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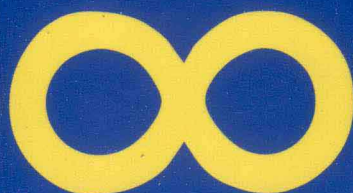
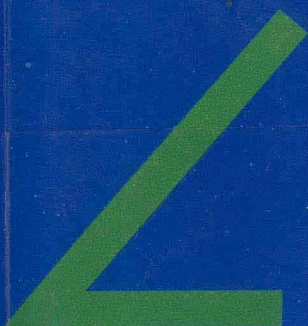
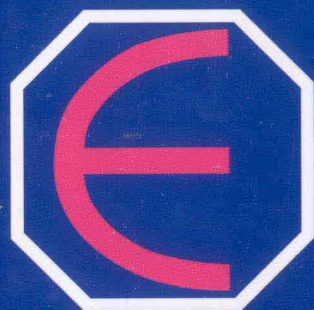
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| 美国原版经典数学课本 |

# RAY'S HIGHER ARITHMETIC

## 美国小学数学 ④

JOSEPH RAY



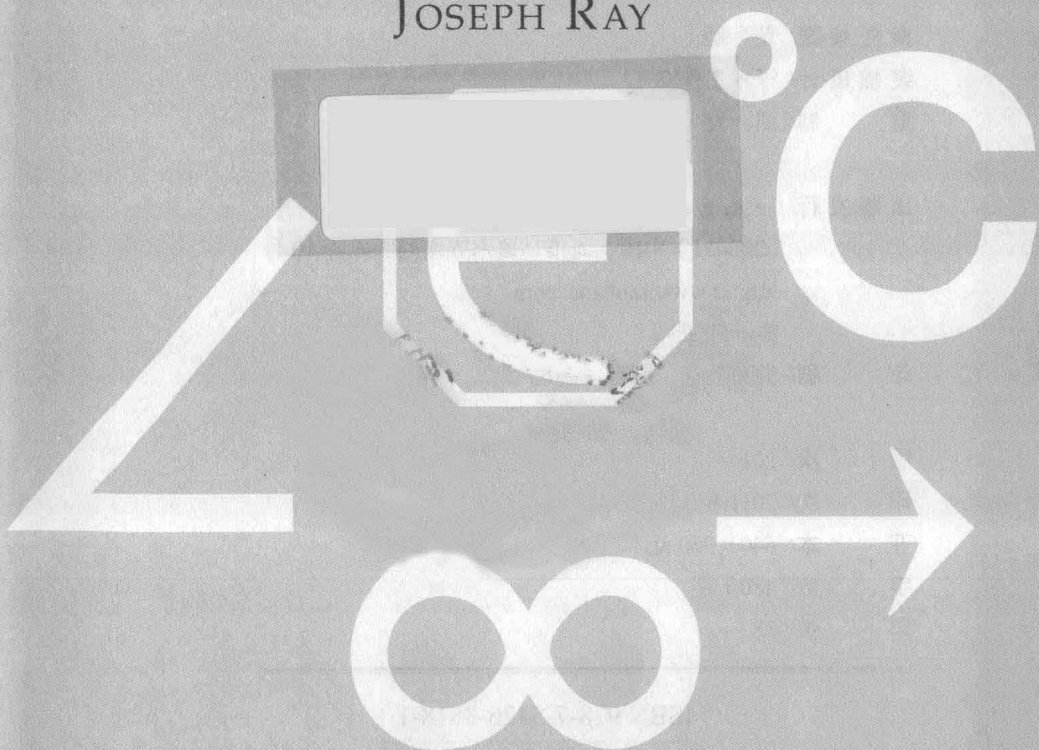
上海三联书店

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呈现于您面前的这套美国数学课本，是一套在西方流行了近半个世纪、至今仍在使用的经典教材。编者约瑟夫·雷伊教授，1807年出生于美国弗吉尼亚俄亥俄县，从小在当地学校接受教育，成绩优秀。16岁时开始其教师职业生涯。18岁，雷伊来到富兰克林学院跟随乔尔·马丁教授学习医学，此后又进入俄亥俄医学院学习。大学毕业后，他在辛辛那提伍德沃德中学任教，讲授数学。1836年，伍德沃德中学由高中升格为辛辛那提伍德学院，雷伊成为该学院教授。1851年，该校又变为一所公立高中，雷伊一直在此担任校长，直至去世。雷伊一生杰出的成就是他倾心编写的系列数学教材，并以此闻名。这套数学课本与他在伍德学院的同事威廉·麦加菲编写的《美国语文读本》，同时被美国近万所学校作为教材，累计销量均超过1.22亿册，对几代美国人的教育产生了很大影响。直至今日，这两套书仍被当作美国家庭教育(Homeschooling)的推荐教材，也是美国学生准备SAT考试的参考用书。

