electromagnetic Fields and Waves

焦其祥 主编

科学出版社

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# 电磁场与电磁波（第二版）

（英文版）

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科学出版社
北　京

## 内 容 简 介

本书中文版是普通高等教育“十一五”国家级规划教材，也是北京邮电大学通信工程国家级特色专业建设点主干教材。全书共 11 章，主要讲述电磁场与电磁波的基本理论和计算方法。本书在叙述上由浅入深、循序渐进，强调数学与物理概念的结合，思路清晰，易于学习。对一些重要内容和例题采用了不同的分析方法，强调分析方法的多样性，拓展思考空间，扩大适应面。书中配有近百道例题，以帮助学生分析问题，引导学生自学。

本书可作为高等院校电子信息、通信工程、微波工程及相关专业本科生的教材，也可供相关教学和工程技术人员参考。

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# Preface to the Second Edition

Compared with the first edition of this book, there is no big change in the text structure of the second edition. It maintains the characteristics of the original book, but partially adds and deletes or modifies the content and examples.

In order to broaden the mind, the second edition still tries to keep the feature of "using diverse analysis methods" to solve some important problems, and give a brief summary before each analysis method. This may provide more options with regard to different teaching hours and reserve some room of thinking for the readers who have the interest and potential.

The modifications in the second edition are as follows:

(1) Text content addition and deletion. For example, a new section "the concept of symmetric dipoles and antenna array" has been added into chapter; individual parts like "low loss transmission line" have been deleted in chapter 9; some content like "the Kirchhoff voltage laws" has been modified; the section "the polarization of the electromagnetic wave" has been reused.

(2) Examples and exercises addition and deletion. Some individual examples which are too complex or too simple have been deleted. Meanwhile, some examples which are very conceptual or have good engineering value have been added into the text. Furthermore, about 30 basic concept problems have been added, and some problems have been deleted or adapted; the order of some problems have been rotated, and problem solving "tips" have been added into some of the problems.

Thanks to Ms. Sun Yongmei, Mr. Qiao Yaojun, Mr. Wu Jian and Mr. Zhang Min who gave valuable suggestions on the modification of this book. These four are all young and middle-aged professors or associate professors; thanks to the precious opinions given by the young teacher, Dr. Shen Yuanmao, and especially thanks to the doctoral supervisor Ms. Gu Wanyi and Professor Wang Wenbo who have been giving strong support and backing to the education of the electromagnetic field and the development of the teaching material for many years. Thanks to Dr. Siamak Sorooshyari and Prof. Xue Quan who have made plenty of efforts to touch up the expression.

We hereby express our heart-felt gratitude to the valuable suggestions given by the readers and other universities of the same profession, and hope for more attention and support from them.

The telecommunications engineering specialty of Beijing University of Posts and Telecommunications is the characteristic specialty development point (TS2055) of the first batch which was authorized by the Ministry of Education of the People's Republic of China. The compiling of this book was subsidized by this development point. The aim of compiling this book is to create an elaborative teaching material in telecommunications engineering around the construction of this development point.

October, 2009

Author

Beijing

# Preface for the First Edition

The theory of Electromagnetic fields and electromagnetic waves is an important basis for specialization subjects. By using strong theories, rigorous logic and conceptual abstraction, it profoundly reveals the basic laws of electromagnetic phenomena. This course will not only deepen our understanding of the electromagnetic laws, but also train the right way of thinking and improve the ability to analyze issues.

As far as problem analysis ability is concerned, we emphasize the principle of diversity of analysis. Scientific analysis is an important part of engineering course in university. Based on this principle, an issue analysis can involve a brief physical concept analysis, a more rigorous mathematics analysis, or a combined analysis from different points of view and in different concepts. Thus, for one thing, students will be provided with further exploration of space, for another, teachers will be provided with more choices in analysis methods and lecture contents, for courses in different professions and different class hours.

In terms of the knowledge structure, the theory of electromagnetic field should be a necessary and indispensable part for college students majoring in electronics and communication engineering. The continuous development of science and technology will further demonstrate its importance.

As is known to everyone, the theory of electromagnetic fields and electromagnetic waves is a basis for specialization subjects, such as microwave technology, antennas, wave propagation, and optical fiber transmission. In fact, its application is far beyond this range.

If you know about high-frequency circuits, you'll know that the leads of resistance, inductance, capacitance should be as short as possible. Because these leads will introduce distributed inductance, capacitance, resistance, which will change the element parameters and do harm to the circuit indicators. we have to use the theory of electromagnetic to deal with the problem of "distributed parameter".

