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Extreme Financial Risks

From Dependence to Risk Management

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An error does not become truth by reason of multiplied propagation, nor does truth become error because nobody sees it.

M.K. Gandhi

Preface: Idiosyncratic and Collective Extreme Risks

Modern western societies have a paradoxical relationship with risks. On the one hand, there is the utopian quest for a zero-risk society [120]. On the other hand, human activities may increase risks of all kinds, from collaterals of new technologies to global impacts on the planet. The characteristic multiplication of major risks in modern society and its reflexive impact on its development is at the core of the concept of the "Risk Society" [47]. Correlatively, our perception of risk has evolved so that catastrophic events (earthquakes, floods, droughts, storms, hurricanes, volcanic eruptions, and so on) are no more systematically perceived as unfair outcomes of an implacable destiny. Catastrophes may also result from our own technological developments whose complexity may engender major industrial disasters such as Bhopal, Chernobyl, AZT, as well as irreversible global changes such as global warming leading to climatic disruptions or epidemics from new bacterial and viral mutations. The proliferation of new sources of risks imposes new responsibilities concerning their determination, understanding, and management. Government organizations as well as private institutions such as industrial companies, insurance companies, and banks which have to face such risks, in their role of regulators or of risk bearers, must ensure that the consequences of extreme risks are supportable without endangering the institutions in charge of bearing these risks.

In the financial sector, crashes probably represent the most striking events among all possible extreme phenomena, with an impact and frequency that has been increasing in the last two decades [450]. Consider the worldwide crash in October 1987 which evaporated more than one thousand billion dollars in a few days or the more recent collapse of the internet bubble in which more than one-third of the world capitalization of 1999 disappeared after March 2000. Finance and stock markets are based on the fluid convertibility of stocks into money and vice versa. Thus, to work well, money is requested to be a reliable standard of value, that is, an effective store of value, hence the concerns with the negative impacts of inflation. Similarly, investors look at the various financial assets as carriers of value, like money, but with additional return potentials (accompanied with downturn risks). But for this view to hold so as to promote economic development, fluctuations in values need to be tamed to minimize the risk of losing a lifetime of savings, or to avoid the risks of losing the investment potential of companies, or even to prevent economic and social recessions in whole countries (consider the situation of California after 2002 with a budget gap representing more than one-fourth of the entire State budget resulting essentially from the losses of financial and tax incomes following the collapse of the internet bubble). It is thus highly desirable to have the tools for monitoring, understanding, and limiting the extreme risks of financial markets. Fully aware of these problems, the worldwide banking organizations have promoted a series of advices and norms, known as the recommendations of the Basle committee [41, 42]. The Basle committee has proposed models for the internal management of risks and the imposition of minimum margin requirements commensurate with the risk exposures. However, some criticisms [117, 467] have found these recommendations to be ill-adapted or even destabilizing. This controversy underlines the importance of a better understanding of extreme risks, of their consequences and ways to prevent or at least minimize them.

In our opinion, tackling this challenging problem requires to decompose it into two main parts. First, it is essential to be able to accurately quantify extreme risks. This calls for the development of novel statistical tools going significantly beyond the Gaussian paradigm which underpins the standard framework of classical financial theory inherited from Bachelier [26], Markowitz [347], and Black and Scholes [60] among others. Second, the existence of extreme risks must be considered in the context of the practice of risk management itself, which leads to ask whether extreme risks can be diversified away similarly to standard risks according to the mean-variance approach. If the answer to this question is negative as can be surmized for numerous concrete empirical evidences, it is necessary to develop new concepts and tools for the construction of portfolios with minimum (but unavoidable) exposition of extreme risks. One can think of mixing equities and derivatives, as long as derivatives themselves do not add an extreme risk component and can really provide an insurance against extreme moves, which has been far from true in recent dramatic instances such as the crash of October 1987. Another approach could involve mutualism as in insurance.

Risk management, and to the same extent portfolio management, thus requires a precise and rigorous analysis of the distribution of the returns of the portfolio of risks. Taking into account the moderate sizes of standard portfolios (from tens to thousands of assets typically) and the non-Gaussian nature of the distributions of the returns of assets constituting the portfolios, the distributions of the returns of typical portfolios are far from Gaussian, in contradiction with the expectation from a naive use of the central limit theorem (see for instance Chap. 2 of [451] and other chapters for a discussion of the deviations from the central limit theorem). This breakdown of universality then requires a careful estimation of the specific case-dependent distribution of the returns of a given portfolio. This can be done directly using the time series of the returns of the portfolio for a given capital allocation. A more constructive approach consists in estimating the joint distribution of the returns of all assets constituting the portfolio. The first approach is much simpler and rapid to implement since it requires solely the estimation of a monovariate distribution. However, it lacks generality and power by neglecting the observable information available from the basket of all returns of the assets. Only the multivariate distribution of the returns of the assets embodies the general information of all risk components and their dependence across assets. However, the two approaches become equivalent in the following sense: the knowledge of the distribution of the returns for all possible portfolios for all possible allocations of capital between assets is equivalent to the knowledge of the multivariate distributions of the asset returns. All things considered, the second approach appears preferable on a general basis and is the method mobilizing the largest efforts both in academia and in the private sector.