与其他数学书相比，雷伊数学教材至少有以下几个明显特点：

第一，强调在“学”中掌握“数”。例如，《小学数学》不完全按难度分册，而是根据其实际应用范围分为四册：初级算术、智力算术、实用算术与高级算术。让学生从对数的认知、运算法则的掌握，延伸到数学在实际生活中的广泛应用，如购物、记账、存款、利息等，并向更高的学术层次过渡。



第二，将数学问题融于文字题（Word Problem）之中。即便最简单的加减运算，它也通过讲故事的方式呈现出来。这样孩子们在学习数学时，不仅可以训练其数学思维，语言能力也可以同步提高。

第三，将抽象思维具体化。书中的数学题大都结合现实事物表述出来，让孩子们理解他们所学的数学在现实生活中是如何加以应用的。这对低年级学生来说，尤其帮助很大，他们能更快更清楚地理解那些对其年龄来讲过于抽象的数学概念。

第四，将不同学科知识融入数学问题中。这种编写方法能让学生从数学应用的不同领域来掌握数学科学，帮助学生从低年级数学步入更复杂的数学应用领域，如几何学与会计学等。孩子们在学习数学的同时，又能接受其他学科知识。如书中有这样一道题：“华盛顿将军出生于公元1732年，他活了67岁，那么他是于哪一年去世？”这么一道简单的计算题，便将历史知识与数学结合起来，一举多得。

对于中国孩子来讲，这套数学课本不仅能教孩子学习数学，更是学习英语的很好途径，让他们换个思维学英语。与阅读文学读本相比，这是另一种不同的感觉，或许更能激发孩子学习英语的兴趣。数学的词汇含义固定，也易于理解记忆，孩子在解题的同时也能提高英语水平，可谓一举多得。对于那些将来准备参加出国英语考试的学生来讲，这套书意义更大，对他们将来的求学之路应该大有帮助。

最后，我们需向读者特别说明一点，由于这套书涉及数字与数学符号偏多，考虑到重新录入排版会出现一些难免的错误，给读者学习带来极大不便。于是我们采用了原版影印的办法，以保证内容的高度准确性，但文字清晰度与重新录入相比略有缺陷，敬请读者谅解。

衷心祝愿天下孩子们快乐成长，并期待您的宝贵意见与建议。

出版者

2011年春



CHAPTER I . INTRODUCTION.....	1
CHAPTER II . NUMERATION AND NOTATION .....	6
CHAPTER III . ADDITION.....	13
CHAPTER IV . SUBTRACTION .....	17
CHAPTER V . MULTIPLICATION .....	21
CHAPTER VI . DIVISION .....	32
CHAPTER VII . PROPERTIES OF NUMBERS .....	48
CHAPTER VIII . COMMON FRACTIONS .....	63
CHAPTER IX . DECIMAL FRACTIONS.....	86
CHAPTER XI . COMPOUND DENOMINATE NUMBERS.....	111
CHAPTER XII . RATIO .....	160
CHAPTER XIII . PROPORTION.....	162
CHAPTER XIV . PERCENTAGE .....	173
CHAPTER XV . PERCENTAGE. —APPLICATIONS .....	184
CHAPTER XVI . PERCENTAGE. —APPLICATIONS .....	226
CHAPTER XVII . PARTNERSHIP.....	310
CHAPTER XVIII . ALLIGATION .....	316

CHAPTER XIX. INVOLUTION ..... 325

CHAPTER XX. EVOLUTION ..... 330

CHAPTER XXI. SERIES ..... 352

CHAPTER XXII. MENSURATION ..... 361

CHAPTER XXIII. MISCELLANEOUS EXERCISES ..... 383

# RAY'S

## HIGHER ARITHMETIC.

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### I. INTRODUCTION.

**Article 1.** A **definition** is a concise description of any object of thought, and must be of such a nature as to distinguish the object described from all other objects.

2. **Quantity** is any thing which can be increased or diminished; it embraces number and magnitude. Number answers the question, "How many?" Magnitude, "How much?"

3. **Science** is knowledge properly classified.

4. The primary truths of a science are called **Principles**.

5. **Art** is the practical application of a principle or the principles of science.

6. **Mathematics** is the science of quantity.

7. The elementary branches of mathematics are Arithmetic, Algebra, and Geometry.

8. **Arithmetic** is the introductory branch of the science of numbers. Arithmetic as a science is composed of defini-



tions, principles, and processes of calculation ; as an art, it teaches how to apply numbers to theoretical and practical purposes.

