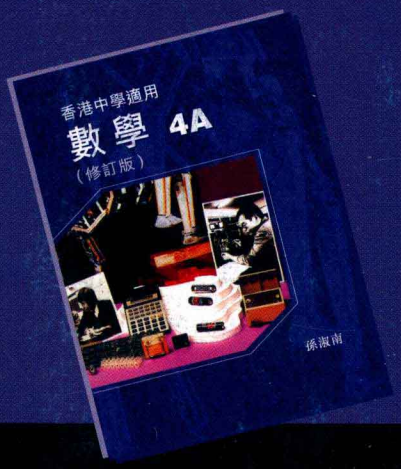
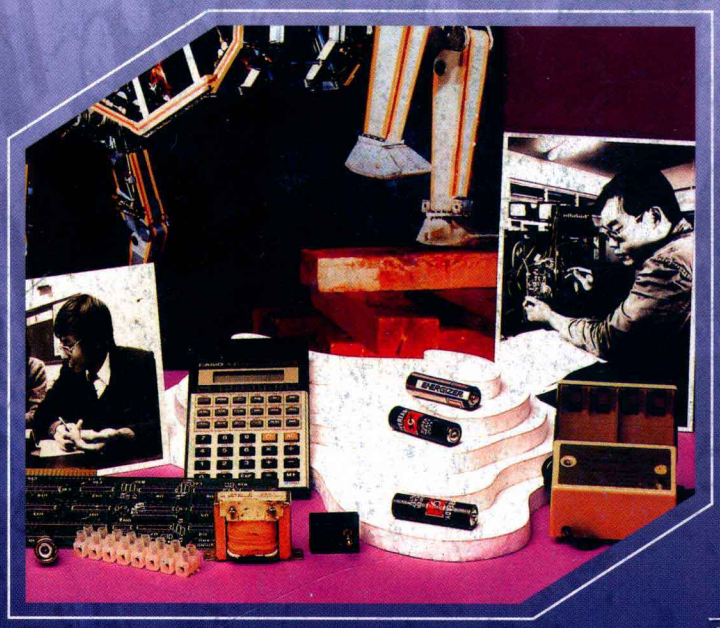


香港中學適用

數學 4A

（修訂版）

學習指引



孫淑南

為課本提供

習題內單數題目的詳細題解

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數學 4A

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學習指引

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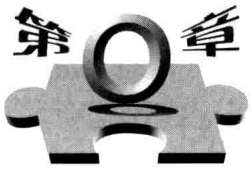
基於香港考試局的規定，本書不會提供有關習題中香港中學會考試題的題解。



指數、恆等式和對數

練習 1 (第 5 頁)

1. **(a)** $4 \cdot 4^5 = 4^{1+5} = \underline{\underline{4^6}}$

(b) $6^3 \cdot 6^2 \cdot 6^{-1} = 6^{3+2-1} = \underline{\underline{6^4}}$

3. **(a)** $\dfrac{x^{12}}{x^5} = x^{12-5} = \underline{\underline{x^7}}$

(b) $\dfrac{3^4 \cdot 3^8}{3^{10}} = 3^{4+8-10} = \underline{\underline{3^2}}$

5. **(a)** $3x^2 \cdot 2x^{-2} = 3 \cdot 2x^{2-2} = 6x^0 = \underline{\underline{6}}$

(b) $\dfrac{x^{-2}}{y^{-1}} = \dfrac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}} = \underline{\underline{\dfrac{y}{x^2}}}$

7. **(a)** $\dfrac{a}{\sqrt[3]{a}} = \dfrac{a}{a^{\frac{1}{3}}} = a^{1-\frac{1}{3}} = \underline{\underline{a^{\frac{2}{3}}}}$

