天元基金影印数学丛书

Analysis I

分析 I （影印版）

Roger Godement

高等教育出版社
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序言

为了更好地借鉴国外数学教育与研究的成功经验，促进我国数学教育与研究事业的发展，提高高等学校数学教育教学质量，本着“为我国热爱数学的青年创造一个较好的学习数学的环境”这一宗旨，天元基金赞助出版“天元基金影印数学丛书”。

该丛书主要包含国外反映近代数学发展的纯数学与应用数学方面的优秀书籍，天元基金邀请国内各个方向的知名数学家参与选题的工作，经专家遴选、推荐，由高等教育出版社影印出版。为了提高我国数学研究生教学的水平，暂把选书的目标确定在研究生教材上。当然，有的书也可作为高年级本科生教材或参考书，有的书则介于研究生教材与专著之间。

欢迎各方专家、读者对本丛书的选题、印刷、销售等工作提出批评和建议。

天元基金领导小组

2007 年 1 月

Preface

Analysis and its Adhesions

Between 1946 and 1990 I had thousands of students; in the very economical French system with its auditoria for two hundred people or more, this was not difficult. On several occasions I felt the desire to write a book which, presupposing only a minimal level of knowledge and a taste for mathematics, would lead the reader to a point from which he (or she) could launch himself without difficulty into the more abstract or more complicated theories of the XXth century. After various attempts I began to write it for Springer-Verlag in the Spring of 1996.

A long-established house, with unrivalled experience in scientific publishing in general and mathematics in particular, Springer seemed to be by far the best possible publisher. My dealings with their mathematical department over six years have quite confirmed this. As, furthermore, Catriona Byrne, who has responsibility for author relations in this sector, has been a friend of mine for a long time, I had no misgivings at confiding my francophone production to a foreign publisher who, though not from our parish, knows its profession superlatively.

My text has been prepared in French on computer, in DOS, with the aid of *Nota Bene*, a perfectly organized, simple and rational American word processor; but it is hardly more adapted to mathematics than the traditional typewriters of yesteryear: greek letters, $\in$, $\int$, Σ have to be written by hand on the printout, something I had been doing anyway since my first 1946 typewriter. I eventually devised a coding system, for instance [[alpha]] for greek letters, that made it easier to translate the NB files into TEX by using global commands. But apart from simple formulae in the main text, most of the others had to be typeset again for the French version.

The excellent English translation has been much easier to do since Dr Spain, who types in TEX, had the TEX version of the French edition. I have taken this opportunity to make some small changes to the French version.

* * *

This is not a standard textbook geared to those many students who have to learn mathematics for other purposes, although it may help them; it is the reader interested in mathematics for its own sake of whom I have thought while writing. To many of the French students and particularly to many of

the brightest, mathematics is merely a lift to the upper strata of society[1]. My goal is not to help bright young people to arrive among the first few in the entry competition for the French *Ecole polytechnique* so as to find themselves thirty years later at the service or at the head of a public or private enterprise producing possibly war planes, missiles, military electronics, or nuclear weapons[2], or who will devise all kinds of financial stunts to make their company grow beyond what they can control, and who, in both cases, will make at least twenty times as much money as the winner of a Fields Medal does.

The sole aim of this book thus is mathematical analysis as it was and as it has become. The fundamental ideas which anyone must know – convergence, continuity, elementary functions, integrals, asymptotics, Fourier series and integrals – are the subject of the first two volumes. Volume II also deals with that part (Weierstrass) of the classical theory of analytic functions which can be explained with the use of Fourier series, while the other part (Cauchy) will be found at the beginning of Volume III. I have not hesitated to introduce, sometimes very early, subjects considered as relatively advanced when they can be explained without technical complications: series indexed by arbitrary countable sets, the definition and elementary properties of Radon measures in $\mathbb{R}$ or $\mathbb{C}$, integrals of semi-continuous functions and even, in an Appendix to Chap. V, a short account of the basic theorems of Lebesgue's theory for those who may care to read it at this early stage, analytic functions, the construction of Weierstrass elliptic functions as a beautiful and useful example of a sophisticated series, etc.