9. A **Proposition** is the statement of a principle, or of something proposed to be done.

10. Propositions are of two kinds, demonstrable and indemonstrable.

*Demonstrable propositions* can be proved by the aid of reason. *Indemonstrable propositions* can not be made simpler by any attempt at proof.

11. An **Axiom** is a self-evident truth.

12. A **Theorem** is a truth to be proved.

13. A **Problem** is a question proposed for solution.

14. Axioms, theorems, and problems are propositions.

15. A process of reasoning, proving the truth of a proposition, is called a **Demonstration**.

16. A **Solution** of a problem is an expressed statement showing how the result is obtained.

17. The term **Operation**, as used in this book, is applied to illustrations of solutions.

18. A **Rule** is a general direction for solving all problems of a particular kind.

19. A **Formula** is the expression of a general rule or principle in algebraic language ; that is, by symbols.

20. A **Unit** is one thing, or one. *One thing* is a concrete unit ; *one* is an abstract unit.

21. **Number** is the expression of a definite quantity. Numbers are either abstract or concrete. An *abstract number* is one in which the kind of unit is not named ; a *concrete number* is one in which the kind of unit is named. Concrete numbers are also called *Denominate Numbers*.

22. Numbers are also divided into Integral, Fractional, and Mixed.

An *Integral number*, or *Integer*, is a whole number; a *Fractional number* is an expression for one or more of the equal parts of a divided whole; a *Mixed number* is an Integer and Fraction united.

23. A **Sign** is a character used to show a relation among numbers, or that an operation is to be performed.

24. The signs most used in Arithmetic are

$$\begin{array}{ccccccccc} + & - & \times & \div & \checkmark & & & & \\ = & : & :: & ( ) & \text{---} & . . . . & \therefore & & \end{array}$$

25. The *sign of Addition* is  $[+]$ , and is called *plus*. The numbers between which it is placed are to be added. Thus,  $3 + 5$  equals 8.

*Plus* is described as a perpendicular cross, in which the bisecting lines are equal.

26. The *sign of Subtraction* is  $[-]$ , and is called *minus*. When placed between two numbers, the one that follows it is to be taken from the one that precedes it. Thus,  $7 - 4$  equals 3.

*Minus* is described as a short horizontal line.

*Plus* and *Minus* are Latin words. *Plus* means *more*; *minus* means *less*.

Michael Steifel, a German mathematician, first introduced  $+$  and  $-$  in a work published in 1544.

27. The *sign of Multiplication* is  $[\times]$ , and is read *multiplied by*, or *times*. Thus,  $4 \times 5$  is to be read, 4 *multiplied by* 5, or 4 *times* 5.

The sign is described as an oblique cross.

William Oughtred, an Englishman, born in 1573, first introduced the sign of multiplication.

28. The *sign of Division* is  $[\div]$ , and is read *divided by*. When placed between two numbers, the one on the left is

to be divided by the one on the right. Thus,  $20 \div 4$  equals 5.

The sign is described as a short horizontal line and two dots: one dot directly above the middle of the line, and the other just beneath the middle of it.

Dr. John Pell, an English analyst, born in 1610, introduced the sign of division.

29. The *Radical sign*,  $[\sqrt{\quad}]$ , indicates that some root is to be found. Thus,  $\sqrt{36}$  indicates that the *square root* of 36 is required;  $\sqrt[3]{125}$ , that the *cube root* of 125 is to be found; and  $\sqrt[4]{625}$  indicates that the *fourth root* of 625 is to be extracted.

The root to be found is shown by the small figure placed between the branches of the Radical sign. The figure is called the *index*.

30. The signs,  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $\sqrt{\quad}$ , are symbols of operation.

31. The *sign of Equality* is  $[=]$ , two short horizontal parallel lines, and is read *equals* or *is equal to*, and signifies that the quantities between which it is placed are equal. Thus,  $3 + 5 = 9 - 1$ . This is called an *equation*, because the quantity  $3 + 5$  is equal to  $9 - 1$ .

32. **Ratio** is the relation which one number bears to another of the same kind. The *sign of Ratio* is  $[:]$ . Ratio is expressed thus,  $6 : 3 = \frac{6}{3} = 2$ , and is read, the ratio of 6 to 3 = 2, or is 2.

The sign of ratio may be described as the sign of division with the line omitted. It has the same force as the sign of division, and is used in place of it by the French.

