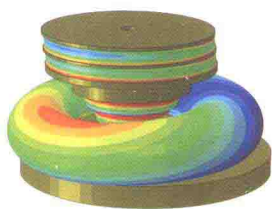


ADVANCED MECHANICS OF TIRE AND
RUBBER PRODUCTS

Proceedings of 1st Tsinghua Tire Mechanics Workshop

轮胎·橡胶制品先进力学
第一届清华轮胎力学国际论坛论文集



Editors

Wei Yintao [Germany] Michael Kaliske Huang Youjian

主编

危银涛 [德] Michael Kaliske 黄友剑

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内 容 简 介

本书共分九章, 力图汇集轮胎与橡胶制品力学的重要进展。各章内容由清华大学汽车安全与节能国家重点实验室组织的国际轮胎力学研讨会上的邀请报告扩充而成。这些报告充分反映了国际轮胎力学领域的研究热点。各章的作者也都是国内外有影响的学者, 内容涉及轮胎结构动力学、轮胎帘线力学行为、轮胎滚动噪声、轮胎滚动六分力、车/轮/路界面、轮胎模型、轮胎结构优化、轮胎性能对车辆行为的影响、轮胎动态硫化、橡胶元件设计计算及疲劳寿命等方面, 基本反映了本领域的热点问题及最新研究方向。

本书的一个重要特点是从系统的观点研究道路-轮胎-车辆相互作用, 从整体的角度分析材料-结构-性能的多尺度关系, 这种整体和系统的观点有助于读者把握轮胎力学这一交叉学科的内涵。

本书可供从事车辆、轮胎、橡胶制品研究、设计及生产部门的技术人员, 以及高等院校与汽车、轮胎和橡胶相关专业师生阅读参考。

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FOREWORD

Tire mechanics is the basis for vehicle dynamics and is also the basis for tire design and development. The former mainly refers to the tire dynamics and the latter refers much more to the tire structural mechanics. For many years, tire dynamics and tire structural mechanics have developed along their own different tracks and almost have not blended. However, last decade the situation has been changing. On the one hand the development needs of advanced vehicle dynamics simulation and control technology makes dynamic modeling of tires must consider the structural characteristics of the tire itself. On the other hand, the pressure of safety, energy conservation and environmental protection makes development of the tire must consider its dynamic mechanical properties from the beginning. The pushing of the two aspects makes tire dynamics and tire mechanics have the trend of reunion.

In this case, in order to discuss and exchange the hot issues in tire dynamics and promote the progress of the tire mechanics in various fields, an international advanced tire mechanics symposium was organized by State Key Laboratory of Automotive Safety and Energy in Tsinghua University and was held in the June 4, 2012. Participating experts are all renowned scholars in the field of international tire mechanics and reports basically reflect the current progress of tire mechanics. To make these valuable knowledge be permanently preserved, we decide to publish a proceedings made from these reports. Professor Yintao Wei in Tsinghua University, Professor Michael Kaliske in Technische Universität Dresden, Germany, and Mrs. Huang Youjian in CSR would spent their precious time in editing and organizing each chapter in the proceedings. To promote exchange of the tire mechanics knowledge, Dresden tire workshop will be held every odd year in Germany and Tsinghua tire workshop every even year in China.

The first chapter of introduction to Tire Dynamics was written by Professor Yintao Wei. Professor Yukio Nakajima from Kogakuin University in Japan composed the second chapter “Vehicle / Tire / Road Interaction—Simulation, Experiment and Sensing—”. After having worked for 30 years in Bridgestone Corporation, he returned to university and this chapter composed by him contains a wealth of practical experience and theoretical knowledge. The third chapter was written by Professor Christian Oertel from FH Brandenburg, Germany in Germany and he introduces theory and application about a series of RMOD-K tire models. In this chapter, different modeling methods are compared and some of application examples of RMOD-K tire models are given. These demonstrate their broad applicability. The fourth chapter was written by Professor Michael Kaliske from Technische Universität Dresden, Germany in Germany and its main contents are about simulation based optimal design of tires. At present, Professor Michael served as chief editor of Tire Science and Technology and he is world-renowned in the field of tire structure analysis.

