Electromagnetic Fields and Waves

电磁场与电磁波

张　育　张福恒　王　磊◎编著

中山大学出版社

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前　言

本书是为普通高等学校电子信息类专业“电磁场理论”基础课所编写的本科双语教材，注重于系统的基础理论推演与实际应用相结合，避免学生在学习过程中对公式和题型的生搬硬套，难度符合国内该课程的要求。本书可作为电子信息类专业本科生一学期课程的教材和教学参考用书，也可作为物理专业本科生和工程技术人员的参考用书。

由于电磁场是一个矢量场，电磁场理论除涉及一般的微积分计算外，还涉及二阶多元偏微分方程组及场变量和源变量的计算，因此给学生的学习带来了一定的困难。本书的特点在于：在矢量分析中，完整地介绍了学习电磁场理论所需的矢量场理论，突出了矢量场理论的物理意义，特别介绍了与本书学习密切相关的戴尔（del）算符的运算；同时，给出了标量场和矢量场的图像描绘方法，并介绍了对学习电磁场理论很有帮助的矢量场唯一性定理和亥姆霍兹定理。在求解静态场边值问题时，本书强调了唯一性定理的数理意义和应用，并利用单调函数以及相关函数的正交完备性来求解。在静态场泊松方程的导出、时变电磁场波动方程的导出、平面电磁波的特性分析、辐射场的推导等内容的论述中，本书特别突出了将戴尔算符作为运算工具，便于学生掌握电磁场理论分析的系统方法，在实际教学体验中，有效地提高了学生求解实际问题的能力。

全书共分为9章，即矢量分析、静电场、静电场的特殊解法、恒定电流场、恒定磁场、时变电磁场、平面电磁波、导行电磁波及天线。每章均配有难度适当、基本覆盖本章内容的习题，大部分章节配有相应的思考题。

在本书的编写过程中，编者参阅了国内外许多优秀教材及相关参考书，在此对参考文献的作者表示感谢。

由于编者水平有限，书中难免存在不当之处，欢迎广大读者批评指正。

张　育　张福恒　王　磊
2013年6月于海口

Contents

Chapter 1

Vector Analysis

1.1 Introduction

Electromagnetic field is a vector field. Thus vector analysis is one of the basic mathematical tools for studying the properties of electromagnetic fields. In this chapter, we mainly introduce the essential knowledge of vector field theory: the vectors operation, the gradient of scalar field, the divergence and curl of vector field, and the operation rule of operator ∇ called del or nabla which is important for the operation of the vector fields. Later, we will introduce some important theorems of the vector fields, and the property of Dirac delta-function δ in this chapter. Although, in the study of the electromagnetic field theory, the all mathematical tools are not only these, what we introduce in this chapter will play an important role in our discussion of electromagnetic field theory.

1.2 Vectors Operation

Most of the quantities encountered in the study of the electromagnetic field theory can be divided into two classes, scalars and vectors.

A quantity, such as mass, length, temperature, energy and electric potential, which only has magnitude, is called *scalar*.

A quantity, such as force, displacement, velocity, electric field intensity and magnetic field intensity, which has both magnitude and direction, is called *vector*. In this book, vectors will be represented by boldface italic types.

A *unit vector* is defined as a vector of unit magnitude and will be written as $\boldsymbol{a}$. If a unit vector $\boldsymbol{a}$ is chosen to have the direction of vector $\boldsymbol{A}$, then we can write vector $\boldsymbol{A}$ as

$$\boldsymbol{A} = A\boldsymbol{a} \quad \text{and} \quad \boldsymbol{a} = \frac{\boldsymbol{A}}{A}$$

We also can say that $\boldsymbol{a}$ is the unit vector of $\boldsymbol{A}$.

