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Discrete Mathematics

离散数学

（英文版）

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华中师范大学出版社

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内容简介

本书是信息与计算科学和计算机科学核心课程——离散数学的基础教材。全书共分七章，分别介绍了离散数学的最基本内容：命题逻辑、谓词逻辑、集合理论、关系、图论、树和代数结构。内容叙述严谨，推理详尽。

本书可作为普通高等学校信息与计算科学专业和计算机专业学生离散数学课程的双语教学教材，亦可作为自动控制、电子工程、管理科学等有关专业的教学用书和工作人员的阅读参考。

新出图证(鄂)字10号

图书在版编目(CIP)数据

离散数学 = Discrete Mathematics：英文/刘红美 编著.
—武汉：华中师范大学出版社，2013.1
ISBN 978-7-5622-5901-5

Ⅰ.①离… Ⅱ.①刘… Ⅲ.①离散数学—高等学校—教材—英文 Ⅳ.①O158

中国版本图书馆 CIP 数据核字(2013)第002839号

离散数学

(英文版)

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作者：刘红美 ©

责任编辑：袁正科　　责任校对：罗　艺　　封面设计：胡　灿

编辑室：第二编辑室　　电话：027－67867362

出版发行：华中师范大学出版社

社址：湖北省武汉市珞喻路152号

电话：027－67863280/67863426(发行部)　027－67861321(邮购)

传真：027－67863291

网址：http://www.ccnupress.com　　电子信箱：hscbs@public.wh.hb.cn

印刷：海军工程学院印刷厂　　督印：章光琼

字数：318千字

开本：787mm×960mm　1/16　　印张：16.25

版次：2013年1月第1版　　印次：2013年1月第1次印刷

定价：35.00元

欢迎上网查询、购书

前　言

为了适应经济全球化和应对科技革命的挑战，全国很多高校相继开设外语或双语教学的专业课程。离散数学在内容、学时、年级等方面有着特殊性，特别适合改革试点课程进行外语或双语教学。

离散数学是研究离散变量的结构及相互关系的一门学科，是计算机科学与技术专业的核心课程，也是其他相关专业的必修课程或重要选修课程。离散数学强调逻辑性和抽象性，注重概念、方法和应用。它的基本概念、基本理论和基本方法大量地应用在数字电路、算法分析与设计、数据结构、程序设计原理、操作系统、数据库系统、人工智能、计算机网络等专业课程中。通过对离散数学的学习，一方面为学生的专业课学习、软件开发和科学研究打下坚实的基础；另一方面培养和提高了学生的抽象思维、逻辑思维和归纳构造能力；同时也可极大地提升学生的数学建模能力，极大地提升学生进行应用研究和解决实际问题的能力。用英语或双语进行离散数学的教学，能够使学生在获得上述专业技能的同时，在专业英语方面也得到锻炼，对学生以后的职业生涯有莫大的好处。

本书作为一个学期离散数学英语教学的教材，是作者经过多年教学实践，对讲义反复增删提炼的结晶。考虑到课时的限制，本书仅包含离散数学的最基本内容：命题逻辑、谓词逻辑、集合理论、关系、图论、树和基本的代数结构。

本书力求语言流畅、通俗易懂、简明精练。为方便初次接触数学英语的同学，书中配有部分常用专业词汇的中文解释。本书适合作为普通高等学校计算机类和工程类本、专科生的离散数学外语或双语教学教材，亦可作为其他相关专业的教学用书和工作人员的阅读参考用书。

在编写过程中，作者参阅了大量国内外目前广泛使用的优秀原版外文教材和优秀中文教材以及其他相关文献和资料。书中一些定理的证明，以及部分例题和练习题采用自 Kenneth H. Rosen 的“Discrete Mathematics and Its Application(Fifth Edition)”，James L. Hein 的“Discrete Mathematics”，J. A. Bondy 和U. S. R. Murty的“Graph Theory with Applications”，Kenneth H. Rosen 的“Handbook of Discrete and

Combinatorial Mathematics”,耿素云和屈婉玲编著的《离散数学》等著作,同时作者根据国内双语教学的实际做了恰当的取舍和编排。另外,河南理工大学计算机科学与技术学院的王建芳老师、三峡大学理学院的熊新华、张钦老师对本书部分章节的修改做了大量的工作,在此向他们表示衷心的感谢。在本书的出版过程中,得到了评审专家的精心指导和编辑的大力支持,借此机会也向他们表示衷心的感谢。

