

Alex Hubert,
Rudolf Schäfer

Magnetic Domains

The Analysis
of Magnetic Microstructures

磁畴



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Professor Dr. Alex Hubert †

Universität Erlangen-Nürnberg
Institut für Werkstoffwissenschaften
Lehrstuhl Werkstoffe der Elektrotechnik
Martensstrasse 7
91058 Erlangen
Germany
e-mail: hubert@ww.uni-erlangen.de

Dr. Rudolf Schäfer

Institut für Festkörper- und Werkstoffforschung
Helmholtzstrasse 20
01069 Dresden
Germany
e-mail: r.schaefer@ifw-dresden.de

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Preface

Magnetic domains are the elements of the *microstructure* of magnetic materials that link the basic physical properties of a material with its macroscopic properties and applications. The analysis of magnetization curves requires an understanding of the underlying domains. In recent years there has been a rising interest in domain analysis, probably due to the increasing perfection of materials and miniaturization of devices. In small samples, measurable domain effects can arise, which tend to average out in larger samples. This book is intended to serve as a reference for all those who are confronted with the fascinating world of magnetic domains. Naturally, domain pictures form an important part of this book.

After a historical introduction (Chap. 1) emphasis is laid on a thorough discussion of domain observation techniques (Chap. 2) and on domain theory (Chap. 3). No domain analysis is possible without knowledge of the relevant material parameters. Their measurement is reviewed in Chap. 4. The detailed discussion of the physical mechanisms and the behaviour of the main types of observed domain patterns in Chap. 5 is subdivided according to the crystal symmetry of the samples, according to the relative strength of induced or crystal anisotropies, and according to the sample dimensions. The last chapter deals with the technical relevance of magnetic domains, which is different for the different fields in which magnetic materials are applied. The discussion reaches from soft magnetic materials forming the core of electrical machines that would not work without the hidden action of magnetic domains, to magnetic sensor elements used in magnetic recording systems, which may suffer from domain-related noise effects.

We emphasize in this book (in Chaps. 5, 6) the analysis of actual magnetization processes, in particular of those that lead to discontinuities and irreversibilities in the magnetization curve. This approach is adequate for several important application areas, such as magnetic sensors and transformer materials.

In other areas, such as those of bulk polycrystalline soft-magnetic materials, the link between observed elementary processes and possible technical descriptions of the hysteresis phenomena is difficult to establish. Formal descriptions of hysteresis phenomena — shortly introduced by reference to actual textbooks in the last section of Chap. 6 — use abstract concepts of magnetic domains in their justification. Whether the discrepancy between the simplified assumptions of hysteresis theories and the complexities of actual domain behaviour is technically relevant remains an interesting open question.

Previous books and reviews touching on the subject of magnetic domains and magnetization processes are listed separately in alphabetic order at the end of the reference list. The few recent works among these cover important, but on the whole rather specialized aspects. One of the aims of this book is to collect the knowledge available from the investigations of many magnetic materials. Although the technical areas of electrical steels, of high frequency cores, of permanent magnets, or of computer storage media have little in common, the magnetic microstructures in most of these materials obey the same rules and differ only in quantitative, not in qualitative aspects.

Despite the preference of many magneticians for the old Gaussian system of units, standard SI units are used throughout. A reference to the old system is made only occasionally. But we prefer to reduce equations to dimensionless units anyway in all practical examples, so that results get a broader range of applicability, and at the same time the mathematical expressions become independent of the unit system.

We take one liberty, however. The magnetic dipole density of a material is given by the vector field $\mathbf{J}(\mathbf{r})$ (measured in Tesla or Vs/m^2). Its official name is *magnetic polarization*, but very often and also in this book, it is called simply *magnetization*, although in the strict SI system this name is reserved to a quantity $\mathbf{M}(\mathbf{r})$ that is measured in A/m and related to $\mathbf{J}$ by $\mathbf{M} = \mathbf{J}/\mu_0$. We will never use the latter quantity, so no confusion is possible. Our choice agrees with the recommendation from P.C. Scholten¹, except that he would prefer to use the abbreviation $\mathbf{M}$ instead of $\mathbf{J}$, which we think might lead to confusion². Anyway, in most cases we use the reduced unit vector of magnetization direction only, and we abbreviate this vector field by $\mathbf{m}(\mathbf{r})$.

The careful reader will discover quite a few original contributions in this book, not published elsewhere. They are intended to improve the grasp of

¹ P.C. Scholten: "Which SI?", J. Magn. Magn. Mat. **149**, 57–59 (1995)

² The fundamental material equation is thus expressed in this book in the form $\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{J}$

otherwise abstract concepts and check their applicability. They also help to fill gaps in the published material. Such gaps may be too small or unimportant to justify an independent scientific publication, but they may form a serious obstacle for newcomers in their attempt to enter the field.

A few hints to the reader: Up to five text organization levels are used. Three of them (chapter, section, subsection) are systematically numbered in a decimal classification. They appear (perhaps in an abridged way) in the right-hand page headers to facilitate orientation. A further subdivision is often needed. It is marked and referred to within the same subsection by “(A), (B) etc.”. In references from other subsections a small capital letter (like in “Sect. 3.4.1A”) is appended. Where needed, an informal fifth level, marked “(i), (ii) etc.”, is added. Parenthesis with numbers like “(3.1)” always refer to equations, while brackets like “[212]” indicate citations. Parenthesis with lower case letters like “(a)” refer to parts of a figure mentioned in the same paragraph.

Some passages in Chap. 3 on domain theory are addressed more to the interested specialist than to the general reader. They are marked by smaller print. All references are collected in the order of occurrence after the text part. Orientation within the references section is supported by an author index that comprises all authors and coauthors of all cited references (explicit text references are listed first). In addition to the regular, numbered and captioned figures we offer integrated “sketches”, that are intended to facilitate reading, but which need no further explanation (thanks to John Chapman for suggesting this tool). The few tables are numbered and treated as if they were figures, which makes it easier to find them.

Erlangen, Dresden, July 1. 1998

Alex Hubert
Rudolf Schäfer

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Prof. Dr. Alex Hubert
Institut für Werkstoffwissenschaften
der Universität Erlangen-Nürnberg
Lehrstuhl Werkstoffe der Elektrotechnik
Martensstr. 7
D-91058 Erlangen
Germany

Dr. Rudolf Schäfer
Institut für Festkörper- und Werkstoffforschung
Helmholtzstr. 20
D-01069 Dresden
Germany
email: r.schaefer@ifw-dresden.de

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