Springer Finance Textbook

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Mathematical Methods for Financial Markets

金融市场用的数学方法

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Preface

We translate to the domain of mathematical finance what F. Knight wrote, in substance, in the preface of his *Essentials of Brownian Motion and Diffusion* (1981): "it takes some temerity for the prospective author to embark on yet another discussion of the concepts and main applications of mathematical finance". Yet, this is what we have tried to do in our own way, after considerable hesitation.

Indeed, we have attempted to fill the gap that exists in this domain between, on the one hand, mathematically oriented presentations which demand quite a bit of sophistication in, say, functional analysis, and are thus difficult for practitioners, and on the other hand, mainstream mathematical finance books which may be hard for mathematicians just entering into mathematical finance.

This has led us, quite naturally, to look for some compromise, which in the main consists of the gradual introduction, at the same time, of a financial concept, together with the relevant mathematical tools.

Interlacing: This program interlaces, on the one hand, the financial concepts, such as arbitrage opportunities, admissible strategies, contingent claims, option pricing, default risk and ruin problems, and on the other hand, Brownian motion, diffusion processes, Lévy processes, together with the basic properties of these processes. We have chosen to discuss essentially continuous-time processes, which in some sense correspond to the real-time efficiency of the markets, although it would also be interesting to study discrete-time models. We have not done so, and we refer the reader to some relevant bibliography in the Appendix at the end of this book. Another feature of our book is that in the first half we concentrate on continuous-path processes, whereas the second half deals with discontinuous processes.

Special features of the book: Intending that this book should be readable for both mathematicians and practitioners, we were led to a somewhat unusual organisation, in particular:

1. in a number of cases, when the discussion becomes too technical, in the Mathematics or the Finance direction, we give only the essence of the argument, and send the reader to the relevant references,
2. we sometimes wanted a given section, or paragraph, to contain most of the information available on the topic treated there. This led us to:
   a) some forward references to topics discussed further in the book, which we indicate throughout the book with an arrow ($\rightarrowtail$)
   b) some repetition or at least duplication of the same kind of topic in various degrees of generality. Let us give an important example: Itô's formula is presented successively for continuous path semi-martingales, Poisson processes, general semi-martingales, mixed processes and Lévy processes.

We understand that this way of writing breaks away with the academic tradition of book writing, but it may be more convenient to access an important result or method in a given context or model.

About the contents: At this point of the Preface, the reader may expect to find a detailed description of each chapter. In fact, such a description is found at the beginning of each chapter, and for the moment we simply refer the reader to the Contents and the user's guide, which follows the Contents.

Numbering: In the following, C,S,B,R are integers. The book consists of two parts, eleven chapters and two appendices. Each chapter C is divided into sections C.S., which in turn are divided into subsections C.S.B. A statement in Subsection C.S.B. is numbered as C.S.B.R. Although this system of numbering is a little heavy, it is the only way we could find of avoiding confusion between the numbering of statements and unrelated sections.

What is missing in this book? Besides discussing the content of this book, let us also indicate important topics that are not considered here: The term structure of interest rate (in particular Heath-Jarrow-Morton and Brace-Gatarek-Musiela models for zero-coupon bonds), optimization of wealth, transaction costs, control theory and optimal stopping, simulation and calibration, discrete time models (ARCH, GARCH), fractional Brownian motion, Malliavin Calculus, and so on.

History of mathematical finance: More than 100 years after the thesis of Bachelier [39, 41], mathematical finance has acquired a history that is only slightly evoked in our book, but by now many historical accounts and surveys are available. We recommend, among others, the book devoted to Bachelier by Courtault and Kabanov [199], the book of Bouleau [114] and the collective book [870], together with introductory papers of Broadie and Detemple [129], Davis [221], Embrechts [321], Girlich [392], Gobet [395, 396], Jarrow and Protter [480], Samuelson [758], Taqqu [819] and Rogers [738], as well as the seminal papers of Black and Scholes [105], Harrison and Kreps [421] and Harrison and Pliska [422, 423]. It is also interesting to read the talks given by the Nobel prize winners Merton [644] and Scholes [764] at the Royal Academy of Sciences in Stockholm.

A philosophical point: Mathematical finance raises a number of problems in probability theory. Some of the questions are deeply rooted in the developments of stochastic processes (let us mention Bachelier once again), while some other questions are new and necessitate the use of sophisticated probabilistic analysis, e.g., martingales, stochastic calculus, etc. These questions may also appear in apparently completely different fields, e.g., Bessel processes are at the core of the very recent Stochastic Loewner Evolutions (SLE) processes. We feel that, ultimately, mathematical finance contributes to the foundations of the stochastic world.

Any relation with the present financial crisis (2007-?)? The writing of this book began in February 2001, at a time when probabilists who had engaged in Mathematical Finance kept developing central topics, such as the no-arbitrage theory, resting implicitly on the "good health of the market", i.e.: its "natural" tendency towards efficiency. Nowadays, "the market" is in quite "bad health" as it suffers badly from illiquidity, lack of confidence, misappreciation of risks, to name a few points. Revisiting previous axioms in such a changed situation is a huge task, which undoubtedly shall be addressed in the future. However, for obvious reasons, our book does not deal with these new and essential questions.