限于作者的学识水平,错误和不当在所难免,诚挚欢迎批评指正。

作　者

2012 年 9 月

CONTENTS

1

Propositional Logic

Logic is the basis for distinguishing what may be correctly inferred from a given collection of facts. Propositional logic, which is also called propositional calculus, studies logical propositions and their combinations using logical connectives. Propositional logic is also named as zero-order logic because it does not use quantifiers, namely quantifiers range over nothing. This chapter defines the meaning of the symbolism and gives various propositional logical properties that are usually used without explicit mention. Only two-valued logic is studied in this chapter, i. e., each statement is either true or false. Multi-valued logic, in which statements have one of more than two values, involves fuzzy sets theory whose detailed discussion is out of the scope of the book. Propositional logic has vast area of applications ranging from natural science to social science. Later in this chapter, some examples of applications in computer science such as circuit design and verification of computer program correctness are presented.

1.1 Propositions and Connectives

In mathematical reasoning we need to use declarative sentences to state conditions and conclusions. These sentences are called propositions, which are the basic building blocks of logic. A **proposition** is a declarative sentence that is either true or false, but not both. We use the lower case letters of the alphabet such as $p, q, r, s, \cdots$ to denote propositions. The truth value of a proposition is true, denoted by T or 1, if it is a true proposition and false, denoted by F or 0, if it is a false proposition. The following are examples of propositions.

EXAMPLE 1.1.1 Each of these declarative sentences is a proposition:

1. $\sqrt{2}$ is irrational.
2. Beijing is the capital of China.
3. $1+3=5$.
4. $9+10 \leqslant 12$.
5. We'll live on the Moon by the end of this century.

Propositions 1 and 2 are true, and therefore their truth values are 1, whereas 3 and 4 are false, so their truth values are 0. The fifth sentence is also a proposition, although at present we do not know its truth value. But by the end of this century, we'll know its truth value. Some sentences that are not propositions are given in the following example.

EXAMPLE 1.1.2 Consider the following sentences:

1. No smoking.
2. Is there life on the Moon?
3. $x=2$.
4. $x>y$.

Sentences 1 and 2 are not propositions because they are not declarative sentences. Sentences 3 and 4 are not propositions because they are neither true nor false, since the variables in these sentences have not been assigned values.

The propositions in EXAMPLE 1.1.1 are called **primitive** propositions, for there is no way to break them down into simpler propositions. **Compound proposition**, on the other hand, is formed by applying logical operators to one or more primitive propositions. The logical operators, also called **connectives**, are denoted by the following symbols and words:

$\neg$ not, negation.

$\wedge$ and, conjunction.

$\vee$ or, disjunction.

$\rightarrow$ conditional, implication.

$\leftrightarrow$ if and only if, biconditional.

DEFINITION 1.1.1 Let p be a proposition. The negation of p, denoted by $\neg p$, is a new proposition. If p is true, then $\neg p$ is false. If p is false, then $\neg p$ is true. The proposition $\neg p$ is read "not p".

EXAMPLE 1.1.3 Let p denote the proposition "Three Gorges Project is located in Yichang". Then $\neg p$ is "Three Gorges Project is not located in Yichang."

The negation of a proposition can be regarded as the result of the operation of Negation Operator on a proposition. The other connectives introduced above can also be used to construct compound propositions from two or more existing propositions.

DEFINITION 1.1.2 Let p and q be propositions. The compound proposition "p and q", denoted by $p \wedge q$, is the proposition that is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the conjunction of p and q.

EXAMPLE 1.1.4 Translate these sentences into logical expressions.

1. Jane likes singing and dancing.
2. Ruth and Richard are classmates.
3. The sun shone on the sea and the waves danced and sparkled.
4. Jack is an intelligent child, but not diligent in his work.

Solution:

1. Let p and q represent the primitive propositions "Jane likes singing" and "Jane likes dancing", respectively. The compound proposition "Jane likes singing and dancing" can be expressed as $p \wedge q$.
2. It is a primitive proposition. We can use a letter p to represent it.
3. Let p, q and r denote "The sun shone on the sea", "The waves danced" and "The waves sparkled", respectively. The compound proposition can be represented as $p \wedge q \wedge r$.
4. Let p and q denote the propositions "Jack is an intelligent child" and "Jack is diligent in his work", respectively. Then this compound proposition can be represented as $p \wedge \neg q$.

