TEXTBOOKS FOR HIGHER EDUCATION

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Matrix Theory with Applications

矩阵论及其应用

Peng Xiongqi

NORTHWESTERN POLYTECHNICAL UNIVERSITY PRESS
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Preface

This book is based on the lecture notes of the course "Matrix Theory and Its Application" given by the author for foreign graduate students at Northwestern Polytechnical University. The textbook is intended for graduate students in engineering and physics and puts an emphasis on the fundamental procedures underlying the applications of matrix theory in engineering fields.

The objective of the book is to give readers a working knowledge of linear algebra and matrix theory, enabling them to analyze engineering systems with the mathematical tools of matrix theory. Readers are expected to obtain skills ranging from the ability to perform insightful analyses and to the ability to develop algorithms for numerical/computer analyses with the combination of numerical calculus. In this latter regard, the book is also intended to serve as an independent study text and as a reference book for beginning graduate students and for practicing engineers.

The book is written to be readily accessible to students and readers having a background in mathematics through calculus. The book itself is divided into eight chapters. The first chapter provides introductory remarks and elementary row operation and solution of linear systems. The second chapter is devoted to matrix determinant and its properties. Chapter 3 discusses vector spaces and matrix ranks, with the last of these focusing upon linear independence. Chapter 4 provides a comprehensive review of linear transformation including its matrix representation and similarity. Fundamental principles of eigenvalue and eigenvector are presented in Chapter 5, along with diagonalization of matrices, exponential of matrices and some applications. Chapter 6 introduces orthogonal subspace and how to obtain orthonormal set with Gram-Schmidt orthogonalization process. Chapter 7 presents an introduction to some special matrices including Hermitian matrices and positive definite matrices etc. The last chapter introduces Singular Value Decomposition and Jordan forms of matrices.

This book's presentation emphasizes motivation and naturalness, driven home by a wide variety of examples and by extensive and careful exercises. Application and illustrative examples are discussed and presented in each chapter, and exercises are provided at the end of each chapter. Although the earlier chapters provide the basis for the latter chapters, each chapter is written to be as self-contained as possible.

As a professor in mechanical engineering, frankly, it is beyond the author's ability to write such a mathematic book for graduate students. However, as the lecturer for the course of "Matrix Theory and Its Application", the author feels that a textbook concentrating on engineering application is in great need for students to better understand matrix theory. So this book is mainly based on the lecture notes with many contents referred from the internet. The author has endeavored to make the book be systematic and hope it can provide some useful references for the students.

The author would like to thank his many former students including Faisal Mahmood, Wajed Zaman, Abd Elmeraim Mohammed, Gribi Abd ellah, Faisal Mushtaq, Afzaal Hassan, Sohail Ahmad, Sabeeh Ahmed, Mubashar Ahmed, Naveed Iqbal Gondal, Waseem Shahzad and many others who have both directly and indirectly contributed to the content of this book.

Special thanks go to Zia-Ur-Rehman and Yu Wang for their contribution on preparation of exercises and proofreading. Any remaining errors are, of course, the responsibility of the author.

In closing, the author wishes to express his sincere gratitude and appreciation to the International Office of Northwestern Polytechnical University. The book could not have been finished without the help and encouragement from the faculty and staff of the International Office.

Peng Xiongqi
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Xi'an, 2011

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Chapter 1 Matrix Basics

1.1 Matrix Definition

Matrices and Determinants were discovered and developed in the eighteenth and nineteenth century. Initially, their development dealt with transformation of geometric objects and solution of systems of linear equations. Matrices provide a theoretically and practically useful way of approaching many types of problems including: Solution of Systems of Linear Equations, Graph Theory, Theory of Games, Computer Graphics, Cryptography, Electrical Networks ..., etc.

Recall the curve-fitting problem: Given three points (x_1, y_1), (x_2, y_2), (x_3, y_3), find a polynomial of degree 2 passing through the three given points.