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All remaining errors are our sole responsibility. We would appreciate comments, suggestions and corrections from readers who may send e-mails to the corresponding author Monique Jeanblanc at monique.jeanblanc@univ-evry.fr.

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Springer Finance is a programme of books addressing students, academics and practitioners working on increasingly technical approaches to the analysis of financial markets. It aims to cover a variety of topics, not only mathematical finance but foreign exchanges, term structure, risk management, portfolio theory, equity derivatives, and financial economics.

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User's Guide

This book consists of two parts: the first part concerns continuous-path processes, and the second part concernes jump processes.

Part I:

Chapter 1 introduces the main results for continuous-path processes and presents many examples, including in particular Brownian motion.

Chapter 2 presents the main tools in finance: self-financing portfolios, valuation of contingent claims, hedging strategies.

Chapter 3 contains some useful information about hitting times and their laws. Closed form expressions are given in the case of (geometric) Brownian motion.

Chapter 4 discusses finer properties of Brownian motion, e.g., local times, bridges, excursions and meanders.

Chapter 5 is devoted mainly to the presentation of one-dimensional diffusions, thus extending the scope of Chapter 4. Filtration problems are also studied.

Chapter 6 focuses on Bessel processes and applications to finance.

Part II:

Chapter 7 is concerned with models of default risk, which involve stochastic processes with a single jump.

Chapter 8 introduces Poisson and compound Poisson processes, which are standard examples of jump processes.

Chapter 9 contains general theory of semi-martingales and aims at unifying results obtained in Chapters 1 and 8.

Chapter 10 presents some jump-diffusion processes and their applications to Finance.

Chapter 11 gives basic results about Lévy processes.

Chapter 12 consists of a list of useful formulae found throughout this book.

Notation and Abbreviations

We shall use the standard notation and abbreviations.

We shall use **increasing** instead of nondecreasing and **positive** instead of non-negative.

u.i.	:	uniformly integrable (for a family of r.v.'s)
BM	:	Brownian motion
r.v.	:	random variable
e.m.m.	:	equivalent martingale measure
a.s.	:	almost surely
w.r.t.	:	with respect to
w.l.g.	:	without loss of generality
SDE	:	Stochastic Differential Equation
BSDE	:	Backward Stochastic Differential Equation
PRP	:	Predictable Representation Property
MCT	:	Monotone Class Theorem
$x \vee y$	=	$\sup(x, y)$
$x \wedge y$	=	$\inf(x, y)$
$x \cdot y$	:	scalar product of the vectors $x, y \in \mathbb{R}^d$
$H \star X$	:	stochastic integral of the process H with respect to the semi-martingale X
x^+	=	$x \vee 0$
x^-	=	$(-x) \vee 0$
$\partial_x f$	=	$\frac{\partial}{\partial x} f = f_x$
C_b^n	:	set of functions with continuous bounded derivatives up to n-th order
$\mu(t), \mu_t$	:	A function (or a process) evaluated at time t. If μ is a deterministic function, $\mu(t)$ is preferably used; if μ is a process, when the subscript is not too large, μ_t is prefered
$\int_a^b ds f(s)$	=	$\int_a^b f(s) ds$ when it seems convenient
$X \overset{\text{law}}{=} Y$	:	the random variables (or the processes) X and Y have the same law
$X \overset{\text{mart}}{=} Y$	:	the process $X - Y$ is a local martingale
$X \in \mathcal{F}$	:	X is a $\mathcal{F}$-measurable r.v., i.e., $X \in L^0(\mathcal{F})$
$X \in b\mathcal{F}$	:	X is a bounded $\mathcal{F}$-measurable random variable
$\mathcal{N}(x)$	=	$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-y^2/2} dy$, the cumulative function for a standard Gaussian law

Other notation can be found in the glossary at the end of the volume.

At the end of the book, the reader will find an extended bibliography, and a list of references, sorted by thema, followed by an index of authors, in which the page number where each author is quoted is specified.

In the text, some important words are in boldface. These words are also found in the subject index. Some notation can be found in the notation index.

To complete this guide, we emphasize some particular features of this book, already mentioned in the Preface:

- in some cases, proofs are sketched and/or omitted, but precise references are given;
- forward references to topics discussed further in the book are indicated with the arrow $\rightarrowtail$;
- we proceed by generalization: an important case/process is discussed, followed (a little later) by a general study.

Throughout this book, the symbol □ indicates the end of a proof, the symbol ◁ indicates the end of an exercise and the symbol ▶ is used to separate a long proof into different parts.

Section 2.1 refers to Chapter 2, Section 1, and Subsection 4.3.7 refers to Chapter 4, Section 3, Subsection 7. Theorem (Proposition, Lemma) 3.2.1.4 is the 4th in Chapter 3, Section 2, Subsection 1.

Begin at the beginning, and go on till you come to the end. Then, stop.

Lewis Carroll, Alice's Adventures in Wonderland.

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