DEFINITION 1.1.3 Let p and q be propositions, the expression $p \vee q$ denotes the disjunction of p and q, and is read "p or q". The proposition $p \vee q$ is false when p and

q are both false, and true otherwise.

We use the connective "or" in disjunction in the **inclusive** sense, that is, a disjunction is true when at least one of the two propositions is true.

EXAMPLE 1.1.5 Let p denote the proposition "Advanced Mathematics is a required course for college freshmen" and q denote "Margaret Mitchell wrote 'Gone with the wind'". Then $p \vee q$ is "Advanced Mathematics is a required course for college freshmen, or Margaret Mitchell wrote 'Gone with the wind'."

Sometimes, we use *or* in an exclusive sense. The **exclusive or** is denoted by $\oplus$. The proposition $p \oplus q$ is true if one or the other but not both of the propositions p and q is true. One way to express $p \oplus q$ for p and q in the above example is "Advanced Mathematics is a required course for college freshmen, or Margaret Mitchell wrote 'Gone with the wind', but not both."

DEFINITION 1.1.4 Let p and q be propositions. The implication $p \rightarrow q$ is the proposition which is false when p is true and q is false, and true otherwise. p is called the antecedent, premise, or hypothesis, and q is called the consequence or conclusion of $p \rightarrow q$.

An implication is sometimes called a **conditional statement**. Some common ways to read the expression $p \rightarrow q$ are "if p then q", "p implies q", "p is sufficient for q", "since p, therefore q", "p only if q", "q whenever p", "q is necessary for p", "q follows from p", "q if p", "$\neg p$ unless q".

EXAMPLE 1.1.6 Denote the following propositions in symbolic forms, and find their truth values.

1. If $2+3=5$, then snow is white.
2. If $2+3 \neq 5$, then snow is white.
3. If $2+3 \neq 5$, then snow is not white.
4. If $2+3=5$, then snow is not white.
5. 4 is a divisor of a only if 2 is a divisor of a, where a is a given positive integer.
6. 4 is not a divisor of a unless 2 is a divisor of a, where a is a given positive integer.

Solution: Let p and q denote "$2+3=5$" and "snow is white", respectively. Then both p and q are true. The propositions 1, 2, 3 and 4 can be expressed as $p\rightarrow q$, $\neg p\rightarrow q$, $\neg p\rightarrow \neg q$ and $p\rightarrow \neg q$, respectively, and they have truth values 1, 1, 1, 0, respectively.

Let r be "4 is a divisor of a", s be "2 is a divisor of a". The proposition 5 and proposition 6 are in the form of $r\rightarrow s$. If a is really divisible by 4, then a is also divisible by 2. Therefore, $r\rightarrow s$ is true. If a is not divisible by 4, then $r\rightarrow s$ is still true no matter whether a is divisible by 2, since r is false.

From EXAMPLE 1.1.6, we see that, the way we have defined implications is more general than the meaning attached to implications in the English language. For instance, proposition 5 is an implication used in normal language, since there is a relationship between the hypothesis and the conclusion. However, in proposition 1, there is no relationship between the hypothesis and conclusion. In mathematical reasoning we consider implications more general than in English.

DEFINITION 1.1.5 The biconditional of two propositions p and q is denoted by $p\leftrightarrow q$, which is read "p if and only if q" or "p is necessary and sufficient for q". The proposition $p\leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

Sometimes "p if and only if q" is abbreviated as "p iff q".

EXAMPLE 1.1.7 Represent the following sentences in logical expressions.

1. 2 is a prime if and only if $\sqrt{5}$ is a rational number.
2. If two lines A and B are parallel, then their corresponding angles are equal, and vice versa.

Solution:

1. Let p denote "2 is a prime" and q denote "$\sqrt{5}$ is a rational number". The proposition 1 can be expressed as $p\leftrightarrow q$, its truth value is 0.
2. Let r and s denote "A and B are parallel" and "the corresponding angles of A and B are equal", respectively. Then the proposition 2 can be represented as $r\leftrightarrow s$, which has truth value 1.

Truth tables are used to display the relationships between the truth values of propositions, where we write "0" for false and "1" for true. As an example, Table 1.1.1 is the truth table for the propositions obtained from p and q by applying the six logical operators defined above.