However, the frontal attack aiming at the determination of the multivariate distribution of the asset returns is a challenging task and, in our opinion, much less instructive and useful than the separate studies of the marginal distributions of the asset returns on the one hand and the dependence structure of these assets on the other hand. In this book, we emphasize this second approach, with the objective of characterizing as faithfully as possible the diverse origins of risks: the risks stemming from each individual asset and the risks having a collective origin. This requires to determine (i) the distributions of returns at different time scales, or more generally, the stochastic process underlying the asset price dynamics, and (ii) the nature and properties of dependences between the different assets.

The present book offers an original and systematic treatment of these two domains, focusing mainly on the concepts and tools that remain valid for large and extreme price moves. Its originality lies in detailed and thorough presentations of the state of the art on (i) the different distributions of financial returns for various applications (VaR, stress testing), and (ii) the most important and useful measures of dependences, both unconditional and conditional and a study of the impact of conditioning on the size of large moves on the measure of extreme dependences. A large emphasis is thus put on the theory of copulas, their empirical testing and calibration, as they offer intrinsic and complete measures of dependences. Many of the results presented here are novel and have not been published or have been recently obtained by the authors or their colleagues. We would like to acknowledge, in particular, the fruitful and inspiring discussions and collaborations with J.V. Andersen, U. Frisch, J.-P. Laurent, J.-F. Muzy, and V.F. Pisarenko.

Chapter 1 describes a general framework to develop "coherent measures" of risks. It also addresses the origins of risks and of dependence between assets in financial markets, from the CAPM (capital asset pricing model) generalized to the non-Gaussian case with heterogeneous agents, the APT (arbitrage pricing theory), the factor models to the complex system view suggesting an emergent nature for the risk-return trade-off.

Chapter 2 addresses the problem of the precise estimation of the probability of extreme events, based on a description of the distribution of asset returns endowed with heavy tails. The challenge is thus to specify accurately these heavy tails, which are characterized by poor sampling (large events are rare). A major difficulty is to neither underestimate (Gaussian error) or overestimate (heavy tail hubris) the extreme events. The quest for a precise quantification opens the door to model errors, which can be partially circumvented by using several families of distributions whose detailed comparisons allow one to discern the sources of uncertainty and errors. Chapter 2 thus discusses several classes of heavy tailed distributions: regularly varying distributions (*i.e.*, with asymptotic power law tails), stretched-exponential distributions (also known as Weibull or subexponentials) as well as log-Weibull distributions which extrapolate smoothly between these different families.

The second element of the construction of multivariate distributions of asset returns, addressed in Chaps. 3–6, is to quantify the dependence structure of the asset returns. Indeed, large risks are not due solely to the heavy tails of the distribution of returns of individual assets but may result from a collective behavior. This collective behavior can be completely described by mathematical objects called *copulas*, introduced in Chap. 3, which fully embody the dependence between asset returns.

Chapter 4 describes synthetic measures of dependences, contrasting and linking them with the concept of copulas. It also presents an original estimation method of the coefficient of tail dependence, defined, roughly speaking, as the probability for an asset to lose a large amount knowing that another asset or the market has also dropped significantly. This tail dependence is of great interest because it addresses in a straightforward way the fundamental question whether extreme risks can be diversified away or not by aggregation in portfolios. Either the tail dependence coefficient is zero and the extreme losses occur asymptotically independently, which opens the possibility of diversifying them away. Alternatively, the tail dependence coefficient is non-zero and extreme losses are fundamentally dependent and it is impossible to completely remove extreme risks. The only remaining strategy is to develop portfolios that minimize the collective extreme risks, thus generalizing the mean-variance to a mean-extreme theory [332, 336, 333].

Chapter 5 presents the main methods for estimating copulas of financial assets. It shows that the empirical determination of a copula is quite delicate with significant risks of model errors, especially for extreme events. Specific studies of the extreme dependence are thus required.