Let the polynomial be $y(x) = ax^2 + bx + c$, where a, b and c are to be determined, the solution of the problem can be written as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{1.1}$$

Eq. (1.1) can be transformed to a matrix format as

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{1.2}$$

By transforming to a matrix format, we can present the linear system equation in Eq. (1.1) with a systematic way which will be very convenient for usage with computer sources.

A matrix is simply a rectangular array of elements arranged in rows and columns. The elements can be symbolic expressions or numbers. Matrix $\boldsymbol{A}$ is denoted by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \quad (1.3)$$

Each matrix has rows and columns and this defines the size of the matrix. If a matrix $\boldsymbol{A}$ has "m" rows and "n" columns, the "size of the matrix" is denoted by $m\times n$. The matrix $\boldsymbol{A}$ may also be denoted by $[\boldsymbol{A}]_{m\times n}$ to show that $\boldsymbol{A}$ is a matrix with m rows and n columns. Each entry in the matrix is called the entry or element of the matrix and is denoted by "a_{ij}" where "i" is the row number and "j" is the column number of the element.

A vector is a special type of matrix that has only one row (called a row vector) or one column (called a column vector). Below, $\boldsymbol{u}$ is a column vector while $\boldsymbol{v}$ is a row vector:

$$\boldsymbol{u} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad \boldsymbol{v} = [1 \quad 4 \quad 2]$$

1.2 Type of Matrices

1.2.1 Square matrix

If the number of rows (m) of a matrix is equal to the number of columns (n) of the matrix ($m = n$), it is called a square matrix. The entries $a_{11}, a_{22}, \cdots, a_{nn}$ are called the diagonal elements of a square matrix. Sometimes the diagonal of the matrix is also called the principal or main of the matrix, for example

$$\begin{bmatrix} 5 & 0 & 0.3 \\ 0.2 & 13 & 0 \\ -4 & 6 & -1 \end{bmatrix}$$

The sum of the diagonal entries of a square matrix $\boldsymbol{A}$ is called the trace of the matrix, that is,

$$\mathrm{tr}\boldsymbol{A} = \sum_{i=1}^{n} a_{ii} \quad (1.4)$$

1.2.2 Upper triangular matrix

A square matrix for which $a_{ij}=0$, $i>j$ is called an upper triangular matrix. That is, all the elements below the diagonal entries are zero, for example

$$A=\begin{bmatrix}9 & 3 & 6\\ 0 & 7 & 9\\ 0 & 0 & 2\end{bmatrix}$$

1.2.3 Lower triangular matrix

A square matrix for which $a_{ij}=0$, $j>i$ is called a lower triangular matrix. That is, all the elements above the diagonal entries are zero, for example

$$A=\begin{bmatrix}9 & 0 & 0\\ 1 & 7 & 0\\ 0 & 7 & 2\end{bmatrix}$$

1.2.4 Diagonal matrix

A square matrix with all non-diagonal elements equal to zero is called a diagonal matrix. That is, only the diagonal entries of the square matrix can be non-zero ($a_{ij}=0$, $j\neq i$), for example

$$A=\begin{bmatrix}9 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 2\end{bmatrix}$$

1.2.5 Identity matrix

One especially important diagonal matrix is termed the identity matrix. The identity matrix is, of course, always a square matrix, and its diagonal elements are all ones, while its off-diagonal elements are all zeros. A diagonal matrix with all diagonal elements equal to one is called an identity matrix ($a_{ij}=0$, $j\neq i$; and $a_{ii}=0$ for all i). We denote it as, $I_n=(\delta_{ij})$, the entries δ_{ij} is the Kronecker delta:

$$\delta_{ij}=\begin{cases}1 & \text{if } i=j\\ 0 & \text{otherwise}\end{cases} \tag{1.5}$$

A 4×4 identity matrix would thus look like this:

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The identity matrix is the functional equivalent of the number 1, because multiplying a matrix by its identity matrix yields the same matrix, i. e. , $\boldsymbol{AI} = \boldsymbol{A} = \boldsymbol{IA}$.

