【云南财经大学博士论丛】

Designing Accelerated Life Test Plans under Progressive Interval Censoring with Random Removals

带有随机移走的逐步删失模型下的加速寿命试验设计

Ding Chang

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Preface

Usually it is required to conduct a thorough assessment on the quality of a newly developed product before putting it into mass production. As to many industrial products, the most important quality is their life time. Life test can be used to achieve this goal, and it has been applied widely in industry in the past several decades.

However, due to the continuing improvement of technical process, the life time of industrial products has been greatly increased in recent years. Of course this is a good news to both the producer and the consumer, but at the same time it brings a big challenge to the experimenter: how to collect enough data on the product's life time to do statistical inference in a tolerable time duration? The idea of "acceleration" has been introduced into life test to solve this problem. From then on, the design of accelerated life test plans attracted the attention of many researchers.

Even with the help of "acceleration", experimenters may still meet with some difficulties in designing and conducting life test plans. Some of the main concerns are: First of all, sometimes it is still difficult, if not impossible, to observe the life time of all tested units even with the help of "acceleration". As a result, accelerated life tests are often being censored to save experiment time and cost. The most frequently used censoring plans are Type Ⅰ and Type Ⅱ censoring schemes. Secondly, in some studies, it is not uncommon for some non-failed units to be randomly removed from the test because of their dangerous status and/or the reduction of experiment funding and facility. At last, since usually it is too demanding to monitor the process of a test continuously, interval inspection schemes are preferred by experimenters due to their convenience in implementation.

Taking these situations into consideration, it is of great importance to discuss how to develop accelerated life test plans which integrate those features and fit in well with the practical requirements. This is exactly our goal in writing this book. In particular, our study focused on the derivation of optimal accelerated life test plans and the design of accelerated life test sampling plans under progressive Type Ⅰ/Type Ⅱ interval censoring with random removals. This book can be used as an elementary course for graduates who major in industrial statistics. It can also be served as a handbook for researchers in related areas.

I would like to express my deep and sincere gratitude to my supervisor, Dr. Siu-Keung Tse, who provided a lot of valuable feedbacks and suggestions. Also, I deeply appreciate Dr. ChunYan Yang, Prof. Kelvin Yau and Dr. Sammy Yuen for their generous help.

Ding Chang

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Chapter 1 Introduction

1.1 Background

1.1.1 Life test

In industrial applications, the quality of a newly developed product needs to be thoroughly assessed before it is put into mass production. In many cases, the product's lifetime is taken as an important characteristic to reflect the quality of the product, especially for some electronic devices and components. Thus life tests are widely used to assess the life of a product. In particular, the information on the failure times of units is collected during a life test, and the lifetime distribution of the product is inferred.

1.1.2 Censoring

In practice, an experimenter may terminate a life test before all units fail in order to save time and/or cost. Data collected under such circumstances is called censored data. The data contains the exact failure times of the failed units and the running times of the non-failed units. In other words, the exact lifetimes are known for only a portion of tested units and the remaining units' lifetimes are known only to exceed certain values. Although censored data results in a loss of information and is more difficult to analyze, censoring schemes are widely used in life tests because they avoid the situation where a test is prolonged by some extremely long life-span units, and thus help to minimize experiment time and cost.

Some of the most commonly used censoring schemes are Type I censoring schemes and Type II censoring schemes. A time point is predetermined for a test under a Type I censoring scheme, and the test will run only until that time is reached. While the duration of a test under a Type I censoring scheme is fixed, the number of failures that can be observed is a random variable. For a test conducted under a Type II censoring scheme, the testing stops after a predetermined number of tested units fail. Thus the number of failures is fixed but the duration of the test is random. Both Type I and Type II censoring schemes have been studied extensively by researchers, as for example in Mann, Schafer and Singpurwalla (1974), Cohen (1991), Meeker and Escobar (1998) and Lawless (2003).