Chapter 6 presents a general and thorough discussion of different measures of conditional dependences (where the condition can be on the size(s) of one or both returns for two assets). Chapter 6 thus sheds new light on the variations of the strength of dependence between assets as a function of the sizes of the analyzed events. As a startling concrete application of conditional dependences, the phenomenon of contagion during financial crises is discussed in detail.

Chapter 7 presents a synthesis of the six previous chapters and then offers suggestions for future work on dependence and risk analysis, including time-varying measures of extreme events, endogeneity versus exogeneity, regime switching, time-varying lagged dependence and so on.

This book has been written with the ambition to be useful to (a) the student looking for a general and in-depth introduction to the field, (b) financial engineers, economists, econometricians, actuarial professionals and researchers, and mathematicians looking for a synoptic view comparing the pros and cons of different modeling strategies, and (c) quantitative practitioners for the insights offered on the subtleties and many dimensional components of both risk and dependence. The content of this book will also be useful to the broader scientific community in the natural sciences, interested in quantifying the complexity of many physical, geophysical, biophysical etc. processes, with a mounting emphasis on the role and importance of extreme phenomena and their non-standard dependences.

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On the Origin of Risks and Extremes

1.1 The Multidimensional Nature of Risk and Dependence

In finance, the fundamental variable is the return that an investor accrues from his investment in a basket of assets over a certain time period. In general, an investor is interested in maximizing his gains while minimizing uncertainties ("risks") on the expected value of the returns on his investment, at possibly multiple time scales – depending upon the frequency with which the manager monitors the portfolio – and time periods – depending upon the investment horizon. From a general standpoint, the return-risk pair is the unavoidable duality underlying all human activities. The relationship between return and risk constitutes one of the most important unresolved questions in finance. This question permeates practically all financial engineering applications, and in particular the selection of investment portfolios. There is a general consensus among academic researchers that risk and return should be related, but the exact quantitative specification is still beyond our comprehension [414].

Uncertainties come in several forms, which we cite in the order of increasing aversion for most human beings:

(i) stochastic occurrences of events quantified by known probabilities;
(ii) stochastic occurrences of events with poorly quantified or unknown probabilities;
(iii) random events that are "surprises," *i.e.*, that were previously thought to be impossible or unthinkable until they happened and revealed their existence.

Here we address the first form, using the mathematical tools of probability theory.

Within this class of uncertainties, one must still distinguish several branches. In the simplest traditional theory exemplified by Markowitz [347], the uncertainties underlying a given set of positions (portfolio) result from the interplay of two components: risk and dependence.

(a) Risk is embedded in the amplitude of the fluctuations of the returns. its simplest traditional measure is the standard deviation (square-root of the variance).
(b) The dependence between the different assets of a portfolio of positions is traditionally quantified by the correlations between the returns of all pairs of assets.

Thus, in their most basic incarnations, both risk and dependence are thought of, respectively, as one-dimensional quantities: the standard deviation of the distribution of returns of a given asset and the correlation coefficient of these returns with those of another asset of reference (the "market" for instance). The standard deviation (or volatility) of portfolio returns provides the simplest way to quantify its fluctuations and is at the basis of Markowitz's portfolio selection theory [347]. However, the standard deviation of a portfolio offers only a limited quantification of incurred risks (seen as the statistical fluctuations of the realized return around its expected – or anticipated – value). This is because the empirical distributions of returns have "fat tails" (see Chap. 2 and references therein), a phenomenon associated with the occurrence of non-typical realizations of the returns. In addition, the dependences between assets are only imperfectly accounted for by the covariance matrix [309].

The last few decades have seen two important extensions.

- First, it has become clear, as synthesized in Chap. 2, that the standard deviation offers only a reductive view of the genuine full set of risks embedded in the distribution of returns of a given asset. As distributions of returns are in general far from Gaussian laws, one needs more than one centered moment (the variance) to characterize them. In principle, an infinite set of centered moments is required to faithfully characterize the potential for small all the way to extreme risks because, in general, large risks cannot be predicted from the knowledge of small risks quantified by the standard deviation. Alternatively, the full space of risks needs to be characterized by the full distribution function. It may also be that the distributions are so heavy-tailed that moments do not exist beyond a finite order, which is the realm of asymptotic power law tails, of which the stable Lévy laws constitute an extreme class. The Value-at-Risk (VaR) [257] and many other measures of risks [19, 20, 73, 447, 453] have been developed to account for the larger moves allowed by non-Gaussian distributions and non-linear correlations.
- Second and more recently, the correlation coefficient (and its associated covariance) has been shown to only be a partial measure of the full dependence structure between assets. Similarly to risks, a full understanding of the dependence between two or more assets requires, in principle, an infinite number of quantifiers or a complete dependence function such as the copulas, defined in Chap. 3.