The fifth chapter was written by Dr. Lingge Jin from China First Automobile Group Corporation and he discussed the matching problem between vehicle and tire. In the vehicle design process, the main task of chassis engineers is to design the suspension and steering systems to use the tire properties to the largest extent. At the same time, from vehicle point of view, to satisfy the requirement of vehicle performance tires must also meet their design goals. Professor Xiangqiao Yan from Harbin Institute of Technology composed the sixth chapter and a simulator for the tire curing processes is developed based on the finite element method of an axisymmetric heat transfer problem for composites in this paper. The numerical simulation result of a truck tire curing process shows that the simulator successfully describes the variation trends in temperature and in state of cure with tire curing process. The seventh chapter, "Structure design and production process innovation based on the tire mechanics", was composed by Professor Weimin Yang from Beijing University of Chemical Technology. The eighth chapter was written by Senior Engineer Youjian Huang from Zhuzhou Times New Material Technology Co. Ltd. and design-calculation method of rubber component and air spring was introduced in this chapter. The ninth chapter was written by Endurica Company in USA, Chief Editor of Rubber Chemistry and Technology. The chapter is about fatigue life prediction for elastomeric structures. Above several scholars are outstanding in their own fields and their reports will give readers a very valuable content.

In addition to these scholars, Dr. Shinan Jin who is the director of department of tire simulation and experiment in LingLong Tyre Research and Development Center and Dr. Jieke He from TTA tire Technology Company also made a speech In this seminar.

We hope that readers can know the latest developments and trends of tire mechanics and can gain help in development of more safety, energy conservation and environmental protection vehicle and tire from the proceedings.

Editors:

Wei Yintao (Tsinghua University, China)

Michael Kaliske (Technische Universität Dresden, Germany)

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前 言

轮胎力学是汽车动力学的基础，也是轮胎设计与开发的基础。前者主要指的是轮胎动力学，后者更多指的是轮胎结构力学。多年来，轮胎动力学和轮胎结构力学沿着不同的轨道发展，几乎没有交融。但是近十年来情况发生了变化，一方面是先进汽车动力学仿真和控制技术发展的需求，使得轮胎动态建模必须考虑轮胎本身的结构特性。另外一方面，安全、节能和环保的压力使得轮胎开发从一开始就必须考虑其动态力学性能。这两方面的推动使得轮胎动力学和轮胎结构力学有会师的趋势。

在这种情况下，清华大学汽车安全与节能国家重点实验室于 2012 年 6 月 4 日组织了一届国际轮胎力学研讨会，以期研讨与交流轮胎动力学的热点问题，推动轮胎力学各领域的进展。与会的专家都是国际轮胎力学领域的知名学者，报告基本反映了轮胎力学领域的当前进展。为使这些宝贵的知识得以永久保存，我们决定将报告结集成书出版。清华大学的危银涛教授、德累斯顿大学的 Michael Kaliske 教授和南车时代新材的黄友剑高工抽出他们宝贵的时间对各章进行了编辑和整理工作。为促进轮胎与橡胶制品力学知识的交流，德累斯顿轮胎研讨会在德国每逢奇数年召开一次，清华轮胎研讨会在中国每逢偶数年召开一次。

第 1 章“轮胎结构动力学引论”由清华大学汽车工程系的危银涛教授撰写。日本工学院的中岛幸雄教授撰写了第 2 章“车/轮/路界面的仿真、试验与识别”。中岛教授曾在普利司通工作 30 余年，又回到大学任职，他撰写的章节含有丰富的实践经验和理论知识。第 3 章由德国勃兰登堡大学的 Christian Oertel 教授撰写，介绍了“RMOD-K 轮胎模型系列”的理论和应用，在这部分比较了不同的建模方法，给出了一些 RMOD-K 轮胎模型系列的应用实例，展示了其广泛的适用性。第 4 章由德国德累斯顿大学的 Michael Kaliske 教授撰写，主要是关于“轮胎优化设计仿真”。Michael 教授目前是《Tire Science and Technology》的主编，他在轮胎结构分析方面享誉世界。