1.1.3 Progressive censoring

Units are not allowed to be removed from the test under conventional Type I and Type II censoring schemes. However, in practice, some non-failed units may have to be withdrawn for more thorough inspection or saved for use as test specimens in other studies. To accommodate this requirement, a progressive censoring scheme is proposed by researchers, allowing units to be removed during the testing process. The earliest work on progressive censoring was documented by Cohen (1963). He pointed out that it is not uncommon for units in life test experiments to fail for reasons quite unrelated to the normal failure mechanism, and a progressive censoring scheme allows the analysis of data arising from these situations. He also mentioned that one of the primary purposes of progressive censoring is to save some live units for other tests, which is especially useful when the units being tested are very expensive. Subsequently, the idea of progressive censoring has received the attention of many researchers in various disciplines. Mann (1971) and Thomas and Wilson (1972) discussed linear estimation of parameters based on progressively censored samples. Lemon (1975) derived the maximum likelihood estimation for the three-parameter Weibull distribution. Viveros and Balakrishnan (1994) proposed a conditional method of inference to derive exact confidence intervals for location and scale parameters of a general location-scale family of distributions. A simple simulation algorithm for generating progressive Type Ⅱ censored samples was given by Balakrishnan and Sandhu (1995). Balasooriya and Low (2004) discussed progressive Type Ⅰ censored sampling plans for a Weibull lifetime distribution under competing causes of failure. Balakrishnan et al (2004), Ng, Chan and Balakrishnan (2004) and Lin, Wu and Balakrishnan (2006) drew inferences from progressive Type Ⅱ censored data for the extreme value distribution, Weibull distribution and log-normal distribution, respectively. More details on progressive censoring schemes can be found in Balakrishnan and Aggarwala (2000).

The number of units being removed during a test is assumed to be fixed in advance under conventional progressive censoring schemes. However, in practice it might be infeasible to predetermine the removal pattern. For example, some units in the test may lead to dangerous situations, say, overheating, fracture and/or leakage; in consideration of safety, these units should be removed even though they do not completely fail. In this case, the removal pattern would be random. In another scenario where some units have to be removed from the test due to a reduction of the budget and facility, the removal pattern is also random. Under these circumstances, the conventional progressive censoring schemes may not be consistent with practical situations. Yuen and Tse (1996) indicated that an experimenter may find that it is inappropriate or too dangerous to carry on the test on some of the units being tested even though these units have not failed completely, and thus the removal pattern of those units will be random. They discussed the estimation problem when lifetimes are Weibull distributed and are collected under a progressive Type II censoring scheme with random removals, where the number of units removed at each failure follows a discrete uniform distribution. Tse and Yuen (1998) calculated the expected experiment time for a Weibull distribution under progressive Type II censoring with random removals. Wu and Chang (2003) studied progressively Type II censored data with random removals from a Pareto distribution. Note that the number of removals is assumed to follow a discrete uniform distribution in these works.

Tse, Yang and Yuen (2000) pointed out that a discrete uniform removal pattern may not fit in well into the real settings since it dictates that each removal event occurs with an equal probability regardless of the number of units removed. They argued that the probability of only some of the units being removed should be different from, and is usually smaller than, the probability of all units being removed; and, if any individual unit removed from the test is independent of the others but removed with the same probability, then the number of removals would follow a binomial distribution. Under this scenario, they calculated the expected experiment time for a Weibull distribution under progressive Type II censoring with random removals. Subsequently, Tse and Xiang (2003) discussed the interval estimation of parameters for a Weibull distribution under progressive Type II censoring with binomial removals.