In integrated circuits, particularly in high-speed integrated circuits, it is full of electromagnetic field, for example, the coupling between circuits, the coupling between the earth, the loss of material and dispersion, as well as the reflection caused by connectors, cables, turning, through holes and other discontinuities, they will change the circuit parameters, do harm to the integrity of signal. To do analysis of these issues, theory of electromagnetic field is essential.

Electromagnetic field and wave, as the intersection of disciplines, derived many disciplines. Communications discipline contains mobile communications, satellite communications, optical fiber communication, etc. In addition to communications, radar, radio and television, they are all subject to information carried by electromagnetic wave.

As far as development of students' innovative ability is concerned, as a qualified student of electronics, communications and information engineering, one must take efforts to learn the theory of electromagnetic field to deal with the most basic electromagnetic problem. And because the course is interdisciplinary, it helps students to learn inter-disciplines and improve create capacity.

The content of this book refers to the teaching program made by electromagnetic field theory's Steering Group which belongs to Ministry of Education of the People's Republic of China. And we also consult the excellent teaching books at home and abroad and the teaching experiences of the authors. In this book, there are also many examples which are expected to be helpful to improve the readers' abilities in persuing problems.

The book is made up of 11 chapters. The 1st, 9th, 10th and 11en were compiled in turn by Zehua Gao, Yangan Zhang, Shufang Li and Li Li. Yafeng Wang and Xin Zhang compiled the exercises and answers. These six authors above are all young and middle-aged professors or associate professors who have got the doctor's degree. Qixiang Jiao compiled the chapters from 2 to 8. And he is also the editor in chief of this book and planed the draft as a whole. Professor Wanyi Gu who is also a tutor of a Ph.D. student check and approve this book and thanks a lot to her. We are grateful to Huazhi Wang, Maolin Zhang who gave us many advices about the architecture and content of this book. We also want to show our appreciations to the doctors, masters and undergraduates who have given strong support in clearing up the draft, drawing and so on. Thanks to the leaders of Telecommunications Engineering College, wireless communication center and optical communication center of BUPT who have given us so much help and support.

Thanks to editors Min Kuang and Jiang Yu of Science Press. They worked hard for the book's quality assurance.

The book < Electromagnetic Fields and Waves Problem's fine solutions> which is a support of this book has been published by Science Press at the same time. The PPT has been finished and teachers may get it from the press for free. The readers may get the web resources relating to this book at http://jpkc.bupt.edu.cn:4213/dcc/index.htm. If readers have any suggestion about the content of this book, they may get touch with the author.(wangyf@bupt.edu.cn, lili@bupt.edu.cn, zhang@bupt.edu.cn).

July, 2004

Author

# Main Character, Parameters and The Expressions of Gradient, Divergence, Rotation

## Main Symbols

| Symbols | Name | Unit representation |
|---|---|---|
| $\boldsymbol{E}$ | electric-field intensity | V/m(volt per meter) |
| $\boldsymbol{H}$ | magnetic field intensity | A/m(Ampere per meter) |
| $\boldsymbol{D}$ | electric displacement (Electric flux density) | $C/m^2$( Coulomb per square meter) |
| $\boldsymbol{B}$ | magnetic induction (magnetic flux density) | T(Tesla) |
| $\varphi$ | electric potential | V(Volt) |
| $\Psi_e$ | electric flux | C(Coulomb) |
| $\Phi$ | magnetic flux | Wb(Weber) |
| $\boldsymbol{A}$ | magnetic vector potential | Wb/m(Weber per meter) |
| $\rho_l$ | linear charge density | C/m(Coulomb per meter) |
| $\rho_s$ | surface charge density | $C/m^2$( Coulomb per square meter) |
| $\rho$ | volume charge density | $C/m^3$(Coulomb per cubic meter) |
| $n$ | refractive index | |
| $R$ | reflection coefficient | |
| $T$ | transmission coefficient, reflection coefficient | |
| $C_0$ | capacitance per unit length | F/m(Farad per meter) |
| $L_0$ | inductance per unit length | H/m(Henry per meter) |
| $\boldsymbol{F}$ | force | N(Newton) |
| $\boldsymbol{T}$ | moment | N·m (Newton·meter) |
| $w_e$ | energy density of electric field | $J/m^3$(Joule per cubic meter) |
| $w_m$ | energy density of magnetic field | $J/m^3$(Joule per cubic meter) |
| $\boldsymbol{S}$ | power density(Poynting vector) | $W/m^2$(Watt per square meter) |
| $\boldsymbol{J}_s$ | surface current density | A/m(Ampere per meter) |
| $\boldsymbol{J}$ | current density | $A/m^2$(Ampere per square meter) |
| $\gamma$ | propagation constant | 1/m(1 per meter) |
| $\alpha$ | attenuation constant | Np/m, dB/m(Napier per meter, decibel/meter) |
| $\beta$ | phase-shift constant | rad/m(radian per meter) |
| $k$ | wave number, TEM phase-shift constant | rad/m(radian per meter) |
| $\eta$ | wave impedance of TEM wave | $\Omega$(Ohm) |
| $\eta_0$ | wave impedance of TEM wave in vacuum | $\Omega$(Ohm) |
| $Z_{W(TE)}$ | wave impedance of TE wave | $\Omega$(Ohm) |
| $Z_{W(TM)}$ | wave impedance of TM wave | $\Omega$(Ohm) |
| $Z_c$ | characteristic impedance | $\Omega$(Ohm) |
| $Z_s$ | surface impedance | $\Omega$(Ohm) |
| $R_s$ | surface resistance | $\Omega$(Ohm) |
| $X_s$ | surface reactance | $\Omega$(Ohm) |
| $R_r$ | radiation resistance | $\Omega$(Ohm) |
| $\lambda$ | wavelength | m(meter) |
| $\lambda_0$ | wavelength in vacuum | m(meter) |
| $\lambda_g$ | waveguide wavelength | m(meter) |
| $\lambda_c$ | cut-off wavelength | m(meter) |
| $\varepsilon^e$ | complex dielectric constant | |
| $\boldsymbol{P}_e$ | electric dipole moment | C·m (Coulomb ·meter) |
| $\boldsymbol{m}$ | magnetic dipole moment | $A{\cdot}m^2$ (Ampere · square meter) |