第 5 章“轮胎性能目标”由中国第一汽车股份有限公司的金凌鸽博士讨论车辆和轮胎的匹配问题。在车辆设计过程中，底盘工程师最主要的任务就是设计能最大限度地发挥轮胎性能的悬挂和转向系统。同时，从车辆的角度来看，为满足车辆的性能要求，轮胎也有很多必须要达到的设计目标。第 6 章是哈尔滨工业大学的闫相桥教授介绍“轮胎动态硫化过程有限元模型”，它是基于复合材料轴对称热传递的有限元分析方法开发的。对卡车轮胎硫化过程的数值仿真结果显示，该模拟器很好地模拟了轮胎硫化过程中温度和硫化程度的变化趋势。第 7 章是北京化工大学杨卫民教授写的“轮胎结构与生产工艺的逐步创新”，第 8 章是时代新材的黄友剑高工撰写的“橡胶元件及空气弹簧设计计算方法”，第 9 章是美国 Endurica 公司、现任《Rubber Chemistry and Technology》主编关于“弹性结构的疲劳寿命预测”。这几位学者都是各自领域的佼佼者，他们的章节一定会给读者带来非常有价值的内容。

除了以上学者，在本次研讨会发表演讲的学者还有金石男博士，玲珑轮胎研发中心仿真和实验部主任；以及特拓轮胎技术公司的贺杰克博士。

我们期望读者能从本书中了解到轮胎力学的最新进展和发展趋势，对更安全节能和环保汽车轮胎与橡胶制品的开发，有所裨益。

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Chapter 1 An Introduction to Tire Structural Dynamics

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Abstract: This chapter mainly focuses on the progress of the tire structural dynamics, basically based on the authors' group research. First, in order to make the readers gain a preliminary understanding of the tire structural mechanics, a global-local method about the force analysis of cord within a tire is introduced and at the same time a new method of cord modeling is also given. Secondly, three important issues on tire structural dynamics are introduced such as vibration and noise, impact mechanics and rolling six-component forces forecast. Vibration mode of tire, vibration analysis and vibration-acoustics of tire on random road are introduced in the vibration and noise problem; The properties and analysis methods of tire cleat impact are introduced in the impact Mechanics problem; Through a combination of tire structural dynamics and external characteristic dynamics, forecasting methods of tire rolling six-component forces are briefly introduced in the six-component forces problem.

Keywords: global-local method; vibration and noise; cleat impact; forecasting; rolling six-component forces

第 1 章 轮胎结构动力学引论

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摘要: 本章主要结合作者自己的研究, 介绍了轮胎结构动力学若干方面的进展。首先, 为了使读者初步了解轮胎结构力学, 介绍了轮胎中帘线受力分析的整体-局部法, 同时也给出了帘线建模的一种新方法。其次, 主要介绍轮胎结构动力学的三个重要问题, 即振动噪声、冲击力学与滚动六分力预报。在振动噪声问题中, 介绍轮胎的振动模态、随机道路上的振动分析和振动噪声; 在冲击力学问题中, 介绍轮胎过坎冲击特性和分析方法; 在滚动六分力问题中, 通过将轮胎的结构动力学和外特性动力学融合的方式, 简要介绍轮胎滚动六分力的预报方法。

关键词: 整体-局部方法; 振动与噪声; 过坎冲击; 预报; 滚动六分力

1.1 Introduction

Tire mechanics is the basis for vehicle dynamics and is also the basis for tire design and development. The former mainly refers to the tire external characteristic dynamics of and the latter refers much more to the tire structural mechanics. For many years, tire external characteristic

dynamics and tire structural mechanics have developed along their own different tracks and almost have not blended. However, in last decade the situation has been changing. On the one hand the development needs of advanced vehicle dynamics simulation and control technology makes dynamic modeling of tires must consider the structural characteristics of the tire itself. On the other hand, the pressure of safety, energy conservation and environmental protection makes development of the tire must consider its dynamic mechanical properties from the beginning. The pushing of the two aspects makes tire dynamics and tire mechanics have the trend of reunion.