1.2.6 Zero matrix

A matrix whose all entries are zero is called a zero matrix ($a_{ij} = 0$ for all i and j).

1.3 Matrix Operations

1.3.1 Matrix addition and subtraction

Two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ can be added only if they are of the same size, in other words, share the same dimensionality. If they do, they are said to be conformable for addition. If not, they are non-conformable. The addition is shown as

$$\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B} \quad \text{where} \quad c_{ij} = a_{ij} + b_{ij} \tag{1.6}$$

meaning you simply sum the corresponding elements of $\boldsymbol{A}$ and $\boldsymbol{B}$ to get the elements of $\boldsymbol{C}$. Thus, if

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 4 & 5 & -6 \\ -7 & 8 & 9 \\ 1 & -2 & 3 \end{bmatrix}$$

Summing the two matrices yields

$$\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \\ a_{31} + b_{31} & a_{32} + b_{32} & a_{33} + b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} 1+4 & 2+5 & 3-6 \\ 4-7 & 5+8 & 6+9 \\ 7+1 & 8-2 & 9+3 \end{bmatrix} = \begin{bmatrix} 5 & 7 & -3 \\ -3 & 13 & 15 \\ 8 & 6 & 12 \end{bmatrix}$$

Matrix subtraction works in the same way, except that elements are subtracted instead of added. So for the subtraction $\boldsymbol{C} = \boldsymbol{A} - \boldsymbol{B}$, you simply subtract the corresponding elements $c_{ij} = a_{ij} - b_{ij}$.

1.3.2 Vector multiplication

Vectors are multiplied just like any other matrix. This is shown in the following example:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \sum_{i=1}^{3} a_i b_i$$

In this case, we are multiplying a row vector times a column vector. The results are a scalar quantity. It is important to note that this product can only be obtained if $\boldsymbol{a}$ and $\boldsymbol{b}$ have the same number of entries. If they have a different number of entries then this product is not defined. We cannot multiply two row or two column vectors.

1.3.3 Scalar multiplication

In linear algebra, individual numbers are referred to as scalars, from the Latin word for ladder. Multiplication of a matrix by a scalar proceeds element-by-element, with the scalar being multiplied by each element in turn. Mathematically, we express multiplication of a matrix $\boldsymbol{A}$ by a scalar as

$$c\boldsymbol{A} = c[a_{ij}] = [ca_{ij}]$$

where c is the scalar. Thus, multiplication by the scalar 3 is accomplished as:

$$3\begin{bmatrix} 3 & 1 \\ 1.7 & 2 \end{bmatrix} = \begin{bmatrix} 3\times 3 & 3\times 1 \\ 3\times 1.7 & 3\times 2 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ 5.1 & 6 \end{bmatrix}$$

1.3.4 Matrix vector multiplication

A vector can be multiplied by a matrix. The result is a vector as shown below:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_{11}b_1 + a_{12}b_2 + a_{13}b_3 \\ a_{21}b_1 + a_{22}b_2 + a_{23}b_3 \\ a_{31}b_1 + a_{32}b_2 + a_{33}b_3 \end{bmatrix}$$

Here the result is a column vector. We can also multiply a vector by a matrix.

1.3.5 Multiplication of matrices

Multiplication of one matrix by another is more complicated than scalar multiplication, and is carried out in accordance with a strict rule. Two matrices **A** and **B** can be multiplied only if the column numbers of **A** is equal to the row numbers of **B** to give

$$[\boldsymbol{C}]_{m\times n} = [\boldsymbol{A}]_{m\times p}\ [\boldsymbol{B}]_{p\times n}$$

If the two inner numbers are equal then the product is defined and the size of the product will be given by the outside numbers. So how does one calculate the elements of **C** matrix?

$$c_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj} \tag{1.7}$$

for each $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. The choice of the row and column to be used in the multiplication and summation is based on which element of the product matrix **C** that you wish to calculate:

• The left-hand matrix row you work with is the same as the row of the product matrix element you wish to calculate.

