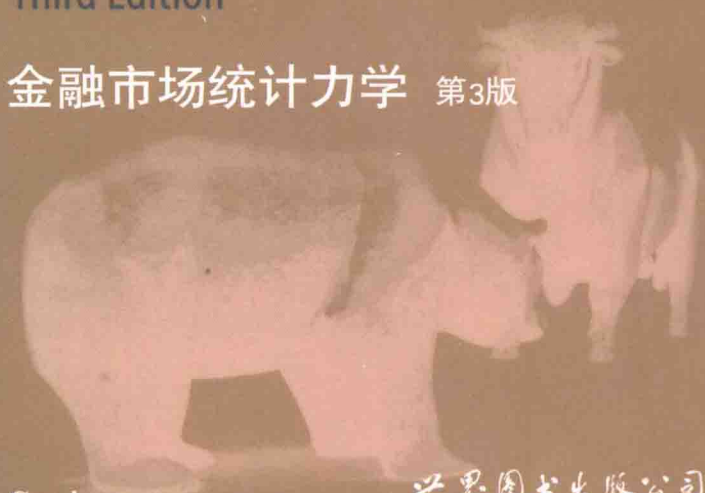


J. Voit

# The Statistical Mechanics of Financial Markets

Third Edition

金融市场统计力学 第3版



Springer

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Johannes Voit

# The Statistical Mechanics of Financial Markets

Third Editon

With 99 Figures

 Springer

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## Preface to the Third Edition

The present third edition of *The Statistical Mechanics of Financial Markets* is published only four years after the first edition. The success of the book highlights the interest in a summary of the broad research activities on the application of statistical physics to financial markets. I am very grateful to readers and reviewers for their positive reception and comments. Why then prepare a new edition instead of only reprinting and correcting the second edition?

The new edition has been significantly expanded, giving it a more practical twist towards banking. The most important extensions are due to my practical experience as a risk manager in the German Savings Banks' Association (DSGV): Two new chapters on risk management and on the closely related topic of economic and regulatory capital for financial institutions, respectively, have been added. The chapter on risk management contains both the basics as well as advanced topics, e.g. coherent risk measures, which have not yet reached the statistical physics community interested in financial markets. Similarly, it is surprising how little research by academic physicists has appeared on topics relating to Basel II. Basel II is the new capital adequacy framework which will set the standards in risk management in many countries for the years to come. Basel II is responsible for many job openings in banks for which physicists are extremely well qualified. For these reasons, an outline of Basel II takes a major part of the chapter on capital.

Feedback from readers, in particular Guido Montagna and Glenn May, has led to new sections on American-style options and the application of path-integral methods for their pricing and hedging, and on volatility indices, respectively. To make them consistent, sections on sensitivities of options to changes in model parameters and variables ("the Greeks") and on the synthetic replication of options have been added, too. Chin-Kun Hu and Bernd Kälber have stimulated extensions of the discussion of cross-correlations in financial markets. Finally, new research results on the description and prediction of financial crashes have been incorporated.

Some layout and data processing work was done in the Institute of Mathematical Physics at the University of Ulm. I am very grateful to Wolfgang Wonneberger and Ferdinand Gleisberg for their kind hospitality and generous

## VIII Preface to the Third Edition

support there. The University of Ulm and Academia Sinica, Taipei, provided opportunities for testing some of the material in courses.

My wife, Jinping Shen, and my daughter, Jiayi Sun, encouraged and supported me whenever I was in doubt about this project, and I would like to thank them very much.

Finally, I wish You, Dear Reader, a good time with and inspiration from this book.

Berlin, July 2005

*Johannes Voit*

# Preface to the First Edition

This book grew out of a course entitled “Physikalische Modelle in der Finanzwirtschaft” which I have taught at the University of Freiburg during the winter term 1998/1999, building on a similar course a year before at the University of Bayreuth. It was an experiment.