I have tried to give the reader an idea of the axiomatic construction of set theory while hoping that he will take Chap. I for what it is: a contribution to his mathematical culture aiming at showing that the whole of mathematics can, in principle, be built from a small number of axioms and definitions. But a full understanding of this Chapter *is not an obligatory prerequisite to an apprenticeship in analysis.* The only thing the reader will have to retain is the naive version of set theory – standard operations on sets and functions to which, anyway, he will get used by merely reading the next chapters – as well as the fact that, even at the simplest level, mathematics rests upon *proofs* of statements, an old art which, in French high schools and probably elsewhere as well, is in the process of becoming obsolete because, we are told, learning to use formulae is much more useful to most people, or because it is too difficult for the many children of the lower strata of society (I was one in the 1930s) who now flood the high schools ...

[1] In XIXth century Cambridge, the winners of the Math Tripos would far more often become judges or bishops than scientists.

[2] One of the brightest students I have known in thirty-five years is today the head of a holding company that controls, among other things, a chain of supermarkets. He sells Camembert, shrink-wrapped meat, Tampax, orange juice, noodles, mustard, etc. If you have to choose, this is a more civilised way to squander your grey matter.

The sequel, in Volumes III and IV, explains subjects which require either a much higher level of abstraction (short introductions to differential varieties and Riemann surfaces, general integration, Hilbert spaces, general harmonic analysis), or, in the last Chap. XII, a much higher level in computation techniques: Dirichlet series of number theory, elliptic and modular functions, connection with Lie groups. While the choice of material in Volumes I to IV represents a coherent and nearly selfcontained block of mathematics, it constitutes nothing more than one particular view of analysis. Other authors could have chosen other views and, for instance, tried to lead their readers into the theory of partial differential equations. I have not even treated differential equations in one variable: one can learn all about them in a myriad of books, and the classical results of the theory, direct applications of the general principles of analysis, should pose no serious problem to the student who has assimilated these reasonably well.

In the two first volumes – Volumes III and IV are written in a much more orthodox fashion – I have firmly emphasised, sometimes with the aid of out of fashion excurses in ordinary language, the ideas at the basis of analysis, and, in some cases, their historical evolution. I am not, far from it, an expert in the history of mathematics; some mathematicians, sensing their end coming, devote themselves to it late in life; others, younger, consider the subject sufficiently interesting to devote a substantial part of their activity to it; they perform a most useful task even from the pedagogical point of view[3] since, at twenty, which I once was, one thinks only of forging ahead without looking behind, and almost always without knowing where one is going: where and when will one learn? I have myself preferred for a quarter of a century to take an interest in a kind of history – science, technology, and armaments in the XX^{th} century – for which mathematics does not prepare one, though there are some indirect connections. Nevertheless I have made some effort to convey to the reader that the ideas and the techniques have evolved, and that it took between one and two centuries for the intuitions of the Founding Fathers to be transformed into perfectly clear concepts founded on unassailable arguments, awaiting the great generalisations of the XX^{th} century.

Adopting this point of view has led me, in these first two volumes, systematically to eschew a perfectly linear exposition, organised like a clockwork and only presenting to the reader the dominant or *à la mode* point of view, with assorted *Blitzbeweise*, lightning proofs in the sense in which we speak of *Blitzkrieg*[4]: one ratifies the result but does not comprehend the strategy until six months after the battle. At the cost of proving the same classical results several times over I have tried to present several methods of arguing to the

[3] E. Hairer and G. Wanner, *Analysis by Its History* (Springer-New York, 1996), is a prime example.

[4] René Etiemble, a great French specialist of comparative literature, once made a study of the styles prevailing in various kinds of activities. He came to the conclusion that mathematical style was the closest there was to the military.

reader, and to make clear the necessity of rigour by evidencing the doubtful arguments, and sometimes false results, due to mathematicians like Newton, the Bernoullis, Euler, Fourier, or Cauchy. Adopting this point of view lengthens the text palpably, but one of the ground principles of N. Bourbaki – no economies of paper – is, I think, mandatory when one addresses students embarking on a subject.

The other principle of this same author – to substitute ideas for computations – appears even more commendable to me whenever it can be applied. All the same, one will, inevitably, find calculations in this book; but I have essentially confined myself to those which, inherited from the great mathematicians of the past, form an integral part of the theory and can be considered as ideas.