(b) $\left(\sqrt[3]{a^3 b}\right)^5 = \left[(a^3 b)^{\frac{1}{3}}\right]^5$

$= \left(a^{3 \times \frac{1}{3}} b^{\frac{1}{3}}\right)^5$

$= \left(a b^{\frac{1}{3}}\right)^5$

$= a^5 b^{\frac{1}{3} \times 5}$

$= \underline{\underline{a^5 b^{\frac{5}{3}}}}$

9. $4m^{1-2x} \div m^{-2x} = \dfrac{4m^{1-2x}}{m^{-2x}}$

$= 4m^{1-2x-(-2x)}$

$= \underline{\underline{4m}}$

11. $x^{\frac{1}{4}} = 3$

$\therefore \quad \left(x^{\frac{1}{4}}\right)^4 = 3^4$

$\therefore \quad x = \underline{\underline{81}}$

13. $7^{2x+1} = 7^3$

$\therefore \quad 2x + 1 = 3$

$\therefore \quad x = \underline{\underline{1}}$

15. **(a)** $(-3)^3(-3)^5(-3)^7 = (-3)^{3+5+7}$

$= (-3)^{15}$

$= (-1)^{15} \times 3^{15}$

$= \underline{\underline{-3^{15}}}$

(b) $(-a)^6 \cdot (-a^6) \cdot a^6$

$= (-1)^6 \cdot a^6 \cdot (-1) \cdot a^6 \cdot a^6$

$= (-1)^{6+1} a^{6+6+6}$

$= \underline{\underline{-a^{18}}}$

17. **(a)** $(-2x^2)^3(3y^3)^2 = (-2)^3(x^2)^3(3)^2(y^3)^2$

$= (-8)(9)x^6 y^6$

$= \underline{\underline{-72x^6 y^6}}$

(b) $[(-2p)^2]^3(-p^2)^0 = [(-2)^2 p^2]^3(1)$

$= (-2)^6 p^6$

$= \underline{\underline{64p^6}}$

練習 I 題解（續）

19. $3^x \cdot 2^{2x} \cdot (6^x)^{-1} = 3^x \cdot 2^{2x} \cdot 6^{-x}$

$= 3^x \cdot 2^{2x} \cdot (2 \cdot 3)^{-x}$

$= 3^x \cdot 2^{2x} \cdot 2^{-x} \cdot 3^{-x}$

$= 3^{x-x} \cdot 2^{2x-x}$

$= 3^0 \cdot 2^x$

$= \underline{\underline{2^x}}$

21. $4x^{-\frac{1}{3}} = 64^{\frac{2}{3}}$

$4x^{-\frac{1}{3}} = (4^3)^{\frac{2}{3}}$

$x^{-\frac{1}{3}} = \left(4^{3 \times \frac{2}{3}}\right) \times \frac{1}{4}$

$= 4^2 \times 4^{-1}$

$= 4^{2-1}$

$= 4$

$\therefore \left(x^{-\frac{1}{3}}\right)^{-3} = 4^{-3}$

$\therefore x = \underline{\underline{\frac{1}{64}}}$

23. $4^{3x+2} = 256(8^{1-x})$

$2^{6x+4} = 2^8(2^3)^{1-x}$

$2^{6x+4} = 2^{8+3(1-x)}$

$2^{6x+4} = 2^{11-3x}$

$\therefore 6x + 4 = 11 - 3x$

$9x = 7$

$\therefore x = \underline{\underline{\frac{7}{9}}}$

25. $2^{x+2} - 6 \cdot 2^{x-1} - 8 = 0$

$2^2 \cdot 2^x - 6 \cdot 2^{-1} \cdot 2^x - 8 = 0$

$4 \cdot 2^x - 3 \cdot 2^x - 8 = 0$

$2^x = 8$

$2^x = 2^3$

$\therefore x = \underline{\underline{3}}$

練習 II (第 9 頁)

1. (a) $x^2 - 144y^2 = x^2 - (12y)^2$

$= \underline{\underline{(x + 12y)(x - 12y)}}$

(b) $225z^2 - 9 = 9(25z^2 - 1)$

$= 9[(5z)^2 - 1^2]$

$= \underline{\underline{9(5z + 1)(5z - 1)}}$

3. (a) $(2x + 1)^2 = (2x)^2 + 2(2x)(1) + 1^2$

$= \underline{\underline{4x^2 + 4x + 1}}$

(b) $(1 - 5y)^2 = 1^2 - 2(1)(5y) + (5y)^2$

$= \underline{\underline{1 - 10y + 25y^2}}$

5. (a) $25x^2 - 10xy + y^2 = (5x)^2 - 2(5x)(y) + y^2$

$= \underline{\underline{(5x - y)^2}}$

(b) $16u^2 + 72uv + 81v^2$

$= (4u)^2 + 2(4u)(9v) + (9v)^2$

$= \underline{\underline{(4u + 9v)^2}}$

7. (a) $18m^2 - 12mn + 2n^2$

$= 2(9m^2 - 6mn + n^2)$

$= 2[(3m)^2 - 2(3m)(n) + n^2]$

$= \underline{\underline{2(3m - n)^2}}$

(b) $x + 8x^2 + 16x^3$

$= x(16x^2 + 8x + 1)$

$= x[(4x)^2 + 2(4x)(1) + 1^2]$

$= \underline{\underline{x(4x + 1)^2}}$

9. (a) $4x^2 - 25(y - z)^2$

$= (2x)^2 - [5(y - z)]^2$

$= [2x + 5(y - z)][2x - 5(y - z)]$

$= \underline{\underline{(2x + 5y - 5z)(2x - 5y + 5z)}}$

練習 II 題解（續）

(b) $6(a+b)^2 - 96(a-b)^2$

$= 6[(a+b)^2 - 16(a-b)^2]$

$= 6\{(a+b)^2 - [4(a-b)]^2\}$

$= 6[a+b+4(a-b)][a+b-4(a-b)]$

$= \underline{\underline{6(5a-3b)(5b-3a)}}$

11. (a) $\left(\frac{3}{m} - \frac{m}{3}\right)^2 = \left(\frac{3}{m}\right)^2 - 2\left(\frac{3}{m}\right)\left(\frac{m}{3}\right) + \left(\frac{m}{3}\right)^2$

$= \underline{\underline{\frac{9}{m^2} - 2 + \frac{m^2}{9}}}$

(b) $x^2\left(2x - \frac{1}{2x}\right)^2$

$= x^2\left[(2x)^2 - 2(2x)\left(\frac{1}{2x}\right) + \left(\frac{1}{2x}\right)^2\right]$

$= x^2\left[4x^2 - 2 + \frac{1}{4x^2}\right]$

$= \underline{\underline{4x^4 - 2x^2 + \frac{1}{4}}}$

13. (a) $\frac{x^2}{4} + \frac{4}{x^2} + 2 = \left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right)\left(\frac{2}{x}\right) + \left(\frac{2}{x}\right)^2$

$= \underline{\underline{\left(\frac{x}{2} + \frac{2}{x}\right)^2}}$

(b) $2m^4 - 8m + \frac{8}{m^2}$

$= 2\left(m^4 - 4m + \frac{4}{m^2}\right)$

$= 2\left[(m^2)^2 - 2(m^2)\left(\frac{2}{m}\right) + \left(\frac{2}{m}\right)^2\right]$

