Graduate Texts in Mathematics

Reinhold Remmert

Classical Topics in Complex Function Theory

复函数论中的经典论题

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Translated by Leslie Kay

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Reinhold Remmert
Mathematisches Institut
Westfälische Wilhelms–Universität Münster
Einsteinstrasse 62
Münster D-48149
Germany

Leslie Kay (Translator)
Department of Mathematics
Virginia Polytechnic Institute
and State University
Blacksburg, VA 24061-0123
USA

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Preface

Preface to the Second German Edition

In addition to the correction of typographical errors, the text has been materially changed in three places. The derivation of Stirling's formula in Chapter 2, §4, now follows the method of Stieltjes in a more systematic way. The proof of Picard's little theorem in Chapter 10, §2, is carried out following an idea of H. König. Finally, in Chapter 11, §4, an inaccuracy has been corrected in the proof of Szegö's theorem.

Oberwolfach, 3 October 1994 *Reinhold Remmert*

Preface to the First German Edition

> Wer sich mit einer Wissenschaft bekannt machen will, darf nicht nur nach den reifen Früchten greifen — er muß sich darum bekümmern, wie und wo sie gewachsen sind. (Whoever wants to get to know a science shouldn't just grab the ripe fruit — he must also pay attention to how and where it grew.)
>
> — J. C. Poggendorf

Presentation of function theory with vigorous connections to historical development and related disciplines: This is also the leitmotif of this second volume. It is intended that the reader experience function theory personally and participate in the work of the creative mathematician. Of course, the scaffolding used to build cathedrals cannot always be erected afterwards; but a textbook need not follow Gauss, who said that once a good building is completed its scaffolding should no longer be seen.[1] Sometimes even the framework of a smoothly plastered house should be exposed.

The edifice of function theory was built by Abel, Cauchy, Jacobi, Riemann, and Weierstrass. Many others made important and beautiful contributions; not only the work of the kings should be portrayed, but also the life of the nobles and the citizenry in the kingdoms. For this reason, the bibliographies became quite extensive. But this seems a small price to pay. "Man kann der studierenden Jugend keinen größeren Dienst erweisen als wenn man sie zweckmäßig anleitet, sich durch das Studium der Quellen mit den Fortschritten der Wissenschaft bekannt zu machen." (One can render young students no greater service than by suitably directing them to familiarize themselves with the advances of science through study of the sources.) (letter from Weierstrass to Casorati, 21 December 1868)

Unlike the first volume, this one contains numerous glimpses of the function theory of several complex variables. It should be emphasized how independent this discipline has become of the classical function theory from which it sprang.

In citing references, I endeavored — as in the first volume — to give primarily original works. Once again I ask indulgence if this was not always successful. The search for the first appearance of a new idea that quickly becomes mathematical folklore is often difficult. The *Xenion* is well known:

> Allegire der Erste nur falsch, da schreiben ihm zwanzig
> Immer den Irrthum nach, ohne den Text zu besehn. [2]

The selection of material is conservative. The Weierstrass product theorem, Mittag-Leffler's theorem, the Riemann mapping theorem, and Runge's approximation theory are central. In addition to these required topics, the reader will find

- Eisenstein's proof of Euler's product formula for the sine;
- Wielandt's uniqueness theorem for the gamma function;
- an intensive discussion of Stirling's formula;
- Iss'sa's theorem;

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[1] Cf. W. Sartorius von Waltershausen: *Gauß zum Gedächtnis*, Hirzel, Leipzig 1856; reprinted by Martin Sändig oHG, Wiesbaden 1965, p. 82.

[2] Just let the first one come up with a wrong reference, twenty others will copy his error without ever consulting the text. [The translator is grateful to Mr. Ingo Seidler for his help in translating this couplet.]

- Besse's proof that all domains in $\mathbb{C}$ are domains of holomorphy;
- Wedderburn's lemma and the ideal theory of rings of holomorphic functions;
- Estermann's proofs of the overconvergence theorem and Bloch's theorem;
- a holomorphic imbedding of the unit disc in $\mathbb{C}^3$;
- Gauss's expert opinion of November 1851 on Riemann's dissertation.

An effort was made to keep the presentation concise. One worries, however:

> Weiß uns der Leser auch für unsre Kürze Dank?
> Wohl kaum? Denn Kürze ward durch Vielheit leider! lang. [3]

Oberwolfach, 3 October 1994 *Reinhold Remmert*

[3] Is the reader even grateful for our brevity? Hardly? For brevity, through abundance, alas! turned long.

Gratias ago

It is impossible here to thank by name all those who gave me valuable advice. I would like to mention Messrs. R. B. Burckel, J. Elstrodt, D. Gaier, W. Kaup, M. Koecher, K. Lamotke, K.-J. Ramspott, and P. Ullrich, who gave their critical opinions. I must also mention the Volkswagen Foundation, which supported the first work on this book through an academic stipend in the winter semester 1982–83.

Thanks are also due to Mrs. S. Terveer and Mr. K. Schlöter. They gave valuable help in the preparatory work and eliminated many flaws in the text. They both went through the last version critically and meticulously, proofread it, and compiled the indices.

Advice to the reader. Parts A, B, and C are to a large extent mutually independent. A reference 3.4.2 means Subsection 2 in Section 4 of Chapter 3. The chapter number is omitted within a chapter, and the section number within a section. Cross-references to the volume *Funktionentheorie I* refer to the *third* edition 1992; the Roman numeral I begins the reference, e.g. I.3.4.2.[4] No later use will be made of material in small print; chapters, sections and subsections marked by * can be skipped on a first reading. Historical comments are usually given after the actual mathematics. Bibliographies are arranged at the end of each chapter (occasionally at the end of each section); page numbers, when given, refer to the editions listed.

Readers in search of the older literature may consult A. Gutzmer's German-language revision of G. Vivanti's *Theorie der eindeutigen Funktionen*, Teubner 1906, in which 672 titles (through 1904) are collected.

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[4] [In this translation, references, still indicated by the Roman numeral I, are to *Theory of Complex Functions* (Springer, 1991), the English translation by R. B. Burckel of the second German edition of *Funktionentheorie I*. Trans.]

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