33. **Proportion** is an equality of ratios. The *sign of Proportion* is  $[::]$ , and is used thus,  $3 : 6 :: 4 : 8$ ; this may be read, 3 is to 6 as 4 is to 8; another reading, the ratio of 3 to 6 is equal to the ratio of 4 to 8.

34. The signs  $[( ), — ]$ , are *signs of Aggregation*—the first is the *Parenthesis*, the second the *Vinculum*. They are used for the same purpose; thus,  $24 - (8 + 7)$ , or  $24 - \overline{8 + 7}$ , means that the sum of  $8 + 7$  is to be subtracted from 24. The numbers within the parenthesis, or under the vinculum, are considered as one quantity.

35. The dots  $[. . . ]$ , used to guide the eye from words at the left to the right, are called *Leaders*, or the *sign of Continuation*, and are read, *and so on*.

36. The *sign of Deduction* is  $[ \therefore ]$ , and is read *therefore*, *hence*, or *consequently*.

37. The signs,  $=, :, ::, ( ), —, \dots, \therefore$ , are symbols of relation.

38. Arithmetic depends upon this primary proposition: that any number may be increased or diminished. “Increased” comprehends Addition, Multiplication, and Involution; “decreased,” Subtraction, Division, and Evolution.

39. The fundamental operations of Arithmetic in the order of their arrangement, are: Numeration and Notation, Addition, Subtraction, Multiplication, and Division.

## II. NUMERATION AND NOTATION.

40. **Numeration** is the method of reading numbers.

**Notation** is the method of writing numbers. Numbers are expressed in three ways; viz., by *words*, *letters*, and *figures*.

41. The first nine numbers are each represented by a single figure, thus:

1	2	3	4	5	6	7	8	9
one.	two.	three.	four.	five.	six.	seven.	eight.	nine.

All other numbers are represented by combinations of these and another figure, 0, called *zero*, *naught*, or *cipher*.

**REMARK.**—The cipher, 0, is used to indicate *no value*. The other figures are called *significant* figures, because they indicate *some value*.

42. The number next higher than 9 is named *ten*, and is written with two figures, thus, 10: in which the cipher, 0, merely serves to show that the unit, 1, on its left, is different from the unit, 1, standing alone, which represents a *single thing*, while this, 10, represents a *single group of ten things*.

The nine numbers succeeding *ten* are written and named as follows:

11	12	13	14	15	16
eleven.	twelve.	thirteen.	fourteen.	fifteen.	sixteen.
	17	18	19		
	seventeen.	eighteen.	nineteen.		

In each of these, the 1 on the left represents a *group of ten things*, while the figure on the right expresses the units or *single things* additional, required to make up the number.

**REMARK.**—The words *eleven* and *twelve* are supposed to be derived from the Saxon, meaning *one left after ten*, and *two left after ten*. The words *thirteen*, *fourteen*, etc., are contractions of *three and ten*, *four and ten*, etc.

The next number above nineteen (*nine and ten*), is *ten and ten*, or two *groups of ten*, written 20, and called *twenty*.

The next numbers are *twenty-one*, 21; *twenty-two*, 22; etc., up to *three tens*, or *thirty*, 30; *forty*, 40; *fifty*, 50; *sixty*, 60; *seventy*, 70; *eighty*, 80; *ninety*, 90.

The highest number that can be written with two figures is 99, called *ninety-nine*; that is, nine tens and nine units.

The next higher number is 9 tens and ten, or *ten tens*, which is called *one hundred*, and written with *three* figures, 100; in which the two ciphers merely show that the unit on their left is neither a single thing, 1, nor a group of ten things, 10, but a *group of ten tens*, being a unit of a higher order than either of those already known.

In like manner, 200, 300, etc., express *two hundreds*, *three hundreds*, and so on, up to *ten hundreds*, called a *thousand*, and written with *four* figures, 1000, being a unit of a still higher order.

**43.** The **Order** of a figure is the place it occupies in a number.

From what has been said, it is clear that a figure in the 1st place, with no others to the right of it, expresses *units* or single things; but standing on the left of another figure, that is, in the 2d place, expresses *groups of tens*; and standing at the left of two figures, or in the 3d place, expresses *tens of tens*, or *hundreds*; and in the 4th place, expresses *tens of hundreds* or *thousands*. Hence, counting from the right hand,

The order of <i>Units</i>	is in the 1st place,	1
The order of <i>Tens</i>	is in the 2d place,	10
The order of <i>Hundreds</i>	is in the 3d place,	100
The order of <i>Thousands</i>	is in the 4th place,	1000

By this arrangement, *the same figure has different values according to the place, or order, in which it stands*. Thus, 3 in the first place is 3 *units*; in the second place 3 *tens*, or thirty; in the third place 3 *hundreds*; and so on.