$= \underline{\underline{2\left(m^2 - \frac{2}{m}\right)^2}}$

練習 III（第 12 頁）

1. (a) $x^3 + 27 = x^3 + 3^3$

$= \underline{\underline{(x+3)(x^2 - 3x + 9)}}$

(b) $125 + y^3 = 5^3 + y^3$

$= \underline{\underline{(5+y)(25 - 5y + y^2)}}$

3. (a) $a^3b^3 - 1 = (ab)^3 - 1^3$

$= \underline{\underline{(ab-1)(a^2b^2 + ab + 1)}}$

(b) $-p^3 - q^3 = -(p^3 + q^3)$

$= \underline{\underline{-(p+q)(p^2 - pq + q^2)}}$

5. (a) $a^3b^3 - 125c^3$

$= (ab)^3 - (5c)^3$

$= (ab - 5c)[(ab)^2 + (ab)(5c) + (5c)^2]$

$= \underline{\underline{(ab-5c)(a^2b^2 + 5abc + 25c^2)}}$

(b) $a^4b^4 - ab = ab(a^3b^3 - 1)$

$= ab[(ab)^3 - 1^3]$

$= \underline{\underline{ab(ab-1)(a^2b^2 + ab + 1)}}$

7. (a) $8 - \frac{1}{m^3} = 2^3 - \left(\frac{1}{m}\right)^3$

$= \underline{\underline{\left(2 - \frac{1}{m}\right)\left(4 + \frac{2}{m} + \frac{1}{m^2}\right)}}$

(b) $3n^3 + \frac{1}{9} = 3\left(n^3 + \frac{1}{27}\right)$

$= 3\left[n^3 + \left(\frac{1}{3}\right)^3\right]$

$= \underline{\underline{3\left(n + \frac{1}{3}\right)\left(n^2 - \frac{1}{3}n + \frac{1}{9}\right)}}$

9. (a) $a^3(b+2)^3 + a^3b^3$

$= a^3[(b+2)^3 + b^3]$

$= a^3[(b+2) + b][(b+2)^2 - (b+2)b + b^2]$

$= a^3(2b+2)[b^2 + 4b + 4 - b^2 - 2b + b^2]$

$= \underline{\underline{2a^3(b+1)(b^2 + 2b + 4)}}$

練習 III 題解（續）

(b) $(a+b)^3-(a-b)^3$

$=[(a+b)-(a-b)][(a+b)^2+(a+b)(a-b)+(a-b)^2]$

$=2b[(a^2+2ab+b^2)+(a^2-b^2)+(a^2-2ab+b^2)]$

$=\underline{\underline{2b(3a^2+b^2)}}$

練習 IV（第 14 頁）

1. (a) $\log 100-\log 10=2-1$

$=\underline{\underline{1}}$

(b) $\log 25+\log 40=\log(25\times 40)$

$=\log 1\,000$

$=\underline{\underline{3}}$

3. $\log(10x)^3+\log\frac{1}{x^3}=\log\left[(10x)^3\cdot\frac{1}{x^3}\right]$

$=\log 1\,000$

$=\underline{\underline{3}}$

5. $\log\frac{x}{\sqrt{x}}-\frac{1}{2}\log x=\log x^{\frac{1}{2}}-\log x^{\frac{1}{2}}$

$=\underline{\underline{0}}$

7. $\log 3^6+\log\left(\frac{1}{9}\right)^3=\log\left[3^6\cdot\left(\frac{1}{9}\right)^3\right]$

$=\log 1$

$=\underline{\underline{0}}$

9. $3\log 5-2\log\frac{5}{2}+\log 50$

$=\log 5^3-\log\left(\frac{5}{2}\right)^2+\log 50$

$=\log\left[\frac{5^3\times 50}{\left(\frac{5}{2}\right)^2}\right]$

$=\log 1\,000$

$=\underline{\underline{3}}$

11. $\frac{2\log x-\log\frac{1}{x}}{\log x^3+4\log x}=\frac{2\log x+\log x}{3\log x+4\log x}$

$=\frac{3\log x}{7\log x}$

$=\frac{3}{7}$

$=$

練習 V（第 16 頁）

1. $\log x=3\log y+\log 2$

$\log x=\log y^3+\log 2$

$\log x=\log(2y^3)$

$\therefore\quad x=\underline{\underline{2y^3}}$

3. $\log(2x-3)=0$

$\therefore\quad 2x-3=1$

$x=\underline{\underline{2}}$

5. $4^x=13$

$\log 4^x=\log 13$

$x\log 4=\log 13$

$x=\frac{\log 13}{\log 4}$

$=\underline{\underline{1.85}}$（準確至二位小數）

7. $4(9^{4+x})=15$

$9^{4+x}=\frac{15}{4}$

$\log 9^{4+x}=\log\frac{15}{4}$

$(4+x)\log 9=\log\frac{15}{4}$

$4+x=\frac{\log\frac{15}{4}}{\log 9}$

$x=\frac{\log\frac{15}{4}}{\log 9}-4$

$=\underline{\underline{-3.40}}$（準確至二位小數）

練習 V 題解（續）

9. $\log(x+y) = \log(x+1) - 1$

$\log(x+y) = \log(x+1) - \log 10$

$\log(x+y) = \log\dfrac{x+1}{10}$

$\therefore \quad x+y = \dfrac{x+1}{10}$

$10x + 10y = x + 1$

$9x = 1 - 10y$

$x = \dfrac{1-10y}{9}$

11. $\log\sqrt{1+\dfrac{x}{2}} = \dfrac{1}{2}$

$\log\left(1+\dfrac{x}{2}\right)^{\frac{1}{2}} = \dfrac{1}{2}$

$\dfrac{1}{2}\log\left(1+\dfrac{x}{2}\right) = \dfrac{1}{2}$

$\log\left(1+\dfrac{x}{2}\right) = 1$

$\therefore \quad 1+\dfrac{x}{2} = 10$

$\dfrac{x}{2} = 9$

$x = \underline{18}$

13. $5^{2x-1} = 7^{3x+1}$

$\log 5^{2x-1} = \log 7^{3x+1}$

$(2x-1)\log 5 = (3x+1)\log 7$

$2x\log 5 - \log 5 = 3x\log 7 + \log 7$

$x(2\log 5 - 3\log 7) = \log 7 + \log 5$

$x = \dfrac{\log 7 + \log 5}{2\log 5 - 3\log 7}$

$= \underline{-1.36}$（準確至二位小數）

15.