Except occasionally, to round off the text, one will find no exercises here. Working at exercises is indispensable when one learns mathematics, and one will find them in profusion in many other books and specialised collections. The majority of French students, obsessed by the string of examinations imposed on them, have a very exaggerated tendency to consider the "lectures" of little use and that only "practical work" and "formulae" count or pay. The result is that the majority of them are able, up to errors in calculation, to integrate a rational function but incapable of answering questions of a general nature, e.g. why is a rational function integrable? To understand a theorem is to be able to reconstruct its proof. To understand a block of mathematics does not reduce to knowing how to apply its results; to understand a theory is to be able to reconstruct its logical structure. Every mathematician knows this.

One does not learn analysis or anything else from one single book; there is neither Bible, nor Gospel nor Koran in Mathematics. The fact that the spirit of my book is radically different from that of Serge Lang, *Undergraduate Analysis* (Springer, 2nd. ed., 1997) for example, should not dissuade from reading it, quite the contrary; even less the books of E. Hairer and G. Wanner, *Analysis by Its History*, Wolfgang Walter, *Analysis I* (Springer, 1992, in German) or Reinhold Remmert, *Theory of Complex Functions* (Springer-New York, 1991, translation of *Funktionentheorie 1*, 4. Auflage, 1995), which I have often used, and cite when I do so. These excellent books present numerous exercises, as does Jean Dieudonné's *Calcul Infinitésimal* (Hermann, 1968) though his style enthuses me less.

I have not acceded to the new fashion which likes to decorate elementary analysis textbooks with numerical calculations to fifteen decimal places under the pretext they will be useful to future computer scientists or applied mathematicians. Everyone knows that the mathematicians of the XVIIth and XVIIIth centuries loved numerical computations – done by hand, not by tapping the keys of an electronic gadget – that enabled them to verify their theoretical results or to demonstrate the power of their methods. This childhood sickness of analysis disappeared when in the XIXth century one

began addressing rigour of proof and generality of formulations, rather than formulae.

This does not mean that numerical calculations have become pointless: thanks to computers, one can do more and more of them, for better or worse, in all scientific and technical areas that, from medical imaging to the perfecting of nuclear weapons[5], use mathematics. One does the same in certain branches of mathematics too; for example, displaying a large number of curves may open the way to a general theorem or to understanding a topological situation, not to speak of the traditional number theory where numerical experiment always was, and still is, used to formulate or verify conjectures.

This only means that the aim of an exposition of the *principles* of analysis is not to teach numerical techniques. Moreover, the partisans of applied mathematics, of numerical analysis and of computer science in all the universities of the world manifest their imperialist tendencies far too clearly for real mathematicians to take on in their stead a task for which they generally lack both taste and competence.

* * *

The innocent reader and many confirmed mathematicians will probably be surprised, possibly even shocked, to find in my book some very heavy allusions to extramathematical subjects and particularly to the relations between science and weaponry. This is neither politically nor scientifically correct: Science is politically neutral[6], even when someone lets it fall inadvertently on Hiroshima while the future winner of a Nobel Prize in physics is recording the results in a B-29 trailing the *Enola Gay*[7]. Nor is it part of the curriculum: a

[5] Nuclear powers agreed a dozen years ago to stop testing. The reason why was that testing was made unnecessary by the improvement of numerical analysis. The most immediate consequence of this "progress" is that everything is now done in full secrecy, which was not the case when they had to propel into the stratosphere two million tons of radioactive rock and sand in order to check their "gadgets".

[6] An assertion long since demolished by countless studies, notably American, either of particular facets of scientific activity, or of Science in a general way, e.g. in Bernard Barber, *Science and the Social Order* (Collier Books, 1952) and Jean-Jacques Salomon, *Science et Politique* (Paris, Ed. du Seuil, 1970, reed. Economica). Disclosing the influence of politics, for instance of WW II and the Cold War, upon Science and Technology is not the same as going in politics as so many scientists believe without ever having read any serious historical work. And I do not see why being opposed to the military exploitation of mathematics and science should be considered as a more political stand than, for instance, helping Los Alamos or Arzamas to develop their "weapons of genocide" was.