# Common Parameters

| $c \approx 3\times10^{8}$m/s | velocity of light(in vacuum) |
|---|---|
| $\varepsilon_0 = \dfrac{1}{36\pi}\times10^{-9}$F/m | dielectric constant(in vacuum) |
| $\mu_0 = 4\pi\times10^{-7}$H/m | permeability (in vacuum) |
| $\sigma_{\mathrm{Ag}} = 6.17\times10^{7}$S/m | Electroconductibility(silver) |
| $\sigma_{\mathrm{Cu}}=5.80\times10^{7}$S/m | Electroconductibility(copper) |
| $\sigma_{\mathrm{Au}}=4.10\times10^{7}$S/m | Electroconductibility(gold) |
| $\sigma_{\mathrm{Pb}}=3.54\times10^{7}$S/m | Electroconductibility(aluminum) |
| $\sigma_{\mathrm{Cu*Zn}}=1.57\times10^{7}$S/m | Electroconductibility(brass) |
| $\sigma_{\mathrm{Fe}} = 1.00\times10^{7}$S/m | Electroconductibility(Ferrum) |
| $e = -1.602\times10^{-19}$C | quantity of electron charge |
| $m_{\mathrm{e}} = 9.107\times10^{-31}$kg | rest mass of electron |
| $R_{\mathrm{e}} = 2.81\times10^{-15}$m | Radius of electron |
| $m_{\mathrm{p}} = 1.673\times10^{-27}$kg | Rest mass of proton |
| $6.6237\times10^{-34}$J·s | Planck constant |
| $1.38\times10^{-23}$J/K | Boltzmann constant |

# The Expressions of Gradient, Divergence, Rotation and Laplace's Equation in Three Kinds Usual Coordinates

Rectangular coordinate system$(x, y, z)$

$$\nabla\varphi = \boldsymbol{e}_x\frac{\partial\varphi}{\partial x} + \boldsymbol{e}_y\frac{\partial\varphi}{\partial y} + \boldsymbol{e}_z\frac{\partial\varphi}{\partial z}$$

$$\nabla\cdot\boldsymbol{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$\nabla\times\boldsymbol{a} = \boldsymbol{e}_x\left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right) + \boldsymbol{e}_y\left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right) + \boldsymbol{e}_z\left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)$$

$$\nabla^2\varphi = \Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}$$

## Cylindrical-coordinate system $(r, \phi, z)$

$$\nabla\varphi = \boldsymbol{e}_r\frac{\partial\varphi}{\partial r} + \frac{\boldsymbol{e}_\phi}{r}\frac{\partial\varphi}{\partial\phi} + \boldsymbol{e}_z\frac{\partial\varphi}{\partial z}$$

$$\nabla\cdot\boldsymbol{a} = \frac{1}{r}\frac{\partial}{\partial r}(ra_r) + \frac{1}{r}\frac{\partial a_\phi}{\partial\phi} + \frac{\partial a_z}{\partial z}$$