Table 1.1.1 The truth table for the Negation of Proposition, the Conjunction, Disjunction, Exclusive Or, Implication and Biconditional of two Propositions

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \to q$	$p \leftrightarrow q$
0	0	1	0	0	0	1	1
0	1	1	0	1	1	1	0
1	0	0	0	1	1	0	0
1	1	0	1	1	0	1	1

The four possible truth assignments for p and q can be listed by any order. But the particular order presented above will be proved useful.

We can construct compound propositions using the logical operators defined so far. Generally parentheses are used to specify the order in which the logical operators in a compound proposition are to be applied. However, to reduce the number of parentheses, we define the hierarchy of evaluation for the connectives of the propositional calculus as: $\neg$, $\wedge$, $\vee$, $\to$ and $\leftrightarrow$.

EXAMPLE 1.1.8 Let p, q and r denote the propositions "Beijing is the capital of China", "Three Gorges Project is located in Yichang" and "$1+3\neq 4$", respectively. Find the truth values for the following compound propositions:

1. $(\neg p \vee q) \wedge (\neg p \vee \neg q) \leftrightarrow r$
2. $(p \wedge r) \to (q \to \neg r)$
3. $(p \vee r) \wedge (q \vee \neg r) \to (\neg p \vee \neg q)$

Solution: p, q and r have truth values 1, 1 and 0, respectively. Consequently, proposition 1, 2 and 3 have truth values 1, 1 and 0, respectively.

WORDS AND EXPRESSIONS

propositional logic	命题逻辑
proposition	命题
truth value	真值
primitive proposition	原子命题

compound proposition	复合命题
negation	否定
conjunction	合取
disjunction	析取
implication	蕴涵式
biconditional	双条件式
truth table	真值表

EXERCISES 1.1

1. Which of the following sentences are propositions? What are the truth values of those that are propositions?

   a. Every map in the world can be colored using four colors.

   b. Discrete Mathematics is a required course for Computer Science Major.

   c. Is the computer available?

   d. $x+3$ is a positive integer.

   e. $4+x=5$.

   f. 2 and 3 are even.

2. Let p, q be primitive propositions for which the implication $p \to q$ is false. Determine the truth values of

   $\neg p \wedge q$ $\quad q \to p$ $\quad \neg q \to \neg p$ $\quad p \vee \neg q$ $\quad q \leftrightarrow \neg p$

3. Let p, q, r denote the following statements:

   p: It is sunny.

   q: I'll go climbing.

   r: I'm free.

   Convert the following statements into symbolic forms.

   a. I am not free.

   b. If it is sunny, I'll go climbing.

   c. I'm free, but it is not sunny.

   d. I'll go climbing only if I am free and it is sunny.

   e. If I'm free, I will go climbing unless it is not sunny.

   f. Whenever it is sunny, I'll go climbing.

   g. Sunny day and free time are sufficient for going climbing.

4. Write the following statements in symbolic forms.

   a. It is cold and it is windy.

   b. If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

   c. It is necessary to wash the boss's car to get promoted.

   d. Winds from the South imply a spring thaw.

   e. If you watch television, your mind will decay, and vice versa.

   f. Low humidity and sunshine are sufficient for me to play tennis this afternoon.

   g. It is snowing but, we will go out for a work.

5. Determine the truth value of each of the following compound propositions.

   a. If $1+1=2$, then $2+3=5$.

   b. If $1+1=3$, then $2+3=4$.

   c. If $1+1=3$, then $2+3=5$.

   d. If people can fly, then $1+2=4$.

   e. $1+1=2$ if and only if $2+3=4$.

   f. $1>2$ if and only if $3>2$.

6. There are some related implications that can be formed from $p \to q$. For example,

   The proposition $q \to p$ is called the converse of $p \to q$.

   The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$.

   The proposition $\neg q \to \neg p$ is called the contra-positive of $p \to q$.

   What are the contra-positive, converse, and inverse of the implication:

   "You can ask for help whenever you need it"?

In computer programming the If-Then and If-Then-Else decision structures arise in languages such as BASIC and C++. The hypothesis p is often a relational expression (such as $x>5$). This expression is a logical proposition that has truth value 0 or 1, depending on the value of the variables contained in the expression (such as x) at that point in the program. The conclusion q may be an "executive statement" directing the program to another line or causing some results to be printed. (So q is not one of the logical statements.) When dealing with "if p then q", in this text, the computer executes q only on the condition that p is true. For p being false, the computer goes to the next instruction in the program sequence. For the decision structure "if p then q else r", q is executed when p is true and r is executed when p is false.