My interest in the statistical mechanics of capital markets goes back to a public lecture on self-organized criticality, given at the University of Bayreuth in early 1994. Bak, Tang, and Wiesenfeld, in the first longer paper on their theory of self-organized criticality [Phys. Rev. A **38**, 364 (1988)] mention Mandelbrot’s 1963 paper [J. Business **36**, 394 (1963)] on power-law scaling in commodity markets, and speculate on economic systems being described by their theory. Starting from about 1995, papers appeared with increasing frequency on the Los Alamos preprint server, and in the physics literature, showing that physicists found the idea of applying methods of statistical physics to problems of economy exciting and that they produced interesting results. I also was tempted to start work in this new field.

However, there was one major problem: my traditional field of research is the theory of strongly correlated quasi-one-dimensional electrons, conducting polymers, quantum wires and organic superconductors, and I had no prior education in the advanced methods of either stochastics and quantitative finance. This is how the idea of proposing a course to our students was born: learn by teaching! Very recently, we have also started research on financial markets and economic systems, but these results have not yet made it into this book (the latest research papers can be downloaded from my homepage <http://www.phy.uni-bayreuth.de/btp314/>).

This book, and the underlying course, deliberately concentrate on the main facts and ideas in those physical models and methods which have applications in finance, and the most important background information on the relevant areas of finance. They lie at the interface between physics and finance, not in one field alone. The presentation often just scratches the surface of a topic, avoids details, and certainly does not give complete information. However, based on this book, readers who wish to go deeper into some subjects should have no trouble in going to the more specialized original references cited in the bibliography.

Despite these shortcomings, I hope that the reader will share the fun I had in getting involved with this exciting topic, and in preparing and, most of all, actually teaching the course and writing the book.

Such a project cannot be realized without the support of many people and institutions. They are too many to name individually. A few persons and institutions, however, stand out and I wish to use this opportunity to express my deep gratitude to them: Mr. Ralf-Dieter Brunowski (editor in chief, Capital – Das Wirtschaftsmagazin), Ms. Margit Reif (Consors Discount Broker AG), and Dr. Christof Kreuter (Deutsche Bank Research), who provided important information; L. A. N. Amaral, M. Ausloos, W. Breymann, H. Büttner, R. Cont, S. Dresel, H. Eißfeller, R. Friedrich, S. Ghashghaie, S. Hügler, Ch. Jelitto, Th. Lux, D. Obert, J. Peinke, D. Sornette, H. E. Stanley, D. Stauffer, and N. Vandewalle provided material and challenged me in stimulating discussions. Specifically, D. Stauffer's pertinent criticism and many suggestions significantly improved this work. S. Hügler designed part of the graphics. The University of Freiburg gave me the opportunity to elaborate this course during a visiting professorship. My students there contributed much critical feedback. Apart from the year in Freiburg, I am a Heisenberg fellow of Deutsche Forschungsgemeinschaft and based at Bayreuth University. The final correction were done during a sabbatical at Science & Finance, the research division of Capital Fund Management, Levallois (France), and I would like to thank the company for its hospitality. I also would like to thank the staff of Springer-Verlag for all the work they invested on the way from my typo-congested L<sup>A</sup>T<sub>E</sub>X files to this first edition of the book.

However, without the continuous support, understanding, and encouragement of my wife Jinping Shen and our daughter Jiayi, this work would not have got its present shape. I thank them all.

Bayreuth,  
August 2000

*Johannes Voit*



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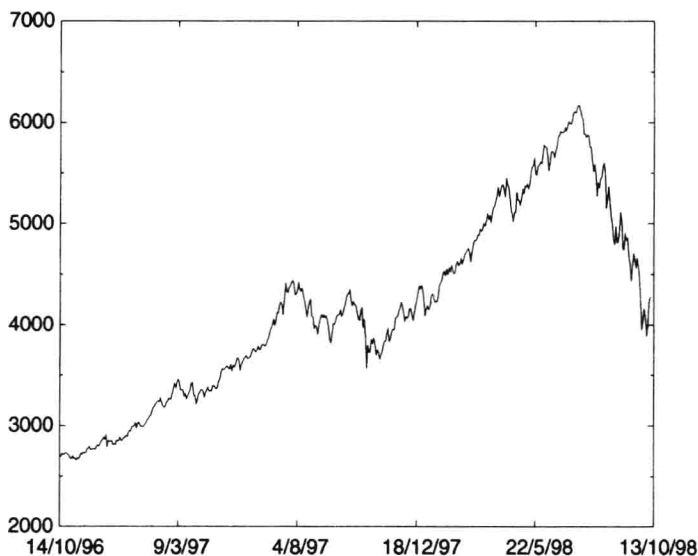
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# 1. Introduction