$$\nabla\times\boldsymbol{a} = \boldsymbol{e}_r\left(\frac{1}{r}\frac{\partial a_z}{\partial\phi} - \frac{\partial a_\phi}{\partial z}\right) + \boldsymbol{e}_\phi\left(\frac{\partial a_r}{\partial z} - \frac{\partial a_z}{\partial r}\right) + \boldsymbol{e}_z\left(\frac{1}{r}\frac{\partial}{\partial r}(ra_\phi) - \frac{1}{r}\frac{\partial a_r}{\partial\phi}\right)$$

$$\nabla^2\varphi = \Delta\varphi = \frac{1}{\mathrm{r}}\frac{\partial}{\partial\mathrm{r}}\left(\mathrm{r}\frac{\partial\varphi}{\partial\mathrm{r}}\right) + \frac{1}{\mathrm{r}^2}\frac{\partial^2\varphi}{\partial\phi^2} + \frac{\partial^2\varphi}{\partial z^2}$$

## Spherical coordinate system $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\phi})$

$$\nabla\varphi = \boldsymbol{e}_r\frac{\partial\varphi}{\partial r} + \frac{\boldsymbol{e}_\theta}{r}\frac{\partial\varphi}{\partial\theta} + \frac{\boldsymbol{e}_\phi}{r\sin\theta}\frac{\partial\varphi}{\partial\phi}$$

$$\nabla\cdot\boldsymbol{a} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 a_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta a_\theta) + \frac{1}{r\sin\theta}\frac{\partial a_\phi}{\partial\phi}$$

$$\nabla\times\boldsymbol{a} = \boldsymbol{e}_r\left(\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta a_\phi) - \frac{1}{r\sin\theta}\frac{\partial a_\theta}{\partial\phi}\right) + \boldsymbol{e}_\theta\left(\frac{1}{r\sin\theta}\frac{\partial a_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(r a_\phi)\right)$$
$$+ \boldsymbol{e}_\phi\left(\frac{1}{r}\frac{\partial}{\partial r}(r a_\theta) - \frac{1}{r}\frac{\partial a_r}{\partial\theta}\right)$$

$$\nabla^2\varphi = \Delta\varphi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\varphi}{\partial\phi^2}$$

# Vector Identities

$$\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c}) = \boldsymbol{b}\cdot(\boldsymbol{c}\times\boldsymbol{a}) = \boldsymbol{c}\cdot(\boldsymbol{a}\times\boldsymbol{b})$$
$$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$$
$$\nabla\cdot\nabla = \nabla^2$$
$$\nabla(\varphi_1\varphi_2) = \varphi_1\nabla\varphi_2 + \varphi_2\nabla\varphi_1$$
$$\nabla\cdot(\varphi\boldsymbol{a}) = \boldsymbol{a}\cdot\nabla\varphi + \varphi\nabla\cdot\boldsymbol{a}$$
$$\nabla\times(\varphi\boldsymbol{a}) = \nabla\varphi\times\boldsymbol{a} + \varphi\nabla\times\boldsymbol{a}$$
$$\nabla(\boldsymbol{a}\cdot\boldsymbol{b}) = (\boldsymbol{a}\cdot\nabla)\boldsymbol{b} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{a} + \boldsymbol{a}\times(\nabla\times\boldsymbol{b}) + \boldsymbol{b}\times(\nabla\times\boldsymbol{a})$$
$$\nabla\cdot(\boldsymbol{a}\times\boldsymbol{b}) = \boldsymbol{b}\cdot\nabla\times\boldsymbol{a} - \boldsymbol{a}\cdot\nabla\times\boldsymbol{b}$$
$$\nabla\times(\boldsymbol{a}\times\boldsymbol{b}) = \boldsymbol{a}\nabla\cdot\boldsymbol{b} - \boldsymbol{b}\nabla\cdot\boldsymbol{a} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{a} - (\boldsymbol{a}\cdot\nabla)\boldsymbol{b}$$
$$\nabla\cdot(\nabla\varphi) = (\nabla\cdot\nabla)\varphi = \nabla^2\varphi = \Delta\varphi$$
$$\nabla\times(\nabla\varphi) = 0$$
$$\nabla\cdot(\nabla\times\boldsymbol{a}) = 0$$
$$\nabla\times(\nabla\times\boldsymbol{a}) = \nabla(\nabla\cdot\boldsymbol{a}) - \nabla^2\boldsymbol{a}$$
$$\oint_S \boldsymbol{a}\cdot\mathrm{d}\boldsymbol{S} = \int_V \nabla\cdot\boldsymbol{a}\mathrm{d}V$$
$$\oint_C \boldsymbol{a}\cdot\mathrm{d}\boldsymbol{l} = \int_S \nabla\times\boldsymbol{a}\cdot\mathrm{d}\boldsymbol{S}$$
$$\oint_S (\boldsymbol{n}\times\boldsymbol{a})\mathrm{d}S = \int_V \nabla\times\boldsymbol{a}\cdot\mathrm{d}V$$
$$\oint_S \varphi\boldsymbol{n}\mathrm{d}S = \int_V \nabla\varphi\mathrm{d}V$$
$$\oint_C \varphi\mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{n}\times\nabla\varphi\mathrm{d}S$$