[7] In *Alvarez: Adventures of a Physicist* (Basic Books, 1987), Luis Alvarez's trip to Hiroshima is the very first thing he relates in his book. He was also one of the main proponents of the H-bomb and, at the end of October, 1949, went to Washington to lobby in favor of it. In 1954, he testified at the Oppenheimer security hearing that Oppenheimer's opposition to the H-bomb was proof of an *exceedingly poor judgment*. Alvarez is one of many similar counterexamples to the "neutrality of Science" theory.

scientist's business is to provide his students or readers, without commentary, the knowledge they will later use, for better or for worse, as suits them. It will be up to them to discover by themselves, possibly years after graduating, that which "has no place" (why, please?) in scientific books or lectures and which was not told them by older scientists well aware of it, or who should have been. Let me give you a few French examples.

As a dozen of people who mostly work in particle physics assured me, you can spend five or eight years learning physics without ever having heard anything about nuclear weapons. I once checked the chemistry library of my Paris university for books by Louis Fieser, a most eminent Harvard chemist who, during WW II, was in charge of improving incendiary weapons and developed napalm; all of his chemistry books are there, but not his account of war work[8].

I found another, particularly caricatural, example in a textbook of physics for high-school finishing students; as required by the French official instructions in 1995, the concluding chapter, on the laser (never mind that an eighteen year old boy or girl can understand practically nothing to it), mentioned a number of civilian applications – ophthalmology, measure of atmospheric pollution, compact discs, energy production by laser-induced thermonuclear fusion[9], etc. – but not a single military use of lasers, a domain in which French industry was always very strong. This is not only dishonest; it is a foolish way to hide the truth since students read newspapers, look at TV, and if they type "laser military history" on www.google.com, they will get about 164,000 documents!

Thirty years ago, in part under the influence of what I had seen on American campuses and read in American newspapers and such reviews as *Science*

[8] Louis Fieser, *The Scientific Method. A Personal Account of Unusual Projects in War and in Peace* (Reinhold, 1964). In the *Biographical Memoirs*, v. 65, 1994, of the (American) National Academy of Science, Fieser's biographer has this to say (p. 165) about his work during WW II: "*With the approach of World War II, Fieser was drawn increasingly into war-related projects. A brief excursion into the area of mixed aliphatic-aromatic polynitro compounds for possible use as exotic explosives was followed by studies of alkali salts of long chain fatty acids as incendiaries, but by far the most important of his war-related work was his long and intensive study of the quinone antimalarials*", to which the author devotes one full page. The word "napalm" is nowhere to be found in this fourteen-page biography, a beautiful example of the art of fooling the reader with opaque technical jargon. All the more remarkable since Fieser was strongly criticised during the Vietnam War for his development of napalm. In his long biography of von Neumann in the *Dictionary of Scientific Biography*, J. Dieudonné devotes two lines to what he calls his "government" work without telling us whether it had to do with, say, the H-bomb or cancer research, two strongly supported domains of "government" work.

[9] This is a very long term project, but the French and American military have justified this very expensive enterprise by pointing out that the new knowledge of fusion processes it will provide will be used to improve nuclear weapons, a fact that is of course not mentioned in the textbook.

and the *Bulletin of the Atomic Scientists*, I succeeded, to my great astonishment, to convince the head of my Paris university library to start a new section that would be devoted to what was then called in America Science and Society studies. Although it received little money, you can now find there several thousands of (mostly American) books and the main reviews in the history of science and technology, including the military side of it, the arms race, economics of research and development, science policy, etc.: no exclusive. But almost all the readers are people who specialise in that field, while most of the 5,000 scientists working at the university don't even know the existence of this library. Since their specialised libraries are practically empty in this respect, the conclusion is inescapable: their only sources of information are their generally narrow personal experience[10], perhaps some historical articles written in scientific reviews by scientists who have no idea of historical writing[11], and cafeteria conversations:

> The humanist who looks at science from the point of view of his own endeavours is bound to be impressed, first of all, by its startling lack of insight into itself. Scientists seem able to go about their business in a state of indifference to, if not ignorance of, anything but the going, currently acceptable doctrines of their several disciplines ... The only thing wrong with scientists is that they don't understand science. They don't know where their institutions come from, what forces shaped and are still shaping them, and they are wedded to an antihistorical way of thinking which threatens to deter them from ever finding out[12].