$3(5^{1-x}) - 4(6^{2-3x}) = 0$

$3(5^{1-x}) = 4(6^{2-3x})$

$\log[3(5^{1-x})] = \log[4(6^{2-3x})]$

$\log 3 + \log 5^{1-x} = \log 4 + \log 6^{2-3x}$

$\log 3 + (1-x)\log 5 = \log 4 + (2-3x)\log 6$

$\log 3 + \log 5 - x\log 5 = \log 4 + 2\log 6 - 3x\log 6$

$(3\log 6 - \log 5)x = \log 4 + 2\log 6 - \log 3 - \log 5$

$x = \dfrac{\log 4 + 2\log 6 - \log 3 - \log 5}{3\log 6 - \log 5}$

$= \underline{0.60}$（準確至二位小數）

補充練習 9C（第 17 頁）

1. (a) $3(3^{-2})^{-\frac{1}{2}} = 3\left[3^{(-2)\times\left(-\frac{1}{2}\right)}\right]$

$= 3 \cdot 3$

$= 3^{1+1}$

$= \underline{3^2}$

(b) $\left(4x^{\frac{1}{6}}\right)\left(\dfrac{1}{8}x^{\frac{1}{3}}\right) = 4 \cdot \dfrac{1}{8} \cdot x^{\frac{1}{6}} \cdot x^{\frac{1}{3}}$

$= \dfrac{1}{2}x^{\frac{1}{6}+\frac{1}{3}}$

$= \underline{\dfrac{1}{2}x^{\frac{1}{2}}}$

3. (a) $\dfrac{9^k \cdot 2^{-3k}}{3^{-k} \cdot 4^k} = \dfrac{(3^2)^k \cdot 2^{-3k}}{3^{-k} \cdot (2^2)^k}$

$= \dfrac{3^{2k} \cdot 2^{-3k}}{3^{-k} \cdot 2^{2k}}$

$= 3^{2k-(-k)} \cdot 2^{-3k-2k}$

$= \underline{2^{-5k} \cdot 3^{3k}}$

(b) $\left(\dfrac{a}{\sqrt[n]{a}}\right)^n \cdot a = \left(\dfrac{a}{a^{\frac{1}{n}}}\right)^n \cdot a$

$= \left(a^{1-\frac{1}{n}}\right)^n \cdot a$

$= a^{\left(1-\frac{1}{n}\right)\times n} \cdot a$

$= a^{n-1} \cdot a$

$= a^{n-1+1}$

$= \underline{a^n}$

補充練習 0 題解（續）

5. $27^{2x-1}\cdot 243^{2x-2}=81^{3x+1}$

$3^{3(2x-1)}\cdot 3^{5(2x-2)}=3^{4(3x+1)}$

$3^{6x-3}\cdot 3^{10x-10}=3^{12x+4}$

$3^{(6x-3)+(10x-10)}=3^{12x+4}$

$3^{16x-13}=3^{12x+4}$

$\therefore\quad 16x-13=12x+4$

$4x=17$

$x=\underline{\underline{\frac{17}{4}}}$

7. $(4x+3)^{-\frac{5}{3}}=-\frac{1}{32}$

$(4x+3)^{-\frac{5}{3}}=-2^{-5}$

$\left[(4x+3)^{-\frac{5}{3}}\right]^{-\frac{3}{5}}=[(-1)^{-5}2^{-5}]^{-\frac{3}{5}}$

$4x+3=(-1)^{-5\left(-\frac{3}{5}\right)}2^{-5\left(-\frac{3}{5}\right)}$

$4x+3=(-1)^3 2^3$

$4x+3=-8$

$x=\underline{\underline{-\frac{11}{4}}}$

9. (a) $\sqrt[7]{a\sqrt{a\sqrt{a}}}=\left[a\left(a\cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{7}}$

$=\left[a\left(a^{1+\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{7}}$

$=\left[a\left(a^{\frac{3}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{7}}$

$=\left[a^{1+\frac{3}{4}}\right]^{\frac{1}{7}}$

$=\left[a^{\frac{7}{4}}\right]^{\frac{1}{7}}$

$=\underline{\underline{a^{\frac{1}{4}}}}$

(b) $\sqrt[7]{a\sqrt{a\sqrt{a}}}=2$

$a^{\frac{1}{4}}=2$ [根據 **(a)**]