# Contents

# Chapter 1

# Vector Analysis

The theory of vector analysis for electromagnetic fields and electromagnetic waves is introduced in Chapter 1. The concepts of scalar, vector, scalar field, vector field are also included. We introduce the concepts of the vector operation, the flux of vector, the divergence of vector, Gauss's law, the circumfluence of vector, the curl of vector, the Stokes's law, the gradient of scalar, and the Helmholtz's law.

## 1.1 Scalar and Vector Fields

### 1.1.1 Scalar

**A scalar** is a physical quantity that can be expresed by magnitude and sign. The scalar is a point in the space, and can be called an absolute scalar if it is independent of coordinate system. For example real numbers such as mass, length, area, time,temperature,voltage, electric charge, current and energy are all scalars.

### 1.1.2 Vector

**A vector** is a physical quantity that can be specified by direction as well as magnitude. A vector may be denoted by $\boldsymbol{a}$ or by line with direction $\overrightarrow{OA}$. It's magnitude can be denoted with $a$ which is called the magnitude of vector $\boldsymbol{a}$ and marked with:

$$|\boldsymbol{a}| = a$$

Vector $\boldsymbol{a}$ can be denoted by a line with direction in three dimensional space shown in Figure 1.1. The length of the line represent the magnitude of vector $\boldsymbol{a}$. The direction of the line represent the direction of vector $\boldsymbol{a}$.

In the three-dimensional Cartesian case of Figure 1.1 that the magnitude of vector $\boldsymbol{a}$ is,

$$a = |\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

The cross angle $\alpha$, $\beta$, $\gamma$ between vector $\boldsymbol{a}$ and the positve direction of $x$ axis, $y$ axis, $z$ axis correspondingly are called the direction angle of vector $\boldsymbol{a}$. The cosine of orientation angle cos$\alpha$, cos$\beta$, cos$\gamma$ is called the direction cosine of vector $\boldsymbol{a}$. It can be seen from the above figure that,

$$a_x = |\boldsymbol{a}| \cos\alpha, \quad a_y = |\boldsymbol{a}| \cos\beta, \quad a_z = |\boldsymbol{a}| \cos\gamma \tag{1.1}$$

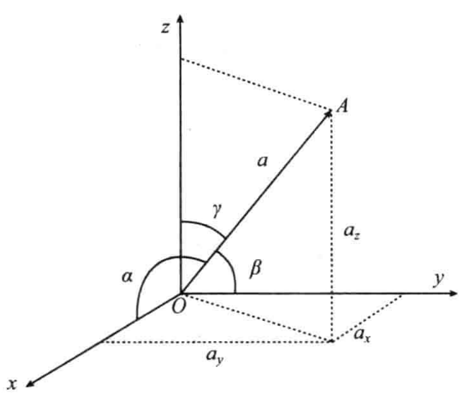


Figure 1.1 Vector $\boldsymbol{a}$ in Cartesian Coordinates

So the orientation cosine satisfy

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \tag{1.2}$$

The vector with a magnitude of 1 is called unit vector and it can be denoted by $\boldsymbol{e}$. The unit vector that has the same direction as vector $\boldsymbol{a}$ with magnitude$a \neq 0$ can be denoted,

$$\boldsymbol{e} = \boldsymbol{a}/a \tag{1.3}$$

The unit vector with the same direction of the positve direction of $x$ axis, $y$ axis, $z$ axis in Cartesian Coordinates is called a base unit vector. The base unit vector can be expressed by $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$. The three-dimensional vector $\boldsymbol{a}$ can be decomposed according to the base unit vectors,

$$\boldsymbol{a} = a_x \boldsymbol{e}_x + a_y \boldsymbol{e}_y + a_z \boldsymbol{e}_z$$

When the tail and the head of a vector is superimposed, the vector can be called zero vector. The length of zero vector is zero and the direction of the zero vector is arbitrary. If the magnitude of two vectors are equal and the direction of the two vectors are opposite, the two vector can be called reverse vector. The reverse vector of vector $\boldsymbol{a}$ is denoted $-\boldsymbol{a}$.

If two vectors satisfy: they lien on the same line or they are parallel and their direction is the same; their magnitude is equal, then the two vector is equal vector.