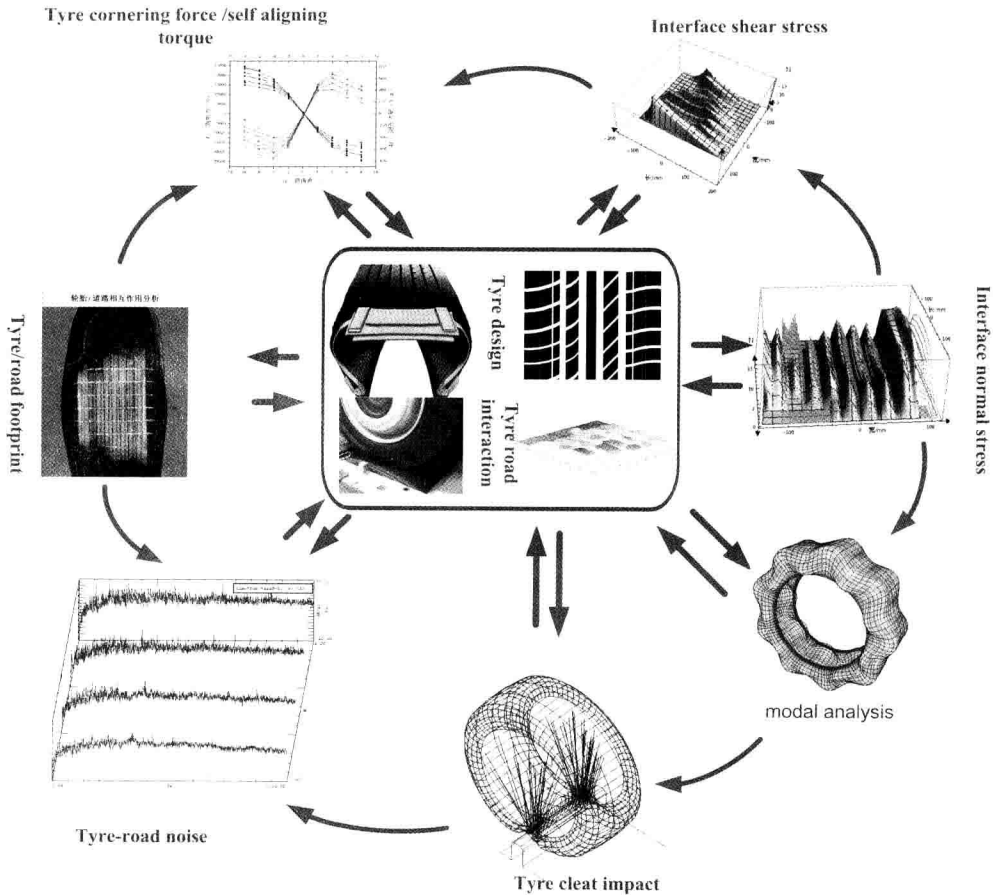


Figure 1.1 Research contents for tire road interaction dynamics

From the point of view of external characteristic mechanics, tires are the only parts in contact with the ground. A small piece of the interface between tire and ground determines the six component forces that can make vehicle realize a variety of motions. The properties of interface are related to the status of tires, road and vehicle and it determines many key system performances such as the handling and stability of vehicle, tire wear, road load, rolling noise, and so on. In the vehicle design process, the main task of chassis engineers is to design the suspension and steering systems to use the tire properties to the largest extent. However, due to the highly non-linear characteristics of the interaction between the tire and road, so far tire mechanics is still the

bottleneck and challenge of vehicle dynamics and there are still many unresolved issues that need in-depth research. From the point of view of the development of vehicle dynamics and control technology to requirement of tire model, the key question of tire model is dynamic, high-frequency, coupling, real-time modeling and solution techniques.

