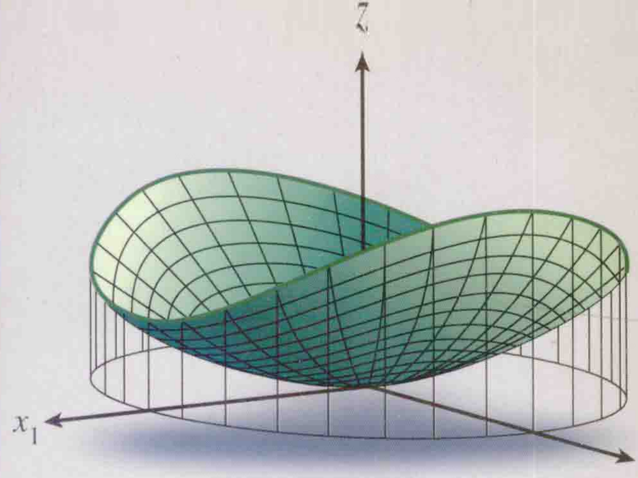
高等学校教材系列

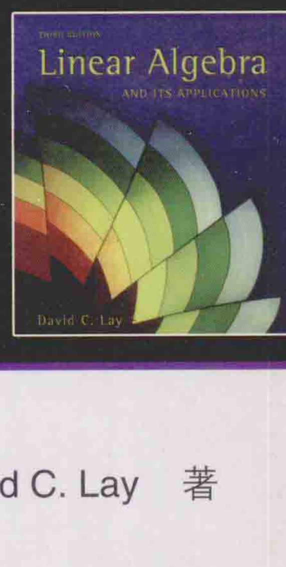


线性代数及其应用

（第三版）（英文版）

Linear Algebra and Its Applications

Third Edition



［美］David C. Lay 著

PEARSON Addison Wesley
PHEI
電子工業出版社
Publishing House of Electronics Industry
http://www.phei.com.cn

教材系列

线性代数及其应用

（第三版）

（英文版）

Linear Algebra and Its Applications

Third Edition

［美］ David C. Lay 著

電子工業出版社
Publishing House of Electronics Industry
北京 · BEIJING

内容简介

线性代数是处理矩阵和向量空间的数学分支科学，在现代数学的各个领域都有应用。本书主要包括线性方程组、矩阵代数、行列式、向量空间、特征值和特征向量、正交性和最小二乘方、对称矩阵和二次型等内容。本书的目的是使学生掌握线性代数最基本的概念、理论和证明。首先以常见的方式，具体介绍了线性独立、子空间、向量空间和线性变换等概念，然后逐渐展开，最后在抽象地讨论概念时，它们就变得容易理解多了。

这是一本介绍性的线性代数教材，内容翔实，层次清晰，适合作为高等院校理工科数学课的教学用书，还可作为公司职员及工程学研究人员的参考书。

版权贸易合同登记号　图字：01-2003-8576

图书在版编目（CIP）数据

线性代数及其应用 = Linear Algebra and Its Applications, Third Edition：第三版 /（美）莱（Lay, D. C.）著. -北京：电子工业出版社，2004.3

（高等学校教材系列）

ISBN 7-5053-9625-0

Ⅰ.线… Ⅱ.①莱… Ⅲ.线性代数－高等学校－教材－英文 Ⅳ. 0151.2

中国版本图书馆CIP数据核字（2004）第004864号

责任编辑：杜闽燕

印　　刷：北京兴华印刷厂

出版发行：电子工业出版社

北京市海淀区万寿路173信箱　邮编：100036

经　　销：各地新华书店

开　　本：787 × 980　1/16　　印张：36　字数：806千字

印　　次：2004年3月第1次印刷

定　　价：49.00元

凡购买电子工业出版社的图书，如有缺损问题，请向购买书店调换；若书店售缺，请与本社发行部联系。联系电话：（010）68279077。质量投诉请发邮件至 zlts@phei.com.cn，盗版侵权举报请发邮件至 dbqq@phei.com.cn。

About the Author

David C. Lay holds a B.A. from Aurora University (Illinois), and an M.A. and Ph.D. from the University of California at Los Angeles. Lay has been an educator and research mathematician since 1966, mostly at the University of Maryland, College Park. He has also served as a visiting professor at the University of Amsterdam, the Free University in Amsterdam, and the University of Kaiserslautern, Germany. He has over 30 research articles published in functional analysis and linear algebra.

