Alexandre J. Chorin, Jerrold E. Marsden

A Mathematical Introduction to Fluid Mechanics

Third Edition

流体力学数学导论 第3版

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Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)*.

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

This book is based on a one-term course in fluid mechanics originally taught in the Department of Mathematics of the University of California, Berkeley, during the spring of 1978. The goal of the course was not to provide an exhaustive account of fluid mechanics, nor to assess the engineering value of various approximation procedures. The goals were:

- to present some of the basic ideas of fluid mechanics in a mathematically attractive manner (which does not mean "fully rigorous");
- to present the physical background and motivation for some constructions that have been used in recent mathematical and numerical work on the Navier–Stokes equations and on hyperbolic systems; and
- to interest some of the students in this beautiful and difficult subject.

This third edition has incorporated a number of updates and revisions, but the spirit and scope of the original book are unaltered.

The book is divided into three chapters. The first chapter contains an elementary derivation of the equations; the concept of vorticity is introduced at an early stage. The second chapter contains a discussion of potential flow, vortex motion, and boundary layers. A construction of boundary layers using vortex sheets and random walks is presented. The third chapter contains an analysis of one-dimensional gas flow from a mildly modern point of view. Weak solutions, Riemann problems, Glimm's scheme, and combustion waves are discussed.

The style is informal and no attempt is made to hide the authors' biases and personal interests. Moreover, references are limited and are by no

means exhaustive. We list below some general references that have been useful for us and some that contain fairly extensive bibliographies. References relevant to specific points are made directly in the text.

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Summer, 1997

Contents

1

The Equations of Motion

In this chapter we develop the basic equations of fluid mechanics. These equations are derived from the conservation laws of mass, momentum, and energy. We begin with the simplest assumptions, leading to Euler's equations for a perfect fluid. These assumptions are relaxed in the third section to allow for viscous effects that arise from the molecular transport of momentum. Throughout the book we emphasize the intuitive and mathematical aspects of vorticity; this job is begun in the second section of this chapter.

1.1 Euler's Equations

Let D be a region in two- or three-dimensional space filled with a fluid. Our object is to describe the motion of such a fluid. Let $\mathbf{x} \in D$ be a point in D and consider the particle of fluid moving through $\mathbf{x}$ at time t. Relative to standard Euclidean coordinates in space, we write $\mathbf{x} = (x, y, z)$. Imagine a particle (think of a particle of dust suspended) in the fluid; this particle traverses a well-defined trajectory. Let $\mathbf{u}(\mathbf{x}, t)$ denote the velocity of the particle of fluid that is moving through $\mathbf{x}$ at time t. Thus, for each fixed time, $\mathbf{u}$ is a vector field on D, as in Figure 1.1.1. We call $\mathbf{u}$ the (***spatial***) ***velocity field of the fluid***.

For each time t, assume that the fluid has a well-defined ***mass density*** $\rho(\mathbf{x}, t)$. Thus, if W is any subregion of D, the mass of fluid in W at time t

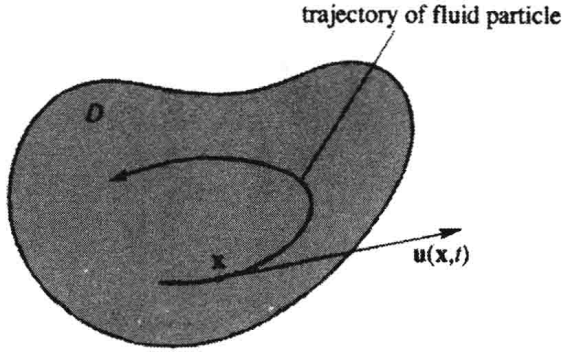

FIGURE 1.1.1. Fluid particles flowing in a region D.

is given by

$$m(W,t) = \int_W \rho(\mathbf{x},t)\,dV,$$

where dV is the volume element in the plane or in space.

In what follows we shall assume that the functions $\mathbf{u}$ and ρ (and others to be introduced later) are smooth enough so that the standard operations of calculus may be performed on them. This assumption is open to criticism and indeed we shall come back and analyze it in detail later.

The assumption that ρ exists is a ***continuum assumption***. Clearly, it does not hold if the molecular structure of matter is taken into account. For most macroscopic phenomena occurring in nature, it is believed that this assumption is extremely accurate.

Our derivation of the equations is based on three basic principles:

i *mass is neither created nor destroyed*;

ii *the rate of change of momentum of a portion of the fluid equals the force applied to it* (***Newton's second law***);

iii *energy is neither created nor destroyed.*

Let us treat these three principles in turn.

i Conservation of Mass

Let W be a fixed subregion of D (W does *not* change with time). The rate of change of mass in W is

$$\frac{d}{dt}m(W,t) = \frac{d}{dt}\int_W \rho(\mathbf{x},t)\,dV = \int_W \frac{\partial \rho}{\partial t}(\mathbf{x},t)\,dV.$$

Let ∂W denote the boundary of W, assumed to be smooth; let $\mathbf{n}$ denote the unit outward normal defined at points of ∂W; and let dA denote the area element on ∂W. The volume flow rate across ∂W per unit area is $\mathbf{u} \cdot n$ and the mass flow rate per unit area is $\rho \mathbf{u} \cdot n$ (see Figure 1.1.2).

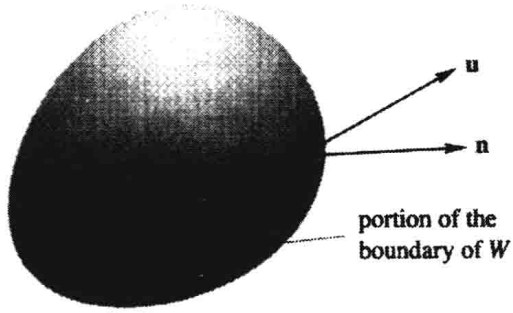

FIGURE 1.1.2. The mass crossing the boundary ∂W per unit time equals the surface integral of $\rho \mathbf{u} \cdot \mathbf{n}$ over ∂W.

The principle of conservation of mass can be more precisely stated as follows: The rate of increase of mass in W equals the rate at which mass is crossing ∂W in the *inward* direction; *i.e.*,

$$\boxed{\frac{d}{dt}\int_W \rho \, dV = -\int_{\partial W} \rho \mathbf{u} \cdot \mathbf{n} \, dA.}$$

This is the ***integral form of the law of conservation of mass.*** By the divergence theorem, this statement is equivalent to

$$\int_W \left[\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u})\right] dV = 0.$$

Because this is to hold for all W, it is equivalent to

$$\boxed{\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0.}$$

The last equation is the ***differential form of the law of conservation of mass***, also known as the ***continuity equation.***

If ρ and $\mathbf{u}$ are not smooth enough to justify the steps that lead to the differential form of the law of conservation of mass, then the integral form is the one to use.

ii Balance of Momentum

Let $\mathbf{x}(t) = (x(t), y(t), z(t))$ be the path followed by a fluid particle, so that the velocity field is given by

$$\mathbf{u}(x(t), y(t), z(t), t) = (\dot{x}(t), \dot{y}(t), \dot{z}(t)),$$

that is,

$$\mathbf{u}(\mathbf{x}(t), t) = \frac{d\mathbf{x}}{dt}(t).$$

This and the calculation following explicitly use standard Euclidean coordinates in space (delete z for plane flow).[1]

The acceleration of a fluid particle is given by

$$\mathbf{a}(t) = \frac{d^2}{dt^2}\mathbf{x}(t) = \frac{d}{dt}\mathbf{u}(x(t), y(t), z(t), t).$$

By the chain rule, this becomes

$$\mathbf{a}(t) = \frac{\partial \mathbf{u}}{\partial x}\dot{x} + \frac{\partial \mathbf{u}}{\partial y}\dot{y} + \frac{\partial \mathbf{u}}{\partial z}\dot{z} + \frac{\partial \mathbf{u}}{\partial t}.$$

Using the notation

$$\mathbf{u}_x = \frac{\partial \mathbf{u}}{\partial x}, \quad \mathbf{u}_t = \frac{\partial \mathbf{u}}{\partial t}, \quad \text{etc.},$$

and

$$\mathbf{u}(x, y, z, t) = (u(x, y, z, t), v(x, y, z, t), w(x, y, z, t)),$$

we obtain

$$\mathbf{a}(t) = u\mathbf{u}_x + v\mathbf{u}_y + w\mathbf{u}_z + \mathbf{u}_t,$$

which we also write as

$$\mathbf{a}(t) = \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u},$$

---

[1] Care must be used if other coordinate systems (such as spherical or cylindrical) are employed. Other coordinate systems can be handled in two ways: first, one can proceed more intrinsically by developing intrinsic (i.e., coordinate free) formulas that are valid in any coordinate system, or, second, one can do all the derivations in Euclidean coordinates and transform final results to other coordinate systems at the end by using the chain rule. The second approach is clearly faster, although intellectually less satisfying. See Abraham, Marsden and Ratiu [1988] (listed in the front matter) for information on the former approach. For reasons of economy we shall do most of our calculations in standard Euclidean coordinates.