It appears more honest to me to violate these miserable and far too comfortable taboos and to put on their guard those innocents who leap into the dark into careers of which they know nothing. Because of their past and potential

[10] It is not always that narrow. As in the USA – the model – there are in France scientists who have been for a long time in top government committees or who have cooperated with industry. They obviously know a lot more than the average researcher, let alone student. But they mostly don't speak, much less write, particularly when defence activities are involved. This striking difference between French and American "Statesmen of Science" can perhaps be explained by the fact that the political spectrum extends much farther to the left in France than in the USA, so that defence work was, at least during most of the Cold War, much more controversial here than on the other side of the Atlantic.

[11] One of the books I have recently read is Gregg Herken, *Brotherhood of the Bomb: The Tangled Lives and Loyalties of Robert Oppenheimer, Ernest Lawrence, and Edward Teller* (Henry Holt, 2002), a superb though very concentrated book. The main text, 334 pages, is followed by over 2,000 notes: an average of six references to sources per page (and a lot more on Internet). No active scientist could spend ten years reading two hundred books and papers already published, interviewing at length eighty colleagues, discovering and reading hundreds of recently declassified government files, and organizing this amount of information into a coherent book.

[12] Eric Larrabee, *Science and the Common Reader* (Commentary, June 1966). As I said above, old scientists who have long been top consultants to their government are not as innocent as Larrabee puts it, but the new generation has not their experience of science politics.

catastrophic consequences, the connections between science, technology and armaments concern all who go into science or technology or who practice them. They have been governed for half a century by the existence of public organisations and private enterprises whose function is *the systematic transformation of scientific and technological progress into military progress* within the limits, often elastic, of the economic capacities of the various countries which take part in it:

> With the attention which is paid in these days to weapons of war, there is probably no known scientific principle that has not already been carefully scrutinized to see whether it is of any significance for defence[13].

In countries – France is a prime example – where discussions on the relations between Science and Defence have been dominated for decades first by silence, then by a thick consensus[14], and have been totally absent from university teaching[15], the thing to say to young people is that *one of the forms of intellectual liberty is not to let oneself be dominated by the dominant ideas.*

But this requires access to other information sources. It would be impossible to thoroughly discuss this subject and its history within the framework of a mathematical treatise. I nevertheless decided to write a few dozen pages – the Postface to Volume II – in order to give the interested reader an idea of it and, in particular, to show that the question and the subject do exist. I have not balked at citing a good number of important bibliographical references

[13] Sir Solly Zuckerman, *Scientists and War* (London, Hamish Hamilton, 1962, p. 80); the author was at the time the British Government chief scientist and had formerly been the head of British military research. There is no reason to believe that Zuckerman's statement is no longer valid, particularly in America.

[14] "Science et Défense" is the title of a French association founded in 1983 by Charles Hernu, then the (socialist) Secretary of Defence and future hero of the Greenpeace affair – the clumsy sinking in Auckland harbour by French agents of a ship that would have interfered with a French nuclear test in the Pacific. Supported by the Armament branch of Defence, the association organises a yearly congress, where, over two days, engineers and scientists lecture on the technical problems of armaments and the closely related sciences. Several hundreds of people attend: military, engineers, industrialists, scientists, and, inevitably, political scientists and metaphysicians of strategy. France is, to my knowledge, the only country where what a number of American historians now call the scientific-military-industrial complex dares to exhibit itself so publicly and without provoking the least reaction. This would not have been possible before the conversion of the Socialist and Communist parties to nuclear weapons when, at the end of the 1970s, they saw a good prospect of winning the 1981 presidential election.

[15] America was, in the 1970s, a notable exception to this general statement: student protests against the Vietnam war and the cooperation of many university departments or laboratories with the DoD led some universities to add to their curriculum lectures on various aspects of "Science and Society" that attracted a sizeable number of science students, while some teachers in the history of science saw their audience suddenly grow. Although the traditional back to normal process did not take very long, many of the present generation of specialists found their calling during this period.