From the perspective of tire structural mechanics, due to the requirement of the high-speed driving comfort, safe driving and energy saving such as the provisions of the Europe tire label law, the design and development of the tire should consider more vibration, noise, rolling resistance, and so on which are regarded as secondary in the past. These are related to structural dynamics problems during tire rolling process.

Hence, no matter to from the point of view of external characteristic mechanics or from the perspective of tire structural mechanics, tire mechanics is developing towards dynamics (see Figure 1.1). Three important problems about tire structural dynamics are introduced in the chapter, namely, vibration and noise, impact mechanics and rolling six-component Forces and Moments forecasts. About the introduction of the external characteristics dynamics, one can refer to Chapter 3. Before introducing vibration, impact and noise, the global-local method about the force analysis of cord within a tire is firstly introduced in Section 1.2 and hence a new method of cord modeling is also given. Through this section tire structural mechanics is briefly introduced. Vibration mode of tire, vibration analysis and vibration-acoustics of tire on random road are introduced in Section 1.3. In Section 1.4, the properties and analysis methods of tire cleat impact are introduced and it is currently more concerned tire characteristics in European. In Section 1.5 forecasting methods of tire rolling six-component forces are briefly introduced and this is used as an attempt of a combination of tire structural dynamics and external characteristic dynamics. Lastly, a summary is given in Section 1.6.

1.2 Tire Stress and Cord Force in Radial Tires

1.2.1 Introduction to tire stress and cord force distribution analysis^[1]

For radial tires, steel cord is the main load bearing component. The layout of the steel reinforcements influences both the cord force distribution and the shear stress/strain on the rubber components, ultimately affecting tire performance parameters such as durability, stiffness, heat generation and rolling resistance. This section focuses on steel cord deformation and force investigation within heavy duty radial tires, serving as an introductory description for tire structural mechanics. To analyze tire deformation, stress, strain and contact behavior, in the 1970s the finite element method (FEM) was begun to be developed for tires^[1-16]. Now FEM has been used to model tire rolling loss^[17-21], tire fracture^[19, 22] and tire dynamics^[23-26]. One interesting and challenging problem addressed in this chapter is: what is the precise deformation and stress map of the steel cords within a tire under realistic loading conditions. This problem is especially meaningful for TBR since the both carcass and belt fibers are made from steel cords (see Figure 1.2).

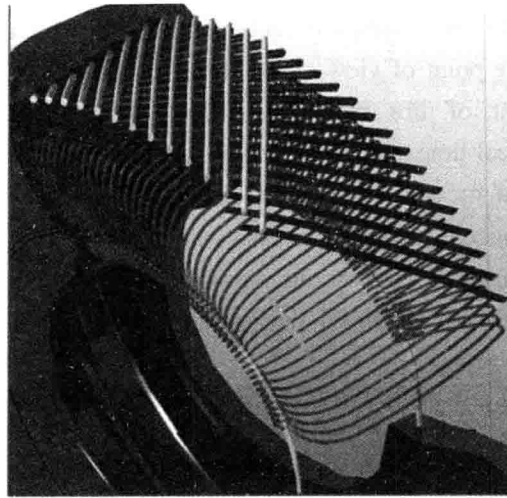


Figure 1.2 Reinforcement for truck and bus radial tires, both carcass and belt fibers are steel cords

1.2.2 Global modeling of a TBR

First a global analysis is conducted to obtain the typical bending deformation and tension forces for the steel cord within a TBR tire, and then a local scale modelling is developed to simulate the stress and strain within the internal cord. The local scale model uses as input the typical deformation and force for the steel cord obtained from the global modeling. A high efficiency modeling approach for layered multi-strand cord structures has been developed that utilizes cord design variables such as lay angle, lay length, and radius of the strand centreline.