## 1.1 Motivation

The public interest in traded securities has continuously grown over the past few years, with an especially strong growth in Germany and other European countries at the end of the 1990s. Consequently, events influencing stock prices, opinions and speculations on such events and their consequences, and even the daily stock quotes, receive much attention and media coverage. A few reasons for this interest are clearly visible in Fig. 1.1 which shows the evolution of the German stock index DAX [1] over the two years from October 1996 to October 1998. Other major stock indices, such as the US Dow Jones Industrial Average, the S&P500, or the French CAC40, etc., behaved in a similar manner in that interval of time. We notice three important features: (i) the continuous rise of the index over the first almost one and a half years which

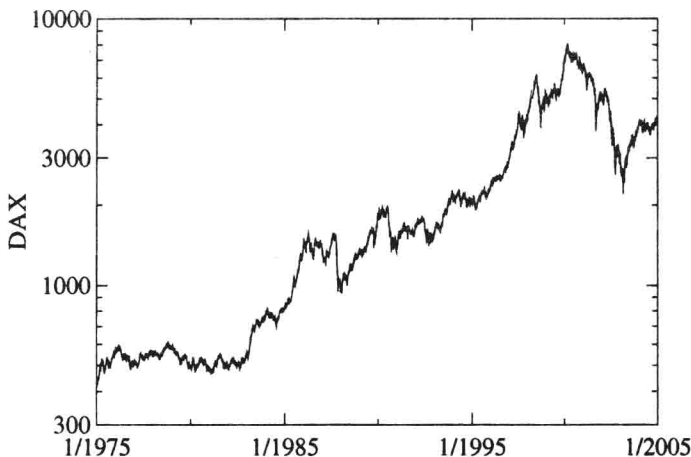


**Fig. 1.1.** Evolution of the DAX German stock index from October 14, 1996 to October 13, 1998. Data provided by Deutsche Bank Research

was interrupted only for very short periods; (ii) the crash on the “second black Monday”, October 27, 1997 (the “Asian crisis”, the reaction of stock markets to the collapse of a bank in Japan, preceded by rumors about huge amounts of foul credits and derivative exposures of Japanese banks, and a period of devaluation of Asian currencies). (iii) the very strong drawdown of quotes between July and October 1998 (the “Russian debt crisis”, following the announcement by Russia of a moratorium on its debt reimbursements, and a devaluation of the Russian rouble), and the collapse of the Long Term Capital Management hedge fund.

While the long-term rise of the index until 2000 seemed to offer investors attractive, high-return opportunities for making money, enormous fortunes of billions or trillions of dollars were annihilated in very short times, perhaps less than a day, in crashes or periods of extended drawdowns. Such events – the catastrophic crashes perhaps more than the long-term rise – exercise a strong fascination.

To place these events in a broader context, Fig. 1.2 shows the evolution of the DAX index from 1975 to 2005. Several different regimes can be distinguished. In the initial period 1975–1983, the returns on stock investments were extremely low, about 2.6% per year. Returns of 200 DAX points, or 12%, per year were generated in the second period 1983–1996. After 1996, we see a marked acceleration with growth rates of 1200 DAX points, or 33%, per year. We also notice that, during the growth periods of the stock market, the losses incurred in a sudden crash usually persist only over a short



**Fig. 1.2.** Long-term evolution of the DAX German stock index from January 1, 1975 to January 1, 2005. Data provided by Deutsche Bank Research supplemented by data downloaded from Yahoo, <http://de.finance.yahoo.com>

time, e.g. a few days after the Asian crash [(ii) above], or about a year after the Russian debt crisis [(iii) above]. The long term growth came to an end, around April 2000 when markets started sliding down. The fourth period in Fig. 1.2 from April 2000 to the end of the time series on March 12, 2003, is characterized by a long-term downward trend with losses of approximately 1400 DAX points, or 20% per year. The DAX even fell through its long-term upward trend established since 1983. Despite the overall downward trend of the market in this period, it recovered as quickly from the crash on September 11, 2001, as it did after crashes during upward trending periods. Finally, the index more or less steadily rose from its low at 2203 points on March 12, 2003 to about 4250 points at the end of 2004. Only the future will show if a new growth period has been kicked off.