– there are plenty more – which will allow those who so desire to complete, verify, or discuss this text. I do not have the naive hope that a twenty year old student of mathematics will plunge into this ocean of literature; it would hardly even be a very good service to encourage him to do so. But maybe this text will find readers who are not so young and no longer have to submit to examinations or competitions for success. Although the French version of this Postface devoted a good deal of space to the French situation, I thought it better, in the English version, to emphasise the American situation more than I did in French, and this for several good reasons.

From Pearl Harbor to the present day, America has been the world leader in this domain – a leader which, for a dozen years, has no longer had any competitor worth naming and seems to be in a technological arms race against itself[16], as was already the case when it spent \$2 000 000 000 (one percent of its 1945 GNP) during WW II in order to get the atomic bomb before the Nazis, who did not believe it could be available in time and devoted very little resources to it. This American polarisation on scientific weapons, more or less faithfully imitated in the Soviet Union, Britain and France, had enormous political consequences; among others, it compelled the much weaker Soviet Union to devote to defence a proportion of its resources which must have strongly contributed to its downfall and to the present American hegemony. On the other hand, the civilian uses of mostly American military innovations in electronics, informatics, aviation, space, telecommunications, nuclear power, etc., had a deep influence on the daily life of people everywhere. Without WW II and the arms race, most of these innovations would have come much later, or never, because the financing of research, development and initial production by defence organisations made it possible for private enterprises to take risks which, otherwise, would have been barred by the return on investment principle that governs civilian innovations. Without World War II, no V-2 missiles and no atomic weapons; without these and the Cold War, no intercontinental ballistic missiles; without ICBMs and the need of the central military authorities for instant worldwide command, control, communication, and intelligence – C^3I as they call it – no satellites; and without satellites and many other innovations propelled by the military - computers, integrated circuits, Arpanet, etc. – then no Internet, to mention only this most spectacular spin-off of the arms race. The idea that civilian industry could have, by itself, spent tens or hundreds of billions in order to invent, produce and market such gigantic amounts of hardware and software at a time when nobody but the military had any proven need for it is foolish. Civilian business does not deal in science fiction.

WW II and the arms race also contributed to propelling the funding of scientific research proper to levels which, before 1939, would have seemed

[16] It has been recently disclosed that America will develop in the next 10 or 15 years an hypersonic cruise missile that will be able to strike anywhere on the Earth in less than two hours from bases in continental America.

unrealistic in the utmost, a fact of which scientists everywhere were the first beneficiaries although never nearly as much as American ones[17]. This not only made it possible for many more young Americans to choose scientific careers than was the case before WW II, it also attracted to America many scientists (and still more engineers) who had been educated elsewhere, a process that is continuing to this day – the famous *brain drain* that was first noticed in the 1950s, not to mention some Russian immigrants after 1917 and the European Jews in the 1930s.

* * *

The French version of this book included many citations and references in English, particularly in the Postface to Volume II, this in order to encourage the reader to use a language that is absolutely indispensable if one wants to inform oneself on anything at all: for clear demographic reasons France accounts for only a small proportion of the literature, for instance from 3% (technology) to 7% (mathematics) in the sciences on the world scale, and although French authors publish excellent books in many domains, scientific or not, they cannot be leaders everywhere. There is for instance nothing of any value on the history of nuclear weapons, not even of French ones, and none of the best American books have been translated. Almost all I know in the Science and Defence domain has been learned from American authors, although a few French historians of Science and Technology are beginning to deal with it.

There is no need to suggest to readers of the English translation of the present book to learn English. One should however warn the beginner that, even though well over 60% of the mathematical literature is now in English, an ability to read French is, at the research level, still needed. Since 1945, the Fields Medal has been awarded to 44 people worldwide; seven of them were French, and two more, although they are not, did all their previous work in France; the first Abel Prize (a recently created substitute to the nonexistent Nobel Prize for Mathematics) has been awarded in 2003 to Jean-Pierre Serre, who won a Fields Medal in 1954, and others won for instance the Wolf Prize. There are in France many more excellent mathematicians than these stars; although some publish in English, still many write in French. And there are of course German and Russian authors, among others, who still publish in the one language they learned as infants, as anglophone authors always did.