Figure 1.3 shows the cord force distribution in the second belt ply and carcass ply in the case of inflation. The tension force in the second belt is clearly a parabolic form with a maximum value of 372 Newton in the centre of the belt (crown) and a minimum value of 4 Newton at the belt edge. For the carcass, because of the hoop effect of the belt plies, the cord force in the crown area is less

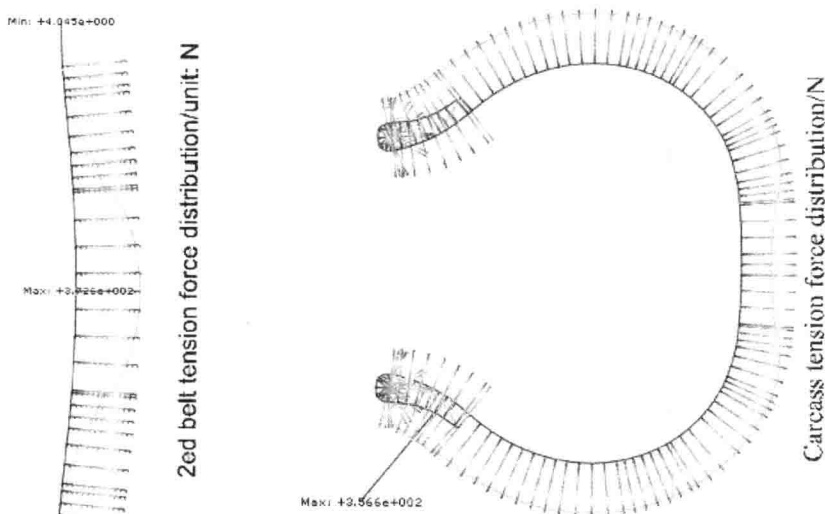


Figure1.3 Belt and carcass force distribution under inflation

than that at the sidewall. Beyond the crown area, the carcass force remains nearly constant from upper sidewall to lower sidewall, whereas in the bead area the carcass tension force increases somewhat then decreases to zero at the carcass end.

The cord force distribution under vertical loading is different from that under inflation. This change mainly occurs near the contact area (see Figure 1.4 and Figure 1.5). The axes of Figure 1.4 and Figure 1.5 are circumferential angle, meridian width from one end to the other end, and the cord force value is intuitively explained in all the figures. Figure 1.4 shows the cord force distribution in the second belt ply, from which one can deduce that the cord force of the second belt ply is much greater than that of the first ply. This result is because the second belt ply is designed to be the main load bearing ply that partakes in most of the inflation pressure and external loading. One can observe from Figure 1.4 that (1) the cord force is anti-symmetric with respect to the circumferential centreline of the tread, (2) in the contact area, the cord force in the crown area decreases as the cord force in the belt end increases, (3) the maximum value of the cord force shifts to the belt end as the minimum value shifts to the centre area. In other words, the contact deformation relaxes the belt in the crown area as it bends the belt in the shoulder area.