$\left(a^{\frac{1}{4}}\right)^4=2^4$

$a=\underline{\underline{16}}$

11. (a) $81x^4-16y^4$

$=(9x^2)^2-(4y^2)^2$

$=(9x^2+4y^2)(9x^2-4y^2)$

$=(9x^2+4y^2)[(3x)^2-(2y)^2]$

$=\underline{\underline{(9x^2+4y^2)(3x+2y)(3x-2y)}}$

(b) $5u^4v-125u^2v^3=5u^2v(u^2-25v^2)$

$=5u^2v[u^2-(5v)^2]$

$=\underline{\underline{5u^2v(u+5v)(u-5v)}}$

13. (a) $25x^4-30x^2y+9y^2$

$=(5x^2)^2-2(5x^2)(3y)+(3y)^2$

$=\underline{\underline{(5x^2-3y)^2}}$

(b) $16a^2b^2+56abc+49c^2$

$=(4ab)^2+2(4ab)(7c)+(7c)^2$

$=\underline{\underline{(4ab+7c)^2}}$

15. (a) $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$

$=(\sqrt{x})^2-2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)+\left(\frac{1}{\sqrt{x}}\right)^2$

$=\underline{\underline{x-2+\frac{1}{x}}}$

(b) (i) $\sqrt{x}-\frac{1}{\sqrt{x}}=\sqrt{2}$

$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2=(\sqrt{2})^2$

$x-2+\frac{1}{x}=2$ [根據 **(a)**]

$\therefore\quad x+\frac{1}{x}=\underline{\underline{4}}$

補充練習 0 題解（續）

(ii) 根據 **(b)(i)**，

$$\left(x+\frac{1}{x}\right)^2 = 4^2$$

$$x^2 + 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = 16$$

$$x^2 + 2 + \frac{1}{x^2} = 16$$

$$\therefore \quad x^2 + \frac{1}{x^2} = \underline{\underline{14}}$$

(iii) 根據 **(b)(ii)**，

$$\left(x^2+\frac{1}{x^2}\right)^2 = 14^2$$

$$(x^2)^2 + 2(x^2)\left(\frac{1}{x^2}\right) + \left(\frac{1}{x^2}\right)^2 = 196$$

$$x^4 + 2 + \frac{1}{x^4} = 196$$

$$\therefore \quad x^4 + \frac{1}{x^4} = \underline{\underline{194}}$$

17. (a) $4x^3 - 32y^3$

$= 4(x^3 - 8y^3)$

$= 4[x^3 - (2y)^3]$

$= 4(x-2y)[x^2 + (x)(2y) + (2y)^2]$

$= \underline{\underline{4(x-2y)(x^2+2xy+4y^2)}}$

(b) $16x - \frac{x^4}{4} = \frac{1}{4}x(64 - x^3)$

$= \frac{1}{4}x(4^3 - x^3)$

$= \underline{\underline{\frac{1}{4}x(4-x)(16+4x+x^2)}}$

19. (a) $8^n + x^{6n}$

$= (2^3)^n + (x^{2n})^3$

$= (2^n)^3 + (x^{2n})^3$

$= (2^n + x^{2n})[(2^n)^2 - (2^n)(x^{2n}) + (x^{2n})^2]$

$= \underline{\underline{(2^n + x^{2n})(2^{2n} - 2^n x^{2n} + x^{4n})}}$

(b) $8x^{6n} - y^{3n+9}$

$= (2x^{2n})^3 - (y^{n+3})^3$

$= \underline{\underline{(2x^{2n} - y^{n+3})(4x^{4n} + 2x^{2n}y^{n+3} + y^{2n+6})}}$

21. (a) $\log\sqrt{625} + 2\log 2 = \log 25 + \log 2^2$

$= \log(25 \cdot 2^2)$

$= \log 100$

$= \underline{\underline{2}}$

(b) $\left(\log 1.6 - \log\frac{4}{25}\right)^2 = \left[\log\left(\frac{1.6}{\frac{4}{25}}\right)\right]^2$

$= [\log 10]^2$

$= 1^2$

$= \underline{\underline{1}}$

23. $\because \quad 10^a = 5 \quad$ 及 $\quad 10^b = 3$

$\therefore \quad a = \log 5 \quad$ 及 $\quad b = \log 3$

(a) $\log 600 = \log(3 \times 2 \times 100)$

$= \log 3 + \log 2 + \log 100$

$= \log 3 + \log\frac{10}{5} + 2$

$= \log 3 + (\log 10 - \log 5) + 2$

$= b + 1 - a + 2$

$\therefore \quad \underline{\log 600 = b - a + 3}$

(b) $\log 40.5 = \log\frac{81}{2}$

$= \log 81 - \log 2$

$= \log 3^4 - \log\frac{10}{5}$

$= 4\log 3 - (\log 10 - \log 5)$

$= 4b - (1 - a)$

$\therefore \quad \underline{\log 40.5 = 4b + a - 1}$

25. (a) $2\log x = 3 - \log y$

$\log x^2 = \log 10^3 - \log y$

$\log x^2 = \log\left(\frac{1\,000}{y}\right)$

$\therefore \quad x^2 = \frac{1\,000}{y}$

$\therefore \quad \underline{y = \frac{1\,000}{x^2}}$

補充練習 0 題解（續）

(b) 根據 **(a)**，$x^2 = \dfrac{1\,000}{y}$

$\therefore\ \underline{x = 10\sqrt{\dfrac{10}{y}}}$ 或 $-10\sqrt{\dfrac{10}{y}}$（捨去）