[17] In 1965, Isidor Rabi, a winner of the Nobel Prize in physics, pointed out that the budget of the Columbia University physics lab had grown from 15,000 dollars before the war to three millions and attributes this to the war which "*did wonderful things in some respects*". Hans Bethe, another Nobel Laureate, remembered in 1962 that before WW II he found it difficult to get some $3,000 for a cyclotron at Cornell, but that, "*Today, $3,000 is pin money. We use it in this laboratory in a day*". To be objective, one should also note the fantastic increase of civilian research funds allocated to Life Sciences, mainly Biology and Medicine; but even in this case, it was WW II, especially the development of penicillin, which at the start demonstrated what could be done in these fields with enough money and a concerted effort.

You fortunately don't need to learn Japanese: Japanese authors do not use it at the international level, a most courteous stand when you think that, for them, learning English is a lot more work than learning French is for the American, or English for the French.

The fact that English has acquired almost the status of an international common language, or lingua franca, has of course its upside, and any other reasonably widespread language, as Latin was three centuries ago, would do. The prevalence of English is often explained by the fact that it is supposedly simpler than, say, German, French, or Russian, and that anyway anglophones now form a large proportion of scientists at the world level. As suggested above, this preponderance of English, which goes far beyond Science, is also, and possibly mainly, a corollary of the enormous resources American government, industry and private foundations have devoted to Science and Technology since the 1940s and more generally of the overwhelming superiority of the American economy[18].

There is in France, and probably elsewhere too, a theory according to which, thanks to the overwhelming power America acquired in 1945 and still more in 1990, the result, or even purpose, of the "invasion of English" is to spread across the whole world the American conceptions of society, politics, economy, technology, mass media, etc. and to help American enterprises to acquire larger and larger parts of foreign markets everywhere, a process that, although or because successful, meets strong opposition in many countries.

Although greatly reinforced by WW II, it started much sooner. The use in America of such typical expressions as "richest in the world", "greatest in the world", "tallest in the world", "fastest in the world", "first in the world", etc. was already widespread in the 1900s and was a plain enough symptom. Standard Oil, General Electric, Ford were models of multinational companies that European enterprises tried (generally without much success at the time, if you except I.G. Farben in the 1920s) to imitate. American sewing machines, typewriters and accounting machines, agricultural machinery, machine tools and, between WW I and WW II, automobiles were invading Europe long before computers did. In the 1920s, jazz had already its fanatics everywhere, Hollywood's movies had already 60%-80% of the French market, most of the best movie theaters were in American hands, and the answer to French attempts to impose import quotas was a near total boycott of French movies in America (19 in 1929, against hundreds of American movies in France), a situation which did not improve after WW II. After 1918 it was Wilson, a U.S. President with the mind and eloquence of a Protestant missionary, who launched the Society of Nations, which Congress rejected. The United States' interventionist policy was already quite plain in the Americas, China and Japan long before the end of the XIXth century, and as a recent book[19]

[18] In some French companies, meetings of the Board are in English because of the presence of one or two American members. At the present time, about 20% of the total capitalisation of the Paris Stock Exchange is American-owned.

[19] Philippe Roger, *L'ennemi américain. Généalogie de l'antiaméricanisme français* (Paris, Seuil, 2002, 600 pp.) puts everything in historical perspective without

reminded us, French hostility toward America was powerfully increased by the war against Spain in 1898, which was viewed by the French right as a threat to European colonial empires, and by the left as a conclusive proof of the transformation of an already unpalatable American capitalism into outright imperialism or economic colonialism. As to the present American taste for firearms, a unique feature among "civilised" countries as they were called in the 1900s, it was Samuel Colt who, during the war against Mexico, triggered the craze by adopting the American system of manufacture invented in arsenals in order to mass produce his celebrated revolvers. Present inequalities in the distribution of income are not worse than they were at the time John D. Rockefeller was worth over one billion dollars, i.e. about 2% of America's GNP: a proportion which, nowadays, would amount to some 200 billion. And New York bar owners pouring French wine on the street[20] were seen already long before March 2003.