It is often more convenient to represent vectors by arrows, with the length and direction of the arrow representing the magnitude and direction of the vector, as shown in Figure 1-2-1. Two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are equal if they have the same magnitude and direction. We can only compare vectors if they have the same physical or geometrical meaning and hence the same dimension. If the magnitude of a vector is zero, the vector is called *null vector* or *zero vector*. This is the only vector that cannot be represented as an arrow because it has zero magnitude.

We also can define the vector area. Suppose that we have a plane surface of scalar area s. We can define a vector area $\boldsymbol{s}$ whose magnitude is s, and whose direction is perpendicular to the plane, in the sense determined by a right-hand grip rule on the rim, see Figure 1 -2 -2. This quantity clearly possesses both magnitude and direction. Similarly, a vector element of area is $d\boldsymbol{s}$. Its direction is determined as a matter of fixed convention by the right-hand-thread rule.

$$d\boldsymbol{s} = ds\boldsymbol{a}_n$$

$\boldsymbol{a}_n$ is the unit vector of the normal direction of ds.

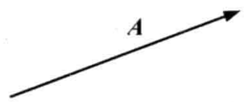


Figure 1 -2 -1 A vector $\boldsymbol{A}$

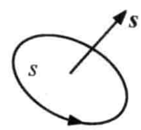


Figure 1 -2 -2 A vector area $\boldsymbol{s}$

1. Vector addition and subtraction

Vectors can be added each other, and it gives another vector. The operations of *vector addition* obey the commutative and associative laws.

Commutative law: $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A}$ (1 -2 -1)

Associative law: $(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C}=\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})$ (1 -2 -2)

To add two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, it gives another vector $\boldsymbol{C}$, $\boldsymbol{C}=\boldsymbol{A}+\boldsymbol{B}$, the triangle of vectors or the parallelogram of vectors gives the rule of two vectors addition, shown in Figure 1 -2 -3 below.

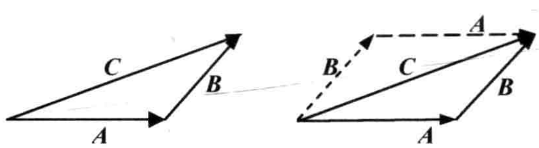


Figure 1 -2 -3 Vector addition: $\boldsymbol{C}=\boldsymbol{A}+\boldsymbol{B}$

By $\boldsymbol{C}=\boldsymbol{A}+\boldsymbol{B}$, it means a vector $\boldsymbol{C}$ can be replaced by two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$. In other words, a single vector can be replaced by two vectors which have the same effect and are called the components of the vector. Any vector can be resolved into components. Conversely, two vectors can be replaced by a single vector which has the same effect.

If $\boldsymbol{B}$ is a vector, then $-\boldsymbol{B}$ (minus $\boldsymbol{B}$) is also a vector with the same magnitude as $\boldsymbol{B}$ but in opposite direction. So we can define *vector subtraction*, $\boldsymbol{C}=\boldsymbol{A}-\boldsymbol{B}$, as

$$\boldsymbol{C}=\boldsymbol{A}+(-\boldsymbol{B})$$

Figure 1 -2 -4 shows the subtraction of $\boldsymbol{B}$ from $\boldsymbol{A}$.

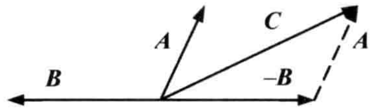


Figure 1-2-4 Vector subtraction: $\boldsymbol{C}=\boldsymbol{A}-\boldsymbol{B}$

2. Multiplication of vector by a scalar

Multiplying a vector $\boldsymbol{A}$ by a scalar k, we obtain a vector $\boldsymbol{B}$ such as

$$\boldsymbol{B}=k\boldsymbol{A}$$

The magnitude of vector $\boldsymbol{B}$ is simply equal to $|k|$ times the magnitude of vector $\boldsymbol{A}$. If $k>0$, $\boldsymbol{B}$ is in the same direction as $\boldsymbol{A}$. If $k<0$, $\boldsymbol{B}$ is in opposite direction from $\boldsymbol{A}$.