27. $2\log\sqrt{2x-3} - \log 10 = \log 1$

$$\log\left[(2x-3)^{\frac{1}{2}}\right]^2 - \log 10 = \log 1$$

$$\log\left(\frac{2x-3}{10}\right) = \log 1$$

$$\therefore\quad \frac{2x-3}{10} = 1$$

$$x = \underline{\underline{\frac{13}{2}}}$$

29. $7^x \cdot 3^{2x+1} = 7$

$$3^{2x+1} = 7^{1-x}$$

$$\log 3^{2x+1} = \log 7^{1-x}$$

$$(2x+1)\log 3 = (1-x)\log 7$$

$$2x\log 3 + \log 3 = \log 7 - x\log 7$$

$$x(2\log 3 + \log 7) = \log 7 - \log 3$$

$$x = \frac{\log 7 - \log 3}{2\log 3 + \log 7}$$

$$= \underline{\underline{0.205}}\ (\text{準確至三位有效數字})$$

31. $\left(\dfrac{7}{3}\right)^{2x-1} \cdot \dfrac{1}{4^x} = 2.8$

$$\left(\frac{7}{3}\right)^{2x-1} \cdot 4^{-x} = 2.8$$

$$\log\left[\left(\frac{7}{3}\right)^{2x-1} \cdot 4^{-x}\right] = \log 2.8$$

$$\log\left(\frac{7}{3}\right)^{2x-1} + \log 4^{-x} = \log 2.8$$

$$(2x-1)\log\frac{7}{3} - x\log 4 = \log 2.8$$

$$2x\log\frac{7}{3} - \log\frac{7}{3} - x\log 4 = \log 2.8$$

$$x\left(2\log\frac{7}{3} - \log 4\right) = \log 2.8 + \log\frac{7}{3}$$

$$x = \frac{\log 2.8 + \log\frac{7}{3}}{2\log\frac{7}{3} - \log 4}$$

$$= \underline{\underline{6.09}}\ (\text{準確至三位有效數字})$$

33. (a) $\left(\dfrac{2a^{-1}b^{-\frac{1}{2}}}{4^2a^{\frac{1}{2}}b^{-2}}\right)^{-\frac{2}{3}} = \left[\dfrac{2a^{-1}b^{-\frac{1}{2}}}{(2^2)^2a^{\frac{1}{2}}b^{-2}}\right]^{-\frac{2}{3}}$

$$= \left[2^{1-4}a^{-1-\frac{1}{2}}b^{-\frac{1}{2}-(-2)}\right]^{-\frac{2}{3}}$$

$$= \left(2^{-3}a^{-\frac{3}{2}}b^{\frac{3}{2}}\right)^{-\frac{2}{3}}$$

$$= 2^2ab^{-1}$$

$$= \underline{\underline{\frac{4a}{b}}}$$

(b) $\dfrac{7^0\sqrt{p}\sqrt[3]{q}}{7p^2\cdot\sqrt[4]{q^3}} = \dfrac{p^{\frac{1}{2}}q^{\frac{1}{3}}}{7p^2q^{\frac{3}{4}}}$

$$= \frac{1}{7p^{2-\frac{1}{2}}q^{\frac{3}{4}-\frac{1}{3}}}$$

$$= \underline{\underline{\frac{1}{7p^{\frac{3}{2}}q^{\frac{5}{12}}}}}$$

35. $6(\sqrt{2x-1})^{-\frac{5}{3}} = 12\times 2^{-6}$

$$\left[(2x-1)^{\frac{1}{2}}\right]^{-\frac{5}{3}} = 2\times 2^{-6}$$

$$(2x-1)^{-\frac{5}{6}} = 2^{-5}$$

$$\left[(2x-1)^{-\frac{5}{6}}\right]^{-\frac{6}{5}} = (2^{-5})^{-\frac{6}{5}}$$

$$2x-1 = 2^6$$

$$2x-1 = 64$$

$$x = \underline{\underline{\frac{65}{2}}}$$

37. (a) $\left(a-\dfrac{b}{2}\right)^2 = a^2 - 2(a)\left(\dfrac{b}{2}\right) + \left(\dfrac{b}{2}\right)^2$

$$= \underline{\underline{a^2 - ab + \frac{b^2}{4}}}$$

補充練習 0 題解（續）

(b) $(a-1)(a+1)+\frac{1}{4}b(b-4a)$

$=a^2-1+\frac{1}{4}b^2-ab$

$=\left(a^2-ab+\frac{b^2}{4}\right)-1$

$=\left(a-\frac{b}{2}\right)^2-1$ [根據 **(a)**]

$=\underline{\underline{\left(a-\frac{b}{2}+1\right)\left(a-\frac{b}{2}-1\right)}}$