This immediately leads us to a few questions:

- Is it possible to earn money not only during the long-term upward moves (that appears rather trivial but in fact is not) *but also during the drawdown periods*? These are questions for investors or speculators.
- What are the factors responsible for long- and short-term price changes of financial assets? How do these factors depend on the type of asset, on the investment horizon, on policy, etc.?
- How do the three growth periods of the DAX index, discussed in the preceding paragraph, correlate with economic factors? These are questions for economists, analysts, advisors to politicians, and the research departments of investment banks.
- What statistical laws do the price changes obey? How smooth are the changes? How frequent are jumps? These problems are treated by mathematicians, econometrists, but more recently also by physicists. The answer to this seemingly technical problem is of great relevance, however, also to investors and portfolio managers, as the efficiency of stop-loss or stop-buy orders [2] directly depends on it.
- How big is the risk associated with an investment? Can this be measured, controlled, limited or even eliminated? At what cost? Are reliable strategies available for that purpose? How big is any residual risk? This is of interest to banks, investors, insurance companies, firms, etc.
- How much fortune is at risk with what probability in an investment into a specific security at a given time?
- What price changes does the evolution of a stock price, resp. an index, imply for “financial instruments” (derivatives, to be explained below, cf. Sect. 2.3)? This is important both for investors but also for the writing bank, and for companies using such derivatives either for increasing their returns or for hedging (insurance) purposes.
- Can price changes be predicted? Can crashes be predicted?



## 1.2 Why Physicists? Why Models of Physics?

This book is about financial markets from a physicist's point of view. Statistical physics describes the complex behavior observed in many physical systems in terms of their simple basic constituents and simple interaction laws. Complexity arises from interaction and disorder, from the cooperation and competition of the basic units. Financial markets certainly are complex systems, judged both by their output (cf., e.g., Fig. 1.1) and their structure. Millions of investors frequent the many different markets organized by exchanges for stocks, bonds, commodities, etc. Investment decisions change the prices of the traded assets, and these price changes influence decisions in turn, while almost every trade is recorded.

When attempting to draw parallels between statistical physics and financial markets, an important source of concern is the complexity of human behavior which is at the origin of the individual trades. Notice, however, that nowadays a significant fraction of the trading on many markets is performed by computer programs, and no longer by human operators. Furthermore, if we make abstraction of the trading volume, an operator only has the possibility to buy or to sell, or to stay out of the market. Parallels to the Ising or Potts models of Statistical Physics resurface!

More specifically, take the example of Fig. 1.1. If we subtract out long-term trends, we are left essentially with some kind of random walk. In other words, the evolution of the DAX index looks like a random walk to which is superposed a slow drift. This idea is also illustrated in the following story taken from the popular book "A Random Walk down Wall Street" by B. G. Malkiel [3], a professor of economics at Princeton. He asked his students to derive a chart from coin tossing.

"For each successive trading day, the closing price would be determined by the flip of a fair coin. If the toss was a head, the students assumed the stock closed  $1/2$  point higher than the preceding close. If the flip was a tail, the price was assumed to be down  $1/2$ . ... The chart derived from the random coin tossing looks remarkably like a normal stock price chart and even appears to display cycles. Of course, the pronounced 'cycles' that we seem to observe in coin tossings do not occur at regular intervals as true cycles do, but neither do the ups and downs in the stock market. In other simulated stock charts derived through student coin tossings, there were head-and-shoulders formations, triple tops and bottoms, and other more esoteric chart patterns. One of the charts showed a beautiful upward breakout from an inverted head and shoulders (a very bullish formation). I showed it to a chartist friend of mine who practically jumped out of his skin. "What is this company?" he exclaimed. "We've got to buy immediately. This pattern's a classic. There's no question the stock will be up 15 points next week." He did not respond kindly to me when I told him the chart had been produced by flipping a coin." Reprinted from B. G. Malkiel: *A Random Walk down Wall Street*, ©1999 W. W. Norton