Thus, nothing very new under the sun, except that American international preponderance and unilateralism have now acquired the status of an official doctrine supported by a host of ideologists invoking a fundamentalist Protestant ethic in order to justify interventions which, in the eyes of a vast majority of people everywhere, are nothing but displays of power even when they rid nations of barbaric rulers or religious oppression in the hope of establishing there a (probably very weak) version of Western democracy.

That being said, nobody has to appreciate the barbaric music and violent movies which presently come from America (the American stars of my youth were Charlie Chaplin, Buster Keaton and the Marx Brothers). Americans do not merely dictate the export of these productions through international commercial agreements and by owning big distribution companies; they also sell them by finding indigenous customers (or imitators) who are only too happy to make money by distributing them among a young and most often uncultured public. And how would local television fill its hours of programmes, how could the cinemas function, without the flow of American productions? The work force in France (say) is not large enough to replace American mediocrity with French mediocrity; and no country is capable of producing a new Shakespeare or a new Bartok every day. One therefore broadcasts what is available or imitates American crass "games".

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himself falling into the trap. It goes without saying that many of the criticisms that some French intellectuals and politicians of the right addressed to America would apply just as well to France. Howard Zinn, *A People's History of the United States, 1492-Present* (HarperCollins, 2003 edition), while or because very one-sided, would be very useful to help understand criticism from the French left, which was never as systematic and well organised as Zinn's, not to mention books by Lewis Mumford, Noam Chomsky, etc.

[20] If a boycott of French wines were to lower prices in France, I, for one, would not shed crocodile tears at the tragic fate of poor American patriots heroically depriving themselves of Chateau Latour at $1,000 a bottle (assuming they don't have a stock of it in their cellar).

Nor is one obliged to approve the Darwinian concepts of economic competition and social relations which, thanks to technologies that have emerged straight from the Cold War and arms race, are presently expanding under the name of "globalisation": the extension to the planet of a "liberal", i.e. capitalist, and "modern" economic system founded on the principles isolated by Adam Smith in 1776 and assimilated erroneously by the robber barons who, at the end of the XIXth century, erected the great American capitalist enterprises, afterwards revised a little and codified. It is now forbidden to shoot the strikers but not to domesticate the unions; to dismiss thousands of employees to improve the competitiveness of companies and in return to exploit the work force at low pay in developing countries; to push for the dismantlement of European social welfare systems hard won after a century of struggle but now judged too expensive – or smacking of Socialism? – by the alumni of the Harvard Business School and its foreign imitations; to suborn the public markets by handing cheques to political parties as is presently the case in France, Germany, Italy, etc., or, in the Third World, to gangsters in high places in order to inundate the countries they rule or own with killing machines under the pretext of lowering the unit price for the countries that produce them, or in order to secure the rights to exploiting their natural resources. It is the reign of money, whose rallying slogan was launched a hundred and fifty years ago by a famous French minister: *Enrichissez-vous*! If you can[21] ...

That said, America possesses, notably in its universities, an intellectual class not to be globally confused with the spokesmen of the Pentagon's warlords or the operators of Wall Street. In particular and as I said above, no one, in France, has revealed the military influence on scientific and technological development since 1940 as a number of American historians, particularly of the younger generation, have done for a quarter of a century with the help of massive documentation; if you are interested in, say, the history of the Cold War, you will find in the American literature all the information, points of view and opinions you want. There is no need either to point out that many American novelists did not wait until 2003 to disseminate unorthodox descriptions of the American society. As to the mathematicians, many of whom have always been very critical of official policy, the years I spent with my family in the 1950s and 1960s at Urbana, Berkeley and Princeton were among the happiest of my life. And when, at the end of October 1961, my Paris flat was destroyed[22] because I had, rather mildly in fact, spoken out during a lecture against the savage repression in Paris of a peaceful Algerian demonstration for independence, I received two days later a telegram from J. Robert Oppenheimer inviting me, on very generous terms, to spend the remainder of the academic year at the Princeton Institute; we went two

[21] What Guizot said is: Get rich, by your work and savings – a cynical precept at a time when the overwhelming majority of people, after working twelve hours a day six days per week, would die as poor as their parents were.