These two fundamental extensions from one-dimensional measures of risk and dependence to infinitely dimensional measures of risk and dependence constitute the core of this book. Chapter 2 reviews our present knowledge and the open challenges in the characterization of distribution of returns. Chapter 3 introduces the notion of copulas which are applied later in Chap. 5 to financial dependences. Chapter 4 describes the main properties of the most important and varied measures of dependence, and underlines their connections with copulas. Finally, Chap. 6 expands on the best methods to capture the dependence between extreme returns.

Understanding the risks of a portfolio of N assets involves the characterization of both the marginal distributions of asset returns and their dependence. In principle, this requires the knowledge of the full (time-dependent) multivariate distribution of returns, which is the joint probability of any given realization of the N asset returns at a given time. This remark entails the two major problems of portfolio theory: (1) to determine the multivariate distribution function of asset returns; (2) to derive from it useful measures of portfolio risks and use them to analyze and optimize the performance of the portfolios. There is a large literature on multivariate distributions and multivariate statistical analysis [363, 468, 282]. This literature includes:

- the use of the multivariate normal distribution on density estimation [428];
- the corresponding random vectors treated with matrix algebra, and thus on matrix methods and multivariate statistical analysis [173, 371];
- the robust determination of multivariate means and covariances [297, 298];
- the use of multivariate linear regression and factor models [160, 161];
- principal component analysis, with excursions in clustering and classification techniques [276, 254];
- methods for data analysis in cases with missing observations [133, 310];
- detecting outliers [249, 250];
- bootstrap methods and handling of multicollinearity [461];
- methods of estimation using the plug-in principles and maximum likelihood [144];
- hypothesis testing using likelihood ratio tests and permutation tests [398];
- discrete multivariate distributions [253];
- computer-aided geometric design, geometric modeling, geodesic applications, and image analysis [464, 105, 426];
- radial basis functions [86], scattered data on spheres, and shift-invariant spaces [139, 433];
- non-uniform spline wavelets [139];
- scalable algorithms in computer graphics [76];
- reverse engineering [139], and so on.

The growing literature on (1) non-stationary processes [85, 210, 222, 361] and (2) regime-switching [172, 180, 215, 269] is not covered here. Nor do we address the more complex issues of embedding financial modeling within economics and social sciences. We do not cover either the consequences for risk assessment coming from the important emerging field of behavioral finance, with its exploration of the impact on decision-making of imperfect bounded subjective probability perceptions [36, 206, 437, 439, 474]. Our book thus uses objective probabilities which can be estimated (with quantifiable errors) from suitable analysis of available data.

1.2 How to Rank Risks Coherently?

The question on how to rank risks, so as to make optimal decisions, is recurrent in finance (and in many other fields) but has not yet received a general solution.

Since the middle of the twentieth century, several paths have been explored. The pioneering work by Von Neuman and Morgenstern [482] has given birth to the mathematical definition of the expected utility function, which provides interesting insights on the behavior of a rational economic agent and has formalized the concept of risk aversion. Based upon the properties of the utility function, Rothschild and Stiglitz [419, 420] have attempted to define the notion of increasing risks. But, as revealed by Allais [4, 5], empirical investigations have proven that the postulates chosen by Von Neuman and Morgenstern are actually often violated by humans. Many generalizations have been proposed for curing the so-called Allais' Paradox, but until now, no generally accepted procedure has been found.

Recently, a theory due to Artzner *et al.* [19, 20] and its generalization by Föllmer and Schied [174, 175] have appeared. Based on a series of postulates that are quite natural, this theory allows one to build *coherent* (resp., convex) measures of risks that provide tools to compare and rank risks [383]. In fact, if this theory seems well-adapted to the assessment of the needed economic capital, that is, of the fraction of capital a company must keep as risk-free assets in order to face its commitments and thus avoid ruin, it seems less natural for the purpose of quantifying the *fluctuations* of the asset returns or equivalently the deviation from a predetermined objective. In fact, as will be exposed in this section, it turns out that the two approaches consisting in assessing the risk in terms of economic capital on the one hand, and in terms of deviations from an objective on the other hand, are actually the two sides of the same coin as recently shown in [407, 408].

1.2.1 Coherent Measures of Risks

According to Artzner *et al.* [19, 20], the risk involved in the variations of the values of a market position is measured by the amount of capital invested in a risk-free asset, such that the market position can be prolonged in the future. In other words, the potential losses should not endanger the future actions of the fund manager of the company, or more generally, of the person or structure which underwrites the position. In this sense, a risk measure