### 1.1.3 Scalar Field

If the location of scalar $\varphi$ is a function of time, then the scalar can be described by the function $\varphi(x, y, z, t)$, where $x, y, z$ determine the scalar's position, and $t$ denotes the time. If the range of scalar function is an infinte set, then the set will represent the field of the scalar. As an example, the distribution of space temperature is a temperature field $T(x, y, z, t)$, and the distribution of electric potential is electric $\varphi(x, y, x, t)$.

If the scalar is independent of time, $\varphi(x, y, x, t)$ describes a static field; If the scalar has a relationship with time, $\varphi(x, y, x, t)$ describes a dynamic field.

### 1.1.4 Vector Field

If a vector $\boldsymbol{F}$ is a function of space, position, and time, it can be expressed by $\boldsymbol{F}(x, y, z, t)$, where $x, y, z$ is space position and $t$ is time. If the range of a vector function is an infinite set, this set represent the field of vector. For instance, the distribution of electric field intensity is given by an electric field vector $\boldsymbol{E}(x, y, z, t)$.

In three-dimensional space, a vector can be represented by three weights in a three-dimensional coordinate system. Without dimension, the three weight would correspond to three scalar values. Thus, one vector can be represented by three scalars. A vector can be written as follows:

$$\boldsymbol{F}(x, y, z) = F_x(x, y, z)\boldsymbol{e}_x + F_y(x, y, z)\boldsymbol{e}_y + F_z(x, y, z)\boldsymbol{e}_z$$

where $F_x(x, y, z)$, $F_y(x, y, z)$ and $F_z(x, y, z)$ are three scalar values.

## 1.2 Operation of Vector

### 1.2.1 Vector Addition

In physics, two forces can be decomposed into a superposition of forces via a parallelogram law. Thus, we can define vector addition as follows.

Suppose there are two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$(such as in Figure 1.2), their start point is $O$, take vector $\boldsymbol{a}$ and $\boldsymbol{b}$ as the side of parallelogram, then vector $\boldsymbol{c}$ corresponding with the diagonal is the resultant vector $\boldsymbol{a}$ and $\boldsymbol{b}$, denote as $\boldsymbol{c}=\boldsymbol{a}+\boldsymbol{b}$. This law is the plus of vector's parallelogram law.

The plus of vector also can be denoted by basic vector.

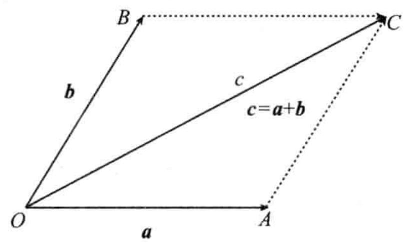

Figure 1.2 Parallelogram law of vector's plus

If

$$\boldsymbol{a} = a_x\boldsymbol{e}_x + a_y\boldsymbol{e}_y + a_z\boldsymbol{e}_z$$

and

$$\boldsymbol{b} = b_x\boldsymbol{e}_x + b_y\boldsymbol{e}_y + b_z\boldsymbol{e}_z$$

then

$$\boldsymbol{a} + \boldsymbol{b} = (a_x + b_x)\boldsymbol{e}_x + (a_y + b_y)\boldsymbol{e}_y + (a_z + b_z)\boldsymbol{e}_z$$

From the above we can see that the coordinate components of the vector plus are the plus of the corresponding coordinate components of the two vectors

The subtraction of vector is the inverse operation of vector plus. Suppose we have $\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{c}$, then

$$\boldsymbol{a} = \boldsymbol{c} - \boldsymbol{b} = \boldsymbol{c} + (-\boldsymbol{b})$$

Vector addition follows commutativity and associativity:

$$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a} \tag{1.4}$$

$$\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})=(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c} \tag{1.5}$$

### 1.2.2 Scalar Vector Multiplication

Multiplication between real number $\lambda$ and vector $\boldsymbol{a}$ defined as multiply of vector between real number $\lambda$ and vector $\boldsymbol{a}$, the result must be a vector, can be written $\lambda\boldsymbol{a}$, and its magnitude is:

$$|\lambda\boldsymbol{a}| = |\lambda||\boldsymbol{a}| \tag{1.6}$$

The vector's direction is determined as follows: if $\lambda > 0$, direction of $\lambda\boldsymbol{a}$ is the same of $\boldsymbol{a}$; if $\lambda < 0$, $\lambda\boldsymbol{a}$ is right-about with $\boldsymbol{a}$; if $\lambda = 0$, $\lambda\boldsymbol{a}= 0$.

when $\lambda = -1$, $\lambda\boldsymbol{a} = -\boldsymbol{a}$ is defined as reverse vector of vector $\boldsymbol{a}$.