• The right-hand matrix column you work with is the same as the column of the product matrix element you wish to calculate.

For example, suppose you define the matrix **C** as the product of the two 3×3 matrices, **A** and **B**, shown above. If you wish to calculate the value of c_{11},

$$\begin{bmatrix} \textcircled{c_{11}} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

You work element-by-element across the first row of the left-hand matrix and element-by-element down the first column of the right-hand matrix as follows:

$$c_{11} = \begin{bmatrix} \textcircled{a_{11}} & \textcircled{a_{12}} & \textcircled{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \textcircled{b_{11}} & b_{12} & b_{13} \\ \textcircled{b_{21}} & b_{22} & b_{23} \\ \textcircled{b_{31}} & b_{32} & b_{33} \end{bmatrix} \Rightarrow c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$$

Similarly, to calculate the value of c_{23}, you work across the second row of the left-hand matrix and down the third column of the right-hand matrix:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \textcircled{c_{23}} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$c_{23}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \textcircled{a_{21}} & \textcircled{a_{22}} & \textcircled{a_{23}} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} & \textcircled{b_{13}} \\ b_{21} & b_{22} & \textcircled{b_{23}} \\ b_{31} & b_{32} & \textcircled{b_{33}} \end{bmatrix} = c_{23}=a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33}$$

1.3.6 Multiplication by the identity matrix

The identity matrix is the matrix equivalent of the number 1 because the result of multiplication of any matrix by its corresponding identity matrix is simply the matrix itself. That is, for any matrix $\boldsymbol{A}$, $\boldsymbol{AI}=\boldsymbol{A}$, $\boldsymbol{IA}=\boldsymbol{A}$.

1.3.7 The inverse of a matrix

For any square matrix $\boldsymbol{A}$, if there exists a matrix $\boldsymbol{X}$ of the same order such that

$$\boldsymbol{XA}=\boldsymbol{AX}=\boldsymbol{I} \tag{1.8}$$

then we call $\boldsymbol{X}$ the inverse of $\boldsymbol{A}$ and denote it by $\boldsymbol{A}^{-1}$. The inverse of a matrix, if exists, must be unique. If $\boldsymbol{A}$ has an inverse then we say $\boldsymbol{A}$ is invertible or nonsingular, otherwise we say $\boldsymbol{A}$ is singular.

1.3.7 Powers of matrices

We will confine ourselves to the situation where the power is an integer, positive or negative, and proceed by first recalling that the nth power (n a positive integer) of a number or a variable is simply the number multiplied by itself $n-1$ times. Similarly, the nth power (n a negative integer) of a number or a variable is simply the reciprocal, or inverse, of its nth power (n positive). It's pretty much the same when working with matrices. If $\boldsymbol{A}$ is a square matrix, then

$$\boldsymbol{A}^3=\boldsymbol{A}\cdot\boldsymbol{A}\cdot\boldsymbol{A}$$
$$\boldsymbol{A}^{-3}=(\boldsymbol{A}^{-1})^3=1/\boldsymbol{A}^3$$

1.3.9 The transpose of a matrix

The transpose of a matrix is an important concept that is frequently encountered when working with matrices, and is represented by A^T. Operationally, the transpose of a matrix is created by "converting" its rows into the corresponding columns of its transpose, meaning the first row of a matrix becomes the first column of its transpose, the second row becomes the second column, and so on. Thus, if

$$\boldsymbol{A}=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$$

the transpose of $\boldsymbol{A}$ is

$$\boldsymbol{A}^T=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}^T=\begin{bmatrix}1&4&7\\2&5&8\\3&6&9\end{bmatrix}$$