3. Scalar product

The *scalar product* of two vectors is also called *dot product* of two vectors, or *inner product*. The dot product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ is written as $\boldsymbol{A}\cdot\boldsymbol{B}$ and is read as "$\boldsymbol{A}$ dot $\boldsymbol{B}$". It is defined as the product of the magnitudes of the two vectors and the cosine of the smaller angle between them, as illustrated in Figure 1-2-5. That is

$$\boldsymbol{A}\cdot\boldsymbol{B} = AB\cos\theta \tag{1-2-3}$$

It is obvious that the dot product is a scalar and it obeys the commutative law

$$\boldsymbol{A}\cdot\boldsymbol{B} = BA\cos\theta = \boldsymbol{B}\cdot\boldsymbol{A} \tag{1-2-4}$$

Equation (1-2-3) is an algebraic expression for the dot product of two vectors. Its geometric meaning is that the scalar product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ is a product of the magnitude of vector $\boldsymbol{A}$ and the projection of vector $\boldsymbol{B}$ on vector $\boldsymbol{A}$, or the magnitude of vector $\boldsymbol{B}$ and the projection of vector $\boldsymbol{A}$ on vector $\boldsymbol{B}$, as shown in Figure 1-2-5.

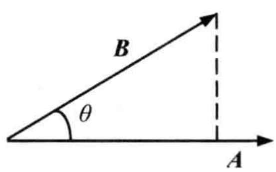


Figure 1-2-5 Illustration for the dot product

The magnitude of a vector $\boldsymbol{A}$ can be obtained by let $\boldsymbol{A}=\boldsymbol{B}$ in Equation (1-2-3),

$$A = \sqrt{\boldsymbol{A}\cdot\boldsymbol{A}} \tag{1-2-5}$$

The scalar product also obeys the distributive law

$$\boldsymbol{A}\cdot(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{A}\cdot\boldsymbol{B}+\boldsymbol{A}\cdot\boldsymbol{C} \tag{1-2-6}$$

Example 1-2-1 If A, B, and C form the three sides of a triangle with angle θ opposites to side C, use vectors to prove the law of cosine for a triangle

$$C = [A^2 + B^2 - 2AB\cos\theta]^{1/2}$$

Solution: From Figure 1-2-6, we have

$$\boldsymbol{C} = \boldsymbol{B} - \boldsymbol{A}$$

The magnitude of vector $\boldsymbol{C}$, from Equation (1-2-5), is

$$C = \sqrt{(\boldsymbol{B} - \boldsymbol{A}) \cdot (\boldsymbol{B} - \boldsymbol{A})}$$

In terms of Equations (1-2-6) and (1-2-4), we have

$$(\boldsymbol{B} - \boldsymbol{A}) \cdot (\boldsymbol{B} - \boldsymbol{A}) = B^2 - \boldsymbol{B} \cdot \boldsymbol{A} - \boldsymbol{A} \cdot \boldsymbol{B} + A^2 = A^2 + B^2 - 2AB\cos\theta$$

Therefore,

$$C = [A^2 + B^2 - 2AB\cos\theta]^{1/2}$$

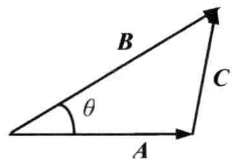


Figure 1-2-6 A triangle formed by $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$

4. Vector product

The *vector product* of two vectors is also called *cross product*, or *exterior product*. It is written as $\boldsymbol{A} \times \boldsymbol{B}$ and is read as "$\boldsymbol{A}$ cross $\boldsymbol{B}$". The cross product is a vector which is directed normal to the plane containing $\boldsymbol{A}$ and $\boldsymbol{B}$ and is equal in magnitude to the product of the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, and the sine of the smaller angle between them. That is

$$\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{a}_{\perp} AB\sin\theta \qquad (1-2-7)$$

where $\boldsymbol{a}_{\perp}$ is a unit vector perpendicular to the plane containing $\boldsymbol{A}$ and $\boldsymbol{B}$, and its direction follows the *right-hand grip rule*: if the fingers of your right hand curl from $\boldsymbol{A}$ to $\boldsymbol{B}$ then the thumb points in the direction of $\boldsymbol{A} \times \boldsymbol{B}$, as shown in Figure 1-2-7. It shows the direction of $\boldsymbol{a}_{\perp}$ is perpendicular to both $\boldsymbol{A}$ and $\boldsymbol{B}$.