1.1.4 Inspection schemes

Two inspection schemes are commonly used in life tests: continuous inspection and interval inspection. In principle, the testing process is monitored continuously in a continuous inspection scheme, and the failure time for each unit is recorded. Although continuous inspection results in exact failure times of tested units, it may be infeasible and inconvenient to employ due to the high cost and the possible danger in monitoring the experiment continuously. Alternatively, the test can be monitored at some specified time points only. The number of failures between two successive inspection points is recorded accordingly. This leads to the interval inspection scheme, which is widely applied in practice since it requires less testing effort and is more convenient to implement, and in some cases it is the only way of checking the status of tested units (Ehrenfeld, 1962). Data collected from interval inspection schemes is called grouped data, which has been studied by many authors such as Cheng and Chen (1988), Lui, Steffey and Pugh (1993), Chen and Mi (1996), Aggarwala (2001), Qian and Correa (2003), Xiong and Ming (2004) and Yang and Tse (2005).

1.1.5 Accelerated life test

With the increasing improvement in manufacturing processes, many products are very reliable with extremely long life-spans under normal conditions. This causes several difficulties in using standard life test methods. It usually requires a long time to complete life tests on reliable products under normal conditions, even with the use of censoring schemes. Consequently, the costs of such kinds of life tests can be considerably high. As a result, it is difficult to observe sufficient failure data to make inferences regarding the lifetime distribution within a tolerable experiment time and cost. Accelerated life test (ALT) plans are often used to remedy this problem.

Under ALTs, units are tested at above average levels of stress (e.g. temperature, voltage, humidity, pressure, vibration, cycling rate and so on) to ensure rapid failures. A model relating a test unit's length of life to stress in operation is derived using the accelerated failure times, and extrapolated to estimate the lifetime distribution under use condition. ALTs can be divided into three types according to the ways that stresses are applied: constant stress ALTs, step stress ALTs and progressive stress ALTs. A constant stress ALT may employ two or more stress levels, and each stress level remains constant during the test. All test units are allocated into several groups according to the number of stress levels to be employed. Each group is tested under a certain stress level. In a step stress ALT, the test can be divided into several stages and each stage employs one stress level. All units are put on test at the first stage to a predetermined time, following which, the non-failed units are further tested in the second stage, which has a different stress level from the first stage. In this way, the test moves step by step until the end. The stress levels in a progressive stress ALT are set to be a consecutive function of time, and all units are tested at these continually changing stress levels until the end. All three types of ALTs have their advantages and disadvantages, as discussed in detail in Nelson (1990). More literatures on ALT plans can be found in Nelson (2005a; 2005b). Most recently, Yang and Tse (2005) studied the design of ALT plans under progressive Type I interval censoring with random removals. Wu, Lin and Chen (2006) considered a step stress ALT under progressive Type I censoring with grouped data. Later, Wu, Lin and Chen (2008a) investigated step stress ALT plans under progressive Type I group censoring with random removals. The design of ALT plans under progressive Type I interval censoring with cost considerations was provided by Wu et al (2008b). Unit lifetimes are assumed to be exponentially distributed in all these works.

1.1.6 Reliability sampling plans

Acceptance sampling plans are commonly used to determine the acceptability of a product with regard to its adherence to the manufacture's specifications. The design of a sampling plan requires an agreement between the consumer and the producer. Lots where the fraction of defectives does not exceed p_α are presumed to be good, and the consumer accepts these lots with a probability of at least $1-\alpha$. Note that good lots still have a chance of being rejected with probability α , thus α is the producer's risk. On the other hand, lots where the fraction of defectives exceeds p_β are presumed to be unacceptable, and the consumer rejects these lots with a probability of at least $1-\beta$. Similarly, unacceptable lots still have a chance of being accepted with probability β , thus β is the consumer's risk. A sampling plan should be designed in such a way that both the consumer's and the producer's risks can be satisfied.