A square matrix $\boldsymbol{A}$ with real elements where $a_{ij}=a_{ji}$ for $i=1,\cdots,n$ and $j=1,\cdots,n$ is called a symmetric matrix. This is same as, if $\boldsymbol{A}=\boldsymbol{A}^T$, then $\boldsymbol{A}$ is a symmetric matrix:

$$\boldsymbol{A}=\begin{bmatrix}1&2&3\\2&5&8\\3&8&9\end{bmatrix}$$

A square matrix $\boldsymbol{A}$ with real elements where $a_{ij}=-a_{ji}$ for $i=1,\cdots,n$ and $j=1,\cdots,n$ is called a skew-symmetric matrix or anti-symmetric matrix. This is same as, if $\boldsymbol{A}=-\boldsymbol{A}^T$, then $\boldsymbol{A}$ is a skew-symmetric matrix:

$$\boldsymbol{A}=\begin{bmatrix}0&2&3\\-2&0&8\\-3&-8&0\end{bmatrix}$$

Since $a_{ii}=-a_{ii}$ only if $a_{ii}=-a_{ii}$, all the diagonal elements of a skew symmetric matrix have to be zero.

The complex conjugate of matrix $\boldsymbol{A}$, denoted by $\bar{\boldsymbol{A}}$, is the matrix formed by taking complex conjugate of $\boldsymbol{A}$ entrywise.

The conjugate transpose (or Hermitian transpose) of $\boldsymbol{A}$, denoted by $\boldsymbol{A}^H$, is the $(m \times n)$ matrix whose (i, j) entry is the complex conjugate $\bar{a}_{ji}$ of the (j, i) entry of $\boldsymbol{A}$.

1.4 Properties of Matrix Operations

1.4.1 Commutative law of addition

If $\boldsymbol{A}$ and $\boldsymbol{B}$ are $m \times n$ matrices, then

$$\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A} \tag{1.9}$$

1.4.2 Associate law of addition

If $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ all are $m \times n$ matrices, then

$$\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})=(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C} \tag{1.10}$$

1.4.3 Associate law of multiplication

If $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are $m \times n$, $n \times p$ and $p \times r$ size matrices, respectively, then

$$\boldsymbol{A}(\boldsymbol{B}\boldsymbol{C})=(\boldsymbol{A}\boldsymbol{B})\boldsymbol{C} \tag{1.11}$$

1.4.4 Distributive law

If $\boldsymbol{A}$ and $\boldsymbol{B}$ are $m \times n$ size matrices, and $\boldsymbol{C}$ and $\boldsymbol{D}$ are $n \times p$ size matrices, then

$$\boldsymbol{A}(\boldsymbol{C}+\boldsymbol{D})=\boldsymbol{A}\boldsymbol{C}+\boldsymbol{A}\boldsymbol{D} \tag{1.12}$$

$$(\boldsymbol{A}+\boldsymbol{B})\boldsymbol{C}=\boldsymbol{A}\boldsymbol{C}+\boldsymbol{B}\boldsymbol{C} \tag{1.13}$$

and the resulting matrix size on both sides is $m \times p$.

Is $\boldsymbol{AB}=\boldsymbol{BA}$?

First both operations $\boldsymbol{AB}$ and $\boldsymbol{BA}$ are only possible if $\boldsymbol{A}$ and $\boldsymbol{B}$ are square matrices of the same size. Why? If $\boldsymbol{AB}$ exists, number of columns of $\boldsymbol{A}$ has to be the same as the number of rows of $\boldsymbol{B}$ and if $\boldsymbol{BA}$ exists, number of columns of $\boldsymbol{B}$ has to be the same as the number of rows of $\boldsymbol{A}$. Even then in general $\boldsymbol{AB} \neq \boldsymbol{BA}$.

Example:

$$\boldsymbol{A}=\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}, \quad \boldsymbol{B}=\begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}$$