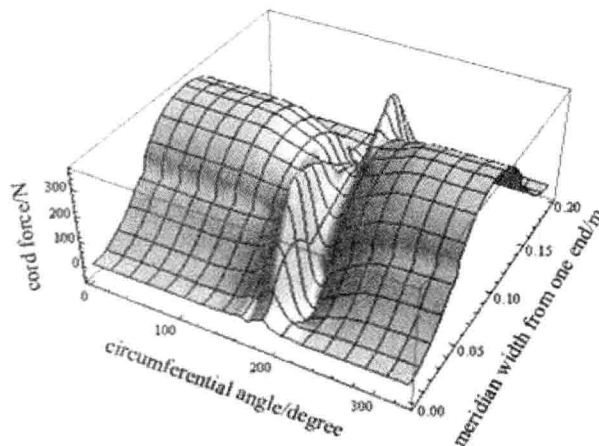


Figure 1.4 Carpet plot of the second belt ply cord force

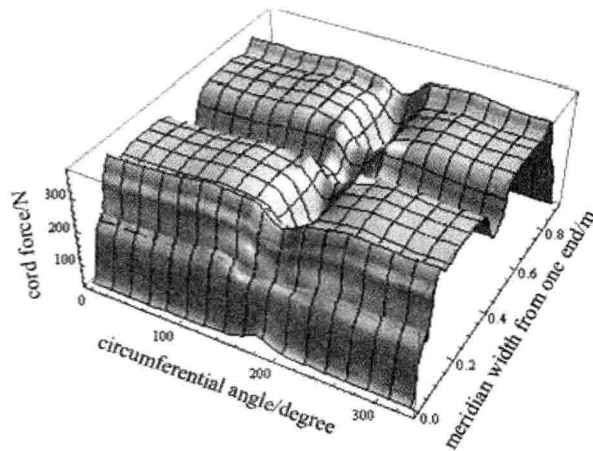


Figure 1.5 Carpet plot of the carcass ply cord force

For the carcass cord force distribution under vertical loading (Figure 1.5), an important characteristic is that the force relaxes in the contact area while it remains nearly equal to the inflation force beyond the contact area. The cord force distribution simulation results are validated by test data which is presented in a later section.

Carcass bending behaviour is of great interest to tire and cord engineers, because one of the main technical requirements for carcass reinforcement is resistance to flex fatigue, especially in the sidewall area. If the bending curvature of the carcass is known, then the stress level in the wire can be calculated with the following equation

$$\sigma_{\max} = \Delta\kappa \cdot E \cdot r_{\text{filament}} \quad (1.1)$$

Where r_{filament} is the radius of a single filament (which is 0.11 mm in this work), the material's Young's modulus E is about 200 MPa.

Equation (1.1) implies that all filaments in the cord will bend independently while having the same bending curvature. According to this assumption, the bending curvature of a single filament can be obtained from the overall deformation of the carcass according to the following formulation:

$$\kappa_0 = \frac{|y''|}{(1+y'^2)^{\frac{3}{2}}}; \kappa_u = \frac{\left| (y+u_y)'' \right|}{\left[1 + (y+u_y)'^2 \right]^{\frac{3}{2}}}; \Delta\kappa = \kappa_u - \kappa_0 \quad (1.2)$$

$$y'_i = \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}}; y''_i = \frac{y'_{i+1} - y'_{i-1}}{x_{i+1} - x_{i-1}}; y'_{i+1} = \frac{y_{i+2} - y_i}{x_{i+2} - x_i}; y'_{i-1} = \frac{y_i - y_{i-2}}{x_i - x_{i-2}};$$

where the central differential method is used to get the first and second order partial differentials of the coordinates y with the coordinate x , and the cord assumed to be stress free in the non-inflated but cured tire shape.

1.2.3 Finite element modeling for complex multi-strand steel cord

In reality the filaments in the cord do not bend in a uniform manner. Relative rotation, shear and contact between filaments during tire deformation can only be captured using a local scale modeling. To mesh a complex multi-strand steel cord, one first has to find a way to describe its geometry mathematically. A single helix centreline for a filament in a specified layer of a steel cord can be described by the following parametric equations

$$\begin{aligned} x_s &= R_s \cos(\theta_s + \theta_{s0}) \\ y_s &= R_s \sin(\theta_s + \theta_{s0}) \\ z_s &= R_s \theta_s \tan(\alpha_s) \end{aligned} \quad (1.3)$$

where R_s is the radius of the center line of the single helix (see Figure 1.6), α_s is the layer angle, θ_s is the rotation angle, and θ_{s0} is the initial phase angle.

For steel cords like 3+9+15×0.22, a single helix formulation is sufficient to describe its geometry because it is basically a single helix structure. However, for steel cord such as 3×4 or 3×7 that involve double helix structures (see Figure 1.7), a mathematical equation based on the