39. (a) $2^{3x+3}=81$

$2^{3x}\cdot 2^3=3^4$

$2^{3x}=\frac{3^4}{2^3}$

$(2^{3x})^{\frac{1}{3}}=\left(\frac{3^4}{2^3}\right)^{\frac{1}{3}}$

$2^x=\underline{\underline{\frac{1}{2}\left(3^{\frac{4}{3}}\right)}}$

(b) $2^{3x+3}=81$

$2^{3x}=\frac{81}{8}$

$\frac{1}{2^{3x}}=\frac{8}{81}$

$\therefore\ \left(\frac{1}{8}\right)^x=\underline{\underline{\frac{8}{81}}}$

41. (a) $(mx)^{-\frac{3}{2}}=n$

$\left[(mx)^{-\frac{3}{2}}\right]^{\frac{2}{3}}=n^{\frac{2}{3}}$

$(mx)^{-1}=m$

$\frac{1}{mx}=m$

$x=\underline{\frac{1}{m^2}}$

(b) 將 $m=\sqrt[3]{n^2}$ 代入 **(a)** 的結果，

$x=\frac{1}{(\sqrt[3]{n^2})^2}$

$=\frac{1}{\left(n^{\frac{2}{3}}\right)^2}$

$\therefore\ x=\underline{\frac{1}{n^{\frac{4}{3}}}}$

43. $\frac{1}{2}\log(x+y)=\log\sqrt{\frac{x}{y}}+1$

$\frac{1}{2}\log(x+y)=\log\sqrt{\frac{x}{y}}+\log 10$

$\log(x+y)^{\frac{1}{2}}=\log\left(10\sqrt{\frac{x}{y}}\right)$

$\therefore\ (x+y)^{\frac{1}{2}}=10\sqrt{\frac{x}{y}}$

$\left[(x+y)^{\frac{1}{2}}\right]^2=\left(10\sqrt{\frac{x}{y}}\right)^2$

$x+y=100\left(\frac{x}{y}\right)$

$xy+y^2=100x$

$y^2=100x-xy$

$y^2=x(100-y)$

$x=\underline{\underline{\frac{y^2}{100-y}}}$

45. (a) $\log\frac{3}{2}=\log 3-\log 2$

$=\log 9^{\frac{1}{2}}-\log 2$

$=\frac{1}{2}\log 9-\log 2$

$\therefore\ \underline{\log\frac{3}{2}=\frac{1}{2}b-a}$

補充練習 0 題解（續）

(b) $\log 54 = \log (27 \times 2)$

$= \log 27 + \log 2$

$= \log 3^3 + \log 2$

$= \log 9^{\frac{3}{2}} + \log 2$

$= \frac{3}{2}\log 9 + \log 2$

$\therefore \quad \underline{\log 54 = \frac{3}{2}b + a}$

(c) $\log 0.12 = \log \frac{12}{100}$

$= \log 12 - \log 100$

$= \log (4 \times 3) - 2$

$= \log 4 + \log 3 - 2$

$= \log 2^2 + \log 9^{\frac{1}{2}} - 2$

$= 2\log 2 + \frac{1}{2}\log 9 - 2$

$\therefore \quad \underline{\log 0.12 = 2a + \frac{1}{2}b - 2}$

47. (a) $x^{3n} + y^{9n} = (x^n)^3 + (y^{3n})^3$

$= \underline{\underline{(x^n + y^{3n})(x^{2n} - x^n y^{3n} + y^{6n})}}$

(b) $\frac{x^{3n} + y^{9n}}{x^n + y^{3n}}$

$= \frac{(x^n + y^{3n})(x^{2n} - x^n y^{3n} + y^{6n})}{x^n + y^{3n}}$ [根據 **(a)**]

$= (x^{2n} - x^n y^{3n} + y^{6n})$

$= x^{2n} - (x^{2n})^{\frac{1}{2}}(y^{6n})^{\frac{1}{2}} + y^{6n}$

$= 4 - 4^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} + 9$

$= \underline{\underline{7}}$

49. $\frac{2(3^{-x} \cdot 4^{2-x}) - 5}{13} = 1$

$2(3^{-x} \cdot 4^{2-x}) - 5 = 13$

$3^{-x} \cdot 4^{2-x} = 9$

$3^{-x} \cdot 4^{2-x} = 3^2$

$4^{2-x} = 3^{2+x}$

$\log 4^{2-x} = \log 3^{2+x}$

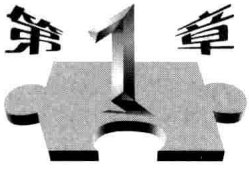
$(2 - x)\log 4 = (2 + x)\log 3$

$2\log 4 - x\log 4 = 2\log 3 + x\log 3$

$2(\log 4 - \log 3) = x(\log 3 + \log 4)$

$x = \frac{2(\log 4 - \log 3)}{\log 3 + \log 4}$

$= \underline{\underline{0.23}}$ (準確至二位小數)



數系和根式

習題 1A (第 28 頁)

1. **(a)** 自然數：5, 1, 253

(b) 整數：5, −5, 1, 0, −1 000, 253

(c) 小數：0.5, −3.174

(d) 分數：$\frac{2}{3}$, $-\frac{5}{4}$, $100\frac{7}{9}$, $\frac{8}{6}$

3. **(a)** $\frac{2}{3} = 0.666\,666\cdots\cdots = 0.\dot{6}$

$\therefore$ $\frac{2}{3}$ 可化成循環小數。

(b) $\frac{3}{5} = 0.6$

$\therefore$ $\frac{3}{5}$ 可化成有盡小數。

(c) $\frac{3}{12} = 0.25$

$\therefore$ $\frac{3}{12}$ 可化成有盡小數。

5. $\sqrt{25} = 5$

$\sqrt{3} = \sqrt{3}$

$\sqrt[3]{27} = 3$

$\sqrt[3]{5} = \sqrt[3]{5}$

$\sqrt[4]{81} = 3$

$\sqrt[3]{-8} = -2$

	(a)	(b)
$\sqrt{25}$	是	否
$\sqrt{3}$	否	是
$\sqrt[3]{27}$	是	否
$\sqrt[3]{5}$	否	是
$\sqrt[4]{81}$	是	否
$\sqrt[3]{-8}$	是	否

7. **(a)** $\sqrt{9} = \underline{\underline{3}}$

$0.\dot{2} = \underline{\underline{\frac{2}{9}}}$

$\cos 60° = \underline{\underline{\frac{1}{2}}}$

$\tan 30° = \underline{\underline{\frac{\sqrt{3}}{3}}}$

(b)

	(i)	(ii)	(iii)	(iv)
$\sqrt{9}$	是	否	否	是
$0.\dot{2}$	否	否	是	是
$\cos 60°$	否	否	是	是
$\tan 30°$	否	是	否	是

習題 1B (第 34 頁)

1. **(a)** $\sqrt{196} = \sqrt{14 \times 14} = \underline{\underline{14}}$

(b) $\sqrt{288} = \sqrt{144 \times 2} = \sqrt{144} \times \sqrt{2} = \underline{\underline{12\sqrt{2}}}$