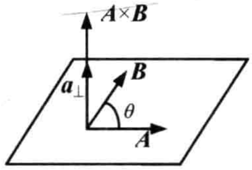


Figure 1-2-7 Illustration for the cross product

It is easy to show that

$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A} \qquad (1-2-8)$$

Thus, the operation of vector product is not commutative. We also can show that the cross product obeys the distributive law

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{C}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{C} \qquad (1-2-9)$$

The cross product of two vectors also has its geometric meaning: If it is a parallelogram with A and B as sides, as shown in Figure 1 – 2 – 8, the vector area $\boldsymbol{s}$ of the parallelogram then is given by

$$\boldsymbol{s} = \boldsymbol{A} \times \boldsymbol{B} \tag{1-2-10}$$

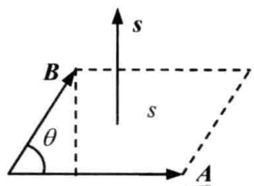


Figure 1 – 2 – 8 A vector area $\boldsymbol{s}$ constructed by vectors $\boldsymbol{A}$ and $\boldsymbol{B}$

It also can be said that the magnitude of the cross product is equal to the area of the parallelogram with A and B as sides.

Example 1 – 2 – 2 If A, B, and C form the three sides of a triangle, as shown in Figure 1 – 2 – 9, use vectors to prove the law of sine for a triangle.

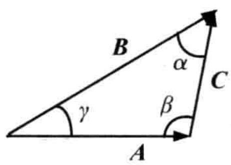


Figure 1 – 2 – 9 A triangle formed by $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$

Solution: From Figure 1 – 2 – 9, we have

$$\boldsymbol{B} = \boldsymbol{C} - \boldsymbol{A} \quad \text{and} \quad \boldsymbol{B} \times \boldsymbol{B} = \boldsymbol{B} \times (\boldsymbol{C} - \boldsymbol{A}) = 0$$

It gives

$$\boldsymbol{B} \times \boldsymbol{C} = \boldsymbol{B} \times \boldsymbol{A} \quad \text{or} \quad BC\sin\alpha = BA\sin\gamma$$

From this, we have

$$\frac{A}{\sin\alpha} = \frac{C}{\sin\gamma}$$

In the same way, we can get

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta}$$

Therefore,

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

5. Product of three vectors

The *scalar triple product* of three vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ is a scalar and written as

$$\boldsymbol{C} \cdot (\boldsymbol{A} \times \boldsymbol{B})$$

Since $\boldsymbol{A}\times\boldsymbol{B}$ is a vector, its direction denoted as $\boldsymbol{a}_n$, $\boldsymbol{a}_n=\dfrac{\boldsymbol{A}\times\boldsymbol{B}}{|\boldsymbol{A}\times\boldsymbol{B}|}$, therefore,

$$\boldsymbol{C}\cdot(\boldsymbol{A}\times\boldsymbol{B})=C(AB\sin\theta)\cos\phi=ABC\sin\theta\cos\phi$$

where θ is the smaller angle between the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, ϕ is the smaller angle between the vectors $\boldsymbol{C}$ and $\boldsymbol{a}_n$. Obviously, the scalar triple product is a scalar. If it is parallelepiped with A, B and C as sides, as shown in Figure 1-2-10, then the scalar triple product yields its volume.