7. What are the values of m, n after each of these statements is encountered in a given C++ program, if $m = 3, n = 8$ before the first statement is reached? [Here the values of m, n following the execution of the statement in part (a) become the values of m, n for the statement in part (b), and so on, through the statement in part (g). The div operation in C++ returns the integer part of a quotient. For example 6 div 2 = 3, 5 div 2 = 2.]
   a. If $n - m == 5$, then $n = n - 2$;
   b. If $((2 * m == n)$ and $(n \text{ div } 4 == 1))$, then $n = 4 * m - 3$;
   c. If $[(n < 8)$ or $(m \text{ div } 2 == 2)]$, then $n = 2 * m$ else $m = 2 * n$;
   d. If $[(m < 20)$ or $(n \text{ div } 6 == 1)]$, then $m = m - n - 5$;
   e. If $[(n == 2 * m)$ or $(n \text{ div } 2 == 5)]$, then $m = m + 2$;
   f. If $[(n \text{ div } 3 == 3)$ and $(m \text{ div } 3 <> 1)]$, then $m = n$;
   g. If $m * n <> 35$ then $n = 3 * m + 7$.

Fuzzy logic is used in artificial intelligence. In fuzzy logic, a proposition has a truth value that is a number between 0 and 1, inclusive. A proposition with a truth value of 0 is false and one with a truth value of 1 is true. Truth values that are between 0 and 1 indicate varying degrees of truth. For instance, the truth value 0.99 can be assigned to the statement "Niomi is happy", since Niomi is very happy, and the truth value 0.3 can be assigned to the statement "Jack is happy", since Jack is happy slightly less than half at the time.

8. The truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition. What are the truth values of the statements "Niomi is not happy" and "Jack is not happy"?
9. The truth value of the conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions. What are the truth values of the statements "Niomi and Jack are happy" and "Neither Niomi nor Jack is happy"?
10. The truth value of the disjunction of two propositions in fuzzy logic is the maximum of the truth values of the two propositions. What are the truth values of the statements "Niomi is happy, or Jack is happy" and "Niomi is not happy, or Jack is not happy"?

1.2 Propositional WFF and Assignment

In Section 1.1, we have defined primitive proposition and compound proposition. Primitive proposition does not contain any connectives, and compound proposition contains at least one connective. For example, if p and q are propositions, then $\neg p$, $\neg p \vee q$, and $(p \leftrightarrow q) \vee \neg q \rightarrow p$ are compound propositions. But if p and q are propositional variables, it means that they have not been assigned any specified propositions. The expressions above then are called well-formed formula or wff for short, which can be pronounced as "woof". A wff is a proposition only if the propositional variables contained in the wff are assigned some specified propositions.

We define the wff by the following inductive definition for the set of propositional wffs.

DEFINITION 1.2.1

1. A propositional variable is a wff.
2. If A is a wff, then $\neg A$ is a wff.
3. If A, B are wffs, then $A \wedge B$, $A \vee B$, $A \rightarrow B$ and $A \leftrightarrow B$ are wffs.

For example, $(p \rightarrow q) \wedge (q \leftrightarrow r)$, $(p \wedge q) \vee \neg r$, and $p \wedge (\neg q \vee r)$ are wffs, but $pq \rightarrow r$ and $(p \rightarrow (q \rightarrow q)$ are not.

We often use capital letters to refer to arbitrary propositional wffs. For example, if we say, A is a wff, we mean that A represents some arbitrary wff. We also use capital letters to denote specific propositional wffs. For example, if we want to discuss the wff $p \wedge q \wedge \neg r$ several times, we may let $W = p \wedge q \wedge \neg r$. Then we can refer to W instead of always writing down the symbols $p \wedge q \wedge \neg r$.

Since there may exist some propositional variables in a wff, we usually don't know its truth value. If all propositional variables in a wff are assigned some specified propositions, the wff becomes a proposition. For example, $(p \vee q) \rightarrow r$ is a wff. Let p be "2 is a prime", q "3 is an even" and r "5 is a rational number". Then p and r are true, but q is false. This wff can be translated as "if 2 is a prime, or 3 is an even, then 5 is a rational number", which is true. In fact, the truth value of a wff depends on the truth values of the propositional variables contained in the wff.