44. The word *Units* may be used in naming all the orders, as follows:

<i>Simple units</i>	are called	<i>Units of the 1st order.</i>
<i>Tens</i>	" "	<i>Units of the 2d order.</i>
<i>Hundreds</i>	" "	<i>Units of the 3d order.</i>
<i>Thousands</i>	" "	<i>Units of the 4th order.</i>
etc.		etc.

45. The following table shows the place and name of each order up to the fifteenth.

TABLE OF ORDERS.

15th.	14th.	13th.	12th.	11th.	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.
Hundreds of Trillions .	Tens of Trillions . .	Trillions . . . .	Hundreds of Billions .	Tens of Billions . .	Billions . . . .	Hundreds of Millions .	Tens of Millions . .	Millions . . . .	Hundreds of Thousands.	Tens of Thousands . .	Thousands . . . .	Hundreds . . . .	Tens . . . .	Units . . . .

46. For convenience in reading and writing numbers, orders are divided into groups of three each, and each group is called a **period**. The following table shows the grouping of the first fifteen orders into five periods:

TABLE OF PERIODS.

Hundreds Tens Units 6 5 4	of Trillions.	Hundreds Tens Units 3 2 1	of Billions.	Hundreds Tens Units 9 8 7	of Millions.	Hundreds Tens Units 6 5 4	of Thousands.	Hundreds Tens Units 3 2 1	of Units.
5th Period.		4th Period.		3d Period.		2d Period.		1st Period.	

47. It will be observed that each period is composed of units, tens, and hundreds of *the same denomination*.

48. List of the Periods, according to the common or French method of Numeration.

First	Period, Units.	Sixth	Period, Quadrillions.
Second	" Thousands.	Seventh	" Quintillions.
Third	" Millions.	Eighth	" Sextillions.
Fourth	" Billions.	Ninth	" Septillions.
Fifth	" Trillions.	Tenth	" Octillions.

The next twelve periods are, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novendecillions, Vigintillions.

PRINCIPLES.—1. *Ten units of any order always make one of the next higher order.*

2. *Removing a significant figure one place to the left increases its value tenfold; one place to the right, decreases its value tenfold.*

3. *Vacant orders in a number are filled with ciphers.*

PROBLEM.—Express in words the number which is represented by 608921045.

SOLUTION.—The number, as divided into periods, is 608·921·045; and is read six hundred and eight million nine hundred and twenty-one thousand and forty-five.

Rule for Numeration.—1. *Begin at the right, and point the number into periods of three figures each.*

2. *Commence at the left, and read in succession each period with its name.*

REMARK.—Numbers may also be read by merely naming each figure with the name of the place in which it stands. This method, however, is rarely used except in teaching beginners. Thus, the numbers expressed by the figures 205, may be read *two hundred and five*, or *two hundreds no tens and five units*.

## EXAMPLES IN NUMERATION.

7	4053	204026	4300201
40	7009	500050	29347283
85	12345	730003	45004024
278	70500	1375482	343827544
1345	165247	6030564	830070320

832045682327825000000321  
 8007006005004003002001000000  
 60030020090080070050060030070  
 504030209102800703240703250207

**PROBLEM.**—Express in figures the number four million twenty thousand three hundred and seven. 4020307.

**SOLUTION.**—Write 4 in *millions* period; place a dot after it to separate it from the next period: then write 20 in *thousands* period; place another dot: then write 307 in *units* period. This gives 4·20·307. As there are but *two* places in the thousands period, a cipher must be put before 20 to complete its orders, and the number correctly written, is 4020307.

**NOTE.**—Every period, except the highest, must have *three* figures; and if any period is not mentioned in the given number, supply its place with three ciphers.

**Rule for Notation.**—*Begin at the left, and write each period in its proper place—filling the vacant orders with ciphers.*

**PROOF.**—Apply to the number, as written, the rule for Numeration, and see if it agrees with the number given.

## EXAMPLES IN NOTATION.

- |                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                               |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ol style="list-style-type: none"> <li>1. Seventy-five.</li> <li>2. One hundred and thirty-four.</li> <li>3. Two hundred and four.</li> <li>4. Three hundred and seventy.</li> <li>5. One thousand two hundred and thirty-four.</li> </ol> | <ol style="list-style-type: none"> <li>6. Nine thousand and seven.</li> <li>7. Forty thousand five hundred and sixty-three.</li> <li>8. Ninety thousand and nine.</li> <li>9. Two hundred and seven thousand four hundred and one.</li> </ol> |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|