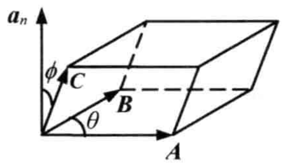


Figure 1-2-10 Illustration for the scalar triple product

From Figure 1-2-10, we can get the *scalar triple product equality* below

$$\boldsymbol{A}\cdot(\boldsymbol{B}\times\boldsymbol{C})=(\boldsymbol{A}\times\boldsymbol{B})\cdot\boldsymbol{C}=\boldsymbol{B}\cdot(\boldsymbol{C}\times\boldsymbol{A}) \tag{1-2-11}$$

The *vector triple product* of three vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ is a vector and is written as $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})$. From above discussion, It is obvious that the $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})$ is not equal to the $(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{C}$. So, the brackets are important in the vector operation.

There is another useful formula called the *vector triple product identity* or *double cross product identity* as below

$$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})=(\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B}-(\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C} \tag{1-2-12}$$

It is used over and over again in our study.

Example 1-2-3 Prove Equation (1-2-12).

Proof: Suppose there are three vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$, as shown in Figure 1-2-11 (a). Let

$$\boldsymbol{A}=\boldsymbol{A}_\perp+\boldsymbol{A}_{//}$$

where $\boldsymbol{A}_\perp$ and $\boldsymbol{A}_{//}$ are the vectors that are respectively normal and parallel to the plane containing the vectors $\boldsymbol{B}$ and $\boldsymbol{C}$. Therefore, we have

$$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})=(\boldsymbol{A}_\perp+\boldsymbol{A}_{//})\times(\boldsymbol{B}\times\boldsymbol{C})=\boldsymbol{A}_{//}\times(\boldsymbol{B}\times\boldsymbol{C})$$

Let

$$\boldsymbol{D}=\boldsymbol{A}_{//}\times(\boldsymbol{B}\times\boldsymbol{C})$$

$\boldsymbol{D}$ and $\boldsymbol{A}_{//}$ are all parallel to the plane containing the vectors $\boldsymbol{B}$ and $\boldsymbol{C}$. From Figure 1-2-11 (b), we have

$$\begin{aligned}\boldsymbol{D}\cdot\boldsymbol{a}_D&=D=A_{//}BC\sin(\theta_2-\theta_1)\\&=-(B\sin\theta_1)(A_{//}C\cos\theta_2)+(C\sin\theta_2)(A_{//}B\cos\theta_1)\\&=[\boldsymbol{B}(\boldsymbol{A}_{//}\cdot\boldsymbol{C})-\boldsymbol{C}(\boldsymbol{A}_{//}\cdot\boldsymbol{B})]\cdot\boldsymbol{a}_D\end{aligned}$$

We know, if $\boldsymbol{b}\cdot\boldsymbol{a}=\boldsymbol{c}\cdot\boldsymbol{a}$, we get $\boldsymbol{b}=\boldsymbol{c}+k\boldsymbol{d}$, where vector $\boldsymbol{d}$ is normal to vector $\boldsymbol{a}$ and k is a scalar. Because $\boldsymbol{A}_{//}$ is normal to $\boldsymbol{D}$, from above we have

$$B(A_{//} \cdot C) - C(A_{//} \cdot B) = D + kA_{//}$$

Now we find out k. Using $A_{//}$ dot above two sides, the left educes

$$(A_{//} \cdot B)(A_{//} \cdot C) - (A_{//} \cdot C)(A_{//} \cdot B) = 0$$

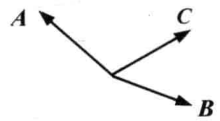
while the right

$$A_{//} \cdot D + kA_{//}^2 = kA_{//}^2$$

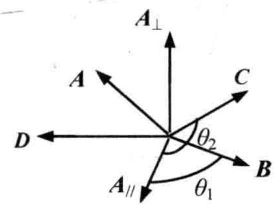
We get $k = 0$. Therefore,

$$D = B(A_{//} \cdot C) - C(A_{//} \cdot B) = B(A \cdot C) - C(A \cdot B)$$

Equation (1-2-12) has been proved.