As a founding member of the NSF-sponsored Linear Algebra Curriculum Study Group, Lay has been a leader in the current movement to modernize the linear algebra curriculum. Lay is also a co-author of several mathematics texts, including *Introduction to Functional Analysis* with Angus E. Taylor, *Calculus and Its Applications*, with L. J. Goldstein and D. I. Schneider, and *Linear Algebra Gems—Assets for Undergraduate Mathematics*, with D. Carlson, C. R. Johnson, and A. D. Porter.

A top-notch educator, Professor Lay has received four university awards for teaching excellence, including, in 1996, the title of Distinguished Scholar–Teacher of the University of Maryland. In 1994, he was given one of the Mathematical Association of America's Awards for Distinguished College or University Teaching of Mathematics. He has been elected by the university students to membership in Alpha Lambda Delta National Scholastic Honor Society and Golden Key National Honor Society. In 1989, Aurora University conferred on him the Outstanding Alumnus award. Lay is a member of the American Mathematical Society, the Canadian Mathematical Society, the International Linear Algebra Society, the Mathematical Association of America, Sigma Xi, and the Society for Industrial and Applied Mathematics. Since 1992, he has served several terms on the national board of the Association of Christians in the Mathematical Sciences.

Preface

The response of students and teachers to the first two editions of *Linear Algebra and Its Applications* has been most gratifying. This Third Edition offers even more visualization of concepts, along with enhanced technology support on the web for both students and instructors. As before, the text provides a modern elementary introduction to linear algebra and a broad selection of interesting applications. The material is accessible to students with the maturity that should come from successful completion of two semesters of college-level mathematics, usually calculus.

The main goal of the text is to help students master the basic concepts and skills they will use later in their careers. The topics here follow the recommendations of the Linear Algebra Curriculum Study Group, which were based on a careful investigation of the real needs of the students and a consensus among professionals in many disciplines that use linear algebra. Hopefully, this course will be one of the most useful and interesting mathematics classes taken as an undergraduate.

DISTINCTIVE FEATURES

Early Introduction of Key Concepts

Many fundamental ideas of linear algebra are introduced within the first seven lectures, in the concrete setting of $\mathbb{R}^n$, and then gradually examined from different points of view. Later generalizations of these concepts appear as natural extensions of familiar ideas, visualized through the geometric intuition developed in Chapter 1. A major achievement of the text, I believe, is that the level of difficulty is fairly even throughout the course.

A Modern View of Matrix Multiplication

Good notation is crucial, and the text reflects the way scientists and engineers actually use linear algebra in practice. The definitions and proofs focus on the columns of a matrix rather than on the matrix entries. A central theme is to view a matrix–vector product $A\mathbf{x}$ as a linear combination of the columns of A. This modern approach simplifies many arguments, and it ties vector space ideas into the study of linear systems.

Linear Transformations

Linear transformations form a "thread" that is woven into the fabric of the text. Their use enhances the geometric flavor of the text. In Chapter 1, for instance, linear transformations provide a dynamic and graphical view of matrix–vector multiplication.

Eigenvalues and Dynamical Systems

Eigenvalues appear fairly early in the text, in Chapters 5 and 7. Because this material is spread over several weeks, students have more time than usual to absorb and review these critical concepts. Eigenvalues are motivated by and applied to discrete and continuous dynamical systems, which appear in Sections 1.10, 4.8, 4.9, and in five sections of Chapter 5. Some courses reach Chapter 5 after about five weeks by covering Sections 2.8 and 2.9 instead of Chapter 4. These two optional sections present all the vector space concepts from Chapter 4 needed for Chapter 5.