3. **(a)** $4\sqrt{3} + 15\sqrt{3} = \underline{\underline{19\sqrt{3}}}$

(b) $\sqrt{8} + 7\sqrt{2} = \sqrt{4 \times 2} + 7\sqrt{2} = 2\sqrt{2} + 7\sqrt{2}$

$= \underline{\underline{9\sqrt{2}}}$

5. **(a)** $\sqrt{8} \times \sqrt{12} = \sqrt{4 \times 2} \times \sqrt{4 \times 3}$

$= 2 \times \sqrt{2} \times 2 \times \sqrt{3}$

$= 2 \times 2 \times \sqrt{2} \times \sqrt{3}$

$= \underline{\underline{4\sqrt{6}}}$

習題 1B 題解（續）

(b) $\sqrt{18}\times\sqrt{72}=\sqrt{9\times2}\times\sqrt{36\times2}$
$=3\times\sqrt{2}\times6\times\sqrt{2}$
$=3\times6\times\sqrt{2}\times\sqrt{2}$
$=\underline{\underline{36}}$

7. (a) $\sqrt{2}(\sqrt{8}+\sqrt{32})=\sqrt{2}(\sqrt{4\times2}+\sqrt{16\times2})$
$=\sqrt{2}(2\sqrt{2}+4\sqrt{2})$
$=\sqrt{2}(6\sqrt{2})$
$=\underline{\underline{12}}$

(b) $(1+\sqrt{8})(1+\sqrt{2})=(1+\sqrt{4\times2})(1+\sqrt{2})$
$=(1+2\sqrt{2})(1+\sqrt{2})$
$=1+\sqrt{2}+2\sqrt{2}+4$
$=\underline{\underline{5+3\sqrt{2}}}$

9. (a) $(1+\sqrt{3})^2=1+2(1)(\sqrt{3})+(\sqrt{3})^2$
$=1+2\sqrt{3}+3$
$=\underline{\underline{4+2\sqrt{3}}}$

(b) $\sqrt{3}(1+\sqrt{3})^2=\sqrt{3}(4+2\sqrt{3})$ [根據 **(a)**]
$=\underline{\underline{6+4\sqrt{3}}}$

11. (a) $\frac{2}{3}\sqrt{\frac{504}{8}}=\frac{2}{3}\sqrt{63}$
$=\frac{2}{3}\sqrt{9\times7}$
$=\frac{2}{3}(3\sqrt{7})$
$=\underline{\underline{2\sqrt{7}}}$

(b) $-\frac{3}{2}\sqrt{\frac{30}{1.6}}=-\frac{3}{2}\sqrt{\frac{300}{16}}$
$=-\frac{3}{2}\sqrt{\frac{75}{4}}$
$=-\frac{3}{2}\left(\frac{\sqrt{25\times3}}{\sqrt{4}}\right)$
$=-\frac{3}{2}\left(\frac{5\sqrt{3}}{2}\right)$
$=\underline{\underline{-\frac{15\sqrt{3}}{4}}}$

13. (a) $\sqrt{48}+\sqrt{\frac{3}{25}}=\sqrt{16\times3}+\frac{\sqrt{3}}{\sqrt{25}}$
$=4\sqrt{3}+\frac{\sqrt{3}}{5}$
$=\underline{\underline{\frac{21\sqrt{3}}{5}}}$

(b) $\sqrt{\frac{75}{4}}-\sqrt{\frac{27}{4}}=\frac{\sqrt{25\times3}}{\sqrt{4}}-\frac{\sqrt{9\times3}}{\sqrt{4}}$
$=\frac{5\sqrt{3}}{2}-\frac{3\sqrt{3}}{2}$
$=\underline{\underline{\sqrt{3}}}$

15. (a) $\sqrt{12}-\sqrt{48}+\sqrt{75}-2\sqrt{27}$
$=\sqrt{4\times3}-\sqrt{16\times3}+\sqrt{25\times3}-2\sqrt{9\times3}$
$=2\sqrt{3}-4\sqrt{3}+5\sqrt{3}-6\sqrt{3}$
$=\underline{\underline{-3\sqrt{3}}}$

(b) $3\sqrt{32}-2\sqrt{20}+\sqrt{50}+4\sqrt{5}$
$=3\sqrt{16\times2}-2\sqrt{4\times5}+\sqrt{25\times2}+4\sqrt{5}$
$=12\sqrt{2}-4\sqrt{5}+5\sqrt{2}+4\sqrt{5}$
$=\underline{\underline{17\sqrt{2}}}$

17. (a) $(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{6})$
$=(\sqrt{2})^2+(\sqrt{2})(\sqrt{6})-(\sqrt{3})(\sqrt{2})-$
$(\sqrt{3})(\sqrt{6})$
$=2+\sqrt{12}-\sqrt{6}-\sqrt{18}$
$=2+\sqrt{4\times3}-\sqrt{6}-\sqrt{9\times2}$
$=\underline{\underline{2+2\sqrt{3}-\sqrt{6}-3\sqrt{2}}}$

(b) $(2\sqrt{5}-\sqrt{3})(3\sqrt{3}+\sqrt{5})$
$=6(\sqrt{5})(\sqrt{3})+2(\sqrt{5})^2-3(\sqrt{3})^2-$
$(\sqrt{3})(\sqrt{5})$
$=6\sqrt{15}+2(5)-3(3)-\sqrt{15}$
$=5\sqrt{15}+10-9$
$=\underline{\underline{5\sqrt{15}+1}}$