(a) Three vectors A, B, and C



(b) $D \perp A_{\perp}, D \perp A_{//}$

Figure 1-2-11 Proving the vector triple product identity

Summary: The operation rule of vectors:

(1) $A + B = B + A$

(2) $A \cdot B = B \cdot A$

(3) $A \cdot (B + C) = A \cdot B + A \cdot C$

(4) $A \times B = -B \times A$

(5) $A \cdot (B \times C) = (A \times B) \cdot C = B \cdot (C \times A)$

(6) $A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$

Review questions:

1) What is a unit vector?

2) What is the significance of a zero vector?

3) How can you determine the area of a triangle with two vectors A and B as two sides?

1.3 Coordinate Systems

The discussion above is quite general and uses graphical representations when operating vectors. From a mathematical point of view it is very convenient to work with the vectors when they are resolved into components along three mutually orthogonal directions. In this section, we will introduce the curvilinear orthogonal coordinate, and the most useful three ordinary orthogonal coordinate systems: the rectangular (or Cartesian) coordinate system, the cylindrical (circular) coordinate system, and the spherical coordinate system.

1. Curvilinear orthogonal coordinates

The electromagnetic field laws and physical quantities are not varied with coordinate system. In reality, they should be expressed in a coordinate system which is suitable to the geometrical shape given in the problem.

In the three-dimensional space, the position of a point P can be determined by the intersection point of three surfaces. The three surfaces are expressed respectively by u_1 = constant, u_2 = constant, and u_3 = constant. Here u_1, u_2, and u_3 denote three coordinates (distances or angles) or coordinate variables. If the three surfaces are mutually orthogonal, we get an *orthogonal coordinate system*. We do not use the non-orthogonal coordinates because the non-orthogonal coordinates will make the problems more complex.

Let $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_3$ represent the unit vectors pointing in the directions of independent positive displacements of u_1, u_2, and u_3 respectively in three-dimensional orthogonal coordinate system, as shown in Figure 1-3-1. These unit vectors form an orthogonal triad at each point in space. They are perpendicular respectively to the three corresponding surfaces, u_1 = constant, u_2 = constant, and u_3 = constant at point $P(u_1, u_2, u_3)$.

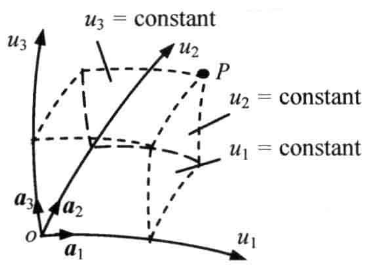


Figure 1-3-1 Curvilinear coordinates

As the three surfaces are perpendicular one another at every point in space, in *right-hand curvilinear orthogonal coordinates*, the relationships of three unit vectors are

$$\boldsymbol{a}_1 \times \boldsymbol{a}_2 = \boldsymbol{a}_3$$
$$\boldsymbol{a}_2 \times \boldsymbol{a}_3 = \boldsymbol{a}_1$$
$$\boldsymbol{a}_3 \times \boldsymbol{a}_1 = \boldsymbol{a}_2$$

and

$$\boldsymbol{a}_i \cdot \boldsymbol{a}_j = \delta_{ij}$$

where

$$i, j = 1, 2, 3 \quad \text{and} \quad \delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

δ_{ij} is called *Kronecker* delta sign. Therefore, in three-dimensional orthogonal coordinates (u_1, u_2, u_3), a vector $\boldsymbol{A}$ at a point in space can be written as

$$\boldsymbol{A} = A_1\boldsymbol{a}_1 + A_2\boldsymbol{a}_2 + A_3\boldsymbol{a}_3 \tag{1-3-1}$$