Orthogonality and Least-Squares Problems

These topics receive a more comprehensive treatment than is commonly found in beginning texts. The Linear Algebra Curriculum Study Group has emphasized the need for a substantial unit on orthogonality and least-squares problems, because orthogonality plays such an important role in computer calculations and numerical linear algebra and because inconsistent linear systems arise so often in practical work.

PEDAGOGICAL FEATURES

Applications

A broad selection of applications illustrates the power of linear algebra to explain fundamental principles and simplify calculations in engineering, computer science, mathematics, physics, biology, economics, and statistics. Some applications appear in separate sections; others are treated in examples and exercises. In addition, each chapter opens with an introductory vignette that sets the stage for some application of linear algebra and provides a motivation for developing the mathematics that follows. Later, the text returns to that application in a section near the end of the chapter.

A Strong Geometric Emphasis

Every major concept in the course is given a geometric interpretation, because many students learn better when they can visualize an idea. There are substantially more drawings here than usual, and some of the figures have never appeared before in a linear algebra text.

Examples

This text devotes a larger proportion of its expository material to examples than do most linear algebra texts. There are more examples than an instructor would ordinarily present in class. But because the examples are written carefully, with lots of detail, students can read them on their own.

Theorems and Proofs

Important results are stated as theorems. Other useful facts are displayed in tinted boxes, for easy reference. Most of the theorems have formal proofs, written with the beginning student in mind. In a few cases, the essential calculations of a proof are exhibited in a carefully chosen example. Some routine verifications are saved for exercises, when they will benefit students.

Practice Problems

A few carefully selected Practice Problems appear just before each exercise set. Complete solutions follow the exercise set. These problems either focus on potential trouble spots in the exercise set or provide a "warm-up" to the exercises, and the solutions often contain helpful hints or warnings about the homework.

Exercises

The abundant supply of exercises ranges from routine computations to conceptual questions that require more thought. A good number of innovative questions pinpoint conceptual difficulties that I have found on student papers over the years. Each exercise set is carefully arranged, in the same general order as the text; homework assignments are readily available when only part of a section is discussed. A notable feature of the exercises is their numerical simplicity. Problems "unfold" quickly, so students spend little time on numerical calculations. The exercises concentrate on teaching understanding rather than mechanical calculations.

True/False Questions

To encourage students to read all of the text and to think critically, I have developed 300 simple true/false questions that appear in 33 sections of the text, just after the computational problems. They can be answered directly from the text, and they prepare students for the conceptual problems that follow. Students appreciate these questions—after they get used to the importance of reading the text carefully. Based on class testing and discussions with students, I decided not to put the answers in the text. (The *Study Guide* tells the students where to find the answers to the odd-numbered questions.) An additional 150 true/false questions (mostly at the ends of chapters) test understanding of the material. The text does provide simple T/F answers to most of these questions, but it omits the justifications for the answers (which usually require some thought).

Writing Exercises

An ability to write coherent mathematical statements in English is essential for all students of linear algebra, not just those who may go to graduate school in mathematics. The text includes many exercises for which a written justification is part of the answer. Conceptual exercises that require a short proof usually contain hints that help a student get started. For all odd-numbered writing exercises, either a solution is included at the back of the text or a hint is given and the solution is in the *Study Guide*, described below.

Computational Topics

The text stresses the impact of the computer on both the development and practice of linear algebra in science and engineering. Frequent Numerical Notes draw attention to issues in computing and distinguish between theoretical concepts, such as matrix inversion, and computer implementations, such as LU factorizations.

SUPPORT ON THE WEB

An icon appears in the text margin whenever expanded or new supplementary material can be accessed through the web sites:

WEB www.laylinalgebra.com or www.mymathlab.com

The inside cover of this text, which lists text references to applications of linear algebra, also shows the locations of many of the icons.

The site *www.laylinalgebra.com* has everything a student needs to begin the course: the first chapter of the text, including answers for odd exercises; data files for the exercises; review sheets and practice exams; and the first chapter of the *Study Guide* (see below). Having all of this material for the first day of class avoids the problems that arise when a bookstore runs out of the text or the *Study Guide*.