Sampling plans to determine the acceptability of a product with respect to lifetime are called reliability sampling plans. The standard reliability sampling plans, such as MIL-STD-414, MIL-STD-105 and ANSI Z1.9 can be used to assess the results against specified requirements. Some early works on reliability sampling plans can be found in Epstein and Sobel (1953, 1955), Epstein (1954), Aroian (1964), Bulgren and Hewette (1973) and Fairbanks (1988). The lifetime of tested units was assumed to be exponentially distributed in these works. Fertig and Mann (1980) discussed life test sampling plans for extreme-value and Weibull distributions. A similar work which considered lognormal and Weibull distributions was authored by Schneider (1989). Balasooriya (1995) considered failure-censored reliability sampling plans for the two-parameter exponential case. Balasooriya and Saw (1998), Balasooriya and Balakrishnan (2000) and Balasooriya, Saw and Gadag (2000) discussed reliability sampling plans for the two-parameter exponential, lognormal and Weibull distributions under progressive Type Ⅱ censoring schemes, respectively. The optimal design of progressive Type Ⅱ censoring plans and the corresponding sampling plans for Weibull distribution was given by Ng, Chan and Balakrishnan (2004). Huang and Wu (2008) studied reliability sampling plans under progressive Type Ⅰ interval censoring using cost functions for exponential distribution. The number of removals is assumed to be predetermined in these works. For progressive censoring with random removals, Tse and Yang (2003) discussed the design of reliability sampling plans for Weibull distribution under progressive Type Ⅱ censoring with binomial rcmovals. One may refer to Schilling and Neubauer (2009) for more details on reliability sampling plans.

1.2 Research motivation and objective

When products that need to be tested have high reliability with long life-spans, ALTs can be used to generate rapid failures within a tolerable experiment time. To terminate a test after adequate information has been collected on a unit's lifetime distribution and to avoid the test being undesirably prolonged by a few extremely long life-span units, both Type Ⅰ and Type Ⅱ censoring schemes are widely used in ALTs. In many practical applications, experimenters prefer to adopt interval inspection schemes in ALTs to make the inspections more convenient and safer to conduct, which also helps to cut down the experiment cost. Furthermore, it is not uncommon for some non-failed test units to be removed in a random pattern during the test as was mentioned in the previous section, especially under the ALT mechanism where those circumstances outlined are more likely to occur. As a result, it is not uncommon to adopt ALT plans which can match all these practical requirements. Thus, it would be of interest to integrate the ideas of interval inspection and progressive censoring with random removals in the design of ALT plans. The first research objective of this book is to explore constant stress ALT plans under progressive Type Ⅰ /Type Ⅱ interval censoring with random removals. The number of units removed at each inspection is assumed to follow a binomial distribution.

Reliability sampling plans are commonly used to determine the acceptability of a product in terms of its life-span. However, if the product to be tested is a high reliability product, it is impractical to employ reliability sampling plans under use condition since they usually require extremely long experiment times. One way to overcome this difficulty is to combine reliability sampling plans with ALTs. Wallace (1985) stressed the need for introducing ALT into reliability sampling plans. Yum and Kim (1990) and Hsieh (1994) developed ALT sampling plans for exponential distributions. Bai, Kim and Chun (1993, 1995) studied Type Ⅱ censored ALT sampling plans for lognormal and Weibull distributions. Most recently, Kim and Yum (2009) discussed the design of Type Ⅰ censored reliability sampling plans for the Weibull distribution with unknown shape and scale parameters. Seo, Jung and Kim (2009) studied ALT sampling plans with a non-constant shape parameter for Weibull distribution.

In regard to the research problem explored in the first objective, the next issue discussed in this book is the design of reliability sampling plans which integrate the features of ALTs, interval inspection and progressive censoring with random removals.

1.3 Outline of the book

The later chapters of this book are organized as follows. Chapter 2 discusses the design of ALT plans under progressive Type Ⅰ interval censoring with random removals. The design of sampling plans with ALT under the same settings is presented in Chapter 3. In Chapter 4, we study the design of ALT plans under progressive Type Ⅱ interval censoring with random removals, while the design of the corresponding sampling plans is investigated in Chapter 5. A discussion and some suggestions for future studies are outlined in Chapter 6.