The multiplication of vector $\boldsymbol{a}$ by a scalar $\lambda$ can be expressed as:

$$\lambda\boldsymbol{a} = \lambda a_x\boldsymbol{e}_x + \lambda a_y\boldsymbol{e}_y + \lambda a_z\boldsymbol{e}_z$$

Supposed $\boldsymbol{a}$ and $\boldsymbol{b}$ are vectors, $\lambda_1$ and $\lambda_2$ are scalar, then we have

$$(\lambda_1 + \lambda_2)\boldsymbol{a} = \lambda_1\boldsymbol{a} + \lambda_2\boldsymbol{a}$$

$$\lambda_1(\boldsymbol{a} + \boldsymbol{b}) = \lambda_1\boldsymbol{a} + \lambda_1\boldsymbol{b}$$

### 1.2.3 Scalar Product of Vector (number product, inner product)

In physics, objects have a diaplacement $\boldsymbol{l}$ under force $\boldsymbol{F}$ , if angle between $\boldsymbol{F}$ and $\boldsymbol{l}$ is $\theta$, then the force $\boldsymbol{F}$'s power on objects is:

$$W = |\boldsymbol{F}||\boldsymbol{l}|\cos\theta \tag{1.7}$$

According to the above calculation, we can define one kind of product of vector: suppose we have two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, $\theta$ is the angle between them, the scalar product of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is denoted by $\boldsymbol{a} \cdot \boldsymbol{b}$. We equate this to the product of magnitudes and the cosine of the angle between the two vectors via

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta \tag{1.8}$$

The scalar product of a vector is also called number product, dot product or inner product.

According to this define, $W$, power of $\boldsymbol{F}$, can be denoted as:

$$W = \boldsymbol{F} \cdot \boldsymbol{l}$$

The scalar product of vector $\boldsymbol{a}$ and $\boldsymbol{b}$, $\boldsymbol{a} \cdot \boldsymbol{b}$ can be denoted by basic unit vector

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z \tag{1.9}$$

According to last expression, the scalar of vector equal to the sum of product of corresponding weight of two vector.

According to $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, we have

$$\cos\theta = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}|\,|\boldsymbol{b}|}$$

Thinking about expression $\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$, then

$$\cos\theta = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}\sqrt{b_x^2 + b_y^2 + b_z^2}}$$

A Scalar Product will satisfy the following properties:

$$\left.\begin{aligned} &\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{b} \cdot \boldsymbol{a} \\ &\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{c} \\ &\lambda_1(\boldsymbol{a} \cdot \boldsymbol{b}) = (\lambda_1 \boldsymbol{a}) \cdot \boldsymbol{b} = \boldsymbol{a} \cdot (\lambda_1 \boldsymbol{b}) = (\boldsymbol{a} \cdot \boldsymbol{b})\lambda_1 \end{aligned}\right\} \tag{1.10}$$

### 1.2.4 Vector Product of vector(cross product, external product)

In the study of turning,moment is a physical quantity that is often used.

Supposed force $\boldsymbol{F}$ is working at a point on object $A$, a moment is produced to a fulcrum with a value that is equal to the product of quantity of the force and distance between $O$ and line of action. This is illustrated in Figure 1.3. Suppose the angle between $\boldsymbol{F}$ and $\overrightarrow{OA}$ is $\theta$, so the moment is:

$$\boldsymbol{T} = \overrightarrow{OA} \times \boldsymbol{F} \tag{1.11}$$

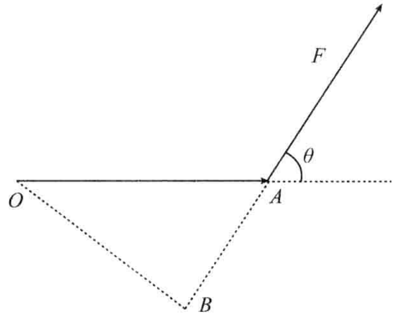

Figure 1.3 Force and moment

The moment is a vector, and its direction is perpendicular to the plane which is specifyed by $\boldsymbol{F}$ and $\overrightarrow{OA}$. And $\overrightarrow{OA}$, $\boldsymbol{F}$ and $\boldsymbol{T}$ meet right-hand system, means if the fingers of right hand are curled from $\overrightarrow{OA}$'s direction toward the control and $\boldsymbol{F}$, the thumb points in the direction of $\boldsymbol{T}$.

According to the above operation, we define another kind of vector operation: suppose there are two vector $\boldsymbol{a}$ and $\boldsymbol{b}$, their included angle is $\theta$, and vector $\boldsymbol{c}$ is vector product of vector $\boldsymbol{a}$ and $\boldsymbol{b}$, denoted as $\boldsymbol{a} \times \boldsymbol{b}$, its value equal to the product of two vector's magnitude and direction sine, namely

$$|\boldsymbol{c}| = |\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}| \sin\theta$$

Vector $\boldsymbol{c}$ is perpendicular to vector $\boldsymbol{a}$ and $\boldsymbol{b}$ , and vector $\boldsymbol{a}$,$\boldsymbol{b}$ and $\boldsymbol{c}$ form the right-handed system.

Vector product ia also named cross product or external product.

According to the above definition, the moment $\boldsymbol{T}$ made by force $\boldsymbol{F}$ also can be denoted as

$$\boldsymbol{T} = \overrightarrow{OA} \times \boldsymbol{F}$$

From the definition of vector product, we can see

$$\left.\begin{array}{l} \boldsymbol{a} \times \boldsymbol{a} = 0 \\ \boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a} \\ (\boldsymbol{a}+\boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{c} + \boldsymbol{b} \times \boldsymbol{a} \\ \lambda_1(\boldsymbol{a} \times \boldsymbol{b}) = (\lambda_1 \boldsymbol{a}) \times \boldsymbol{b} = \boldsymbol{a} \times (\lambda_1 \boldsymbol{b}) = (\boldsymbol{a} \times \boldsymbol{b})\lambda_1 \end{array}\right\} \tag{1.12}$$

We infer the following about a vector product expression with basic unit:

According to the vector computation rules,we get

$$\boldsymbol{a} \times \boldsymbol{b} = (a_x \boldsymbol{e}_x + a_y \boldsymbol{e}_y + a_z \boldsymbol{e}_z) \times (b_x \boldsymbol{e}_x + b_y \boldsymbol{e}_y + b_z \boldsymbol{e}_z)$$

Expand the expression, we get

$$\boldsymbol{a} \times \boldsymbol{b} = (a_y b_z - a_z b_y)\boldsymbol{e}_x + (a_z b_x - a_x b_z)\boldsymbol{e}_y + (a_x b_y - a_y b_x)\boldsymbol{e}_z \tag{1.13}$$

The cross product can be expressed in terms of a determinant as follows

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} \boldsymbol{e}_x + \begin{vmatrix} a_z & a_x \\ b_z & b_x \end{vmatrix} \boldsymbol{e}_y + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \boldsymbol{e}_x$$

or

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \tag{1.14}$$

***Example 1.1*** The vectors $\boldsymbol{a}$=$2\boldsymbol{e}_x - 6\boldsymbol{e}_y - 3\boldsymbol{e}_z$ and $\boldsymbol{b}$=$4\boldsymbol{e}_x$+$3\boldsymbol{e}_y$−$\boldsymbol{e}_z$ specify a plane. What is the unit normal vector?

**Method1** basic normal vector is perpendicular to this plane, so this vector must be perpendicular to vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. Suppose the unit vector $\boldsymbol{c}$=$c_x \boldsymbol{e}_x + c_y \boldsymbol{e}_y + c_z \boldsymbol{e}_z$, so

$$\boldsymbol{c} \cdot \boldsymbol{a} = 2c_x - 6c_y - 3c_z = 0$$

$$\boldsymbol{c} \cdot \boldsymbol{b} = 4c_x + 3c_y - c_z = 0$$

And the same, thinking about $\boldsymbol{c}$=$c_x \boldsymbol{e}_x + c_y \boldsymbol{e}_y + c_z \boldsymbol{e}_z$ is an unit vector, so $c_x^2 + c_y^2 + c_z^2 = 1$. Finally, the solution of this unit vector is

$$\boldsymbol{c} = \pm\left(\frac{3}{7}\boldsymbol{e}_x - \frac{2}{7}\boldsymbol{e}_y + \frac{6}{7}\boldsymbol{e}_z\right)$$

**Method2** because $\boldsymbol{a}\times\boldsymbol{b}$ is perpendicular to the plane specifid by $\boldsymbol{a}$ and $\boldsymbol{b}$, and that

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = 15\boldsymbol{e}_x - 10\boldsymbol{e}_y + 30\boldsymbol{e}_z$$

Because unit vector parallel $\boldsymbol{a}\times\boldsymbol{b}$, this unit vector is

$$\boldsymbol{c} = \frac{\boldsymbol{a} \times \boldsymbol{b}}{|\boldsymbol{a} \times \boldsymbol{b}|} = -\frac{3}{7}\boldsymbol{e}_x + \frac{2}{7}\boldsymbol{e}_y - \frac{6}{7}\boldsymbol{e}_z$$

The reserve direction unit vector is