Practice Tests and Review Sheets

Multiple copies of tests, with solutions, cover all the main topics in the text. They come directly from courses I have taught in recent years. Each review sheet identifies the key definitions, theorems, and skills from a specified portion of the text.

Case Studies and Application Projects

Seven Case Studies expand topics introduced at the beginning of each chapter, adding real-world data and opportunities for further exploration. Students will find these readings interesting. Some faculty may assign the projects contained therein. More than twenty Application Projects either extend existing topics in the text or introduce new applications, such as cubic splines, traffic flow, airline flight routes, dominance matrices in sports competition, and error-correcting codes. Some new mathematical applications are integration techniques, polynomial root location, conic sections, quadric surfaces, and extrema for functions of two variables. Numerical linear algebra topics, such as condition numbers, matrix factorizations, and the QR method for finding eigenvalues, are also included. Woven into each discussion are exercises that often involve large data sets (and thus require technology for their solution.)

Data Files

Hundreds of files contain data for about 900 numerical exercises in the text, Case Studies, and Application Projects. The data is stored in a variety of formats—for MATLAB, Maple, Mathematica, and the TI-83+/86/89 and HP48G graphic calculators. Accessing matrices and vectors for a particular problem requires only a few keystrokes, which eliminates data entry errors and saves time on homework.

SUPPLEMENTS

Study Guide

I wrote this paperback student supplement to be an integral part of the course. The *Study Guide* (ISBN 0-201-77013-X) complements the text in several ways: (1) It shows the students how to learn linear algebra, with suggestions for studying and discussions of the logical structure of various theorems and proofs. (2) It supplies detailed solutions for every third odd-numbered problem (which includes most key exercises) and solutions to every odd-numbered writing exercise for which the text's answer is only a "Hint." (3) It provides a "lab manual" for using technology in the course, with additional help for **[M]** exercises and descriptions of appropriate commands for MATLAB, Maple, Mathematica, and graphic calculators, when those commands are first needed.

Instructor's Edition

For the convenience of instructors, this special edition includes brief answers to all exercises. A *Note to the Instructor* at the beginning of the text provides a commentary on the design and organization of the text, to help instructors plan their courses. It also describes other support available for instructors.

Instructor's Technology Manuals

Each manual provides detailed guidance for integrating a specific software package or graphic calculator throughout the course, written by faculty who have already used the technology with this text.

ACKNOWLEDGMENTS

I am indeed grateful to many groups of people who have helped me over the years with various aspects of this book.

I want to thank Israel Gohberg and Robert Ellis for over fifteen years of research collaboration in linear algebra, which has so greatly shaped my view of linear algebra.

It has been my privilege to work with David Carlson, Charles Johnson, and Duane Porter on the Linear Algebra Curriculum Study Group. Their ideas about teaching linear algebra have influenced this text in several important ways.

For assistance with the chapter opening examples and subsequent discussions, I thank Professor Thomas Polaski, Winthrop University; Professor Wassily Leontief, Institute for Economic Analysis, New York University; Clopper Almon, University of Maryland; David P. Young, Phantom Works, The Boeing Company; Roland Lamberson, Humboldt State University; and Russell Hardie, Chris Peterson, and the Earth Satellite Corporation.

I want to thank the technology experts who labored on the various supplements for the Third Edition, preparing the data, writing notes for the instructors, writing technology notes for the students in the *Study Guide*, and sharing their projects with us: Jeremy

Case (MATLAB), Taylor University; Douglas B. Meade (Maple), University of South Carolina; Lyle Cochran (Mathematica), Whitworth College, Michael Miller (TI calculators), Western Baptist College; and Thomas Polaski (HP-48G), Winthrop University. I also thank the two best undergraduate students I've had in recent years—Barker French and Ariel Weinberger—who updated the Second Edition's MATLAB data, checked all my calculations in the exercise solutions, and wrote drafts of some additional solutions. Finally, I am grateful to Jane Day, San Jose State University, and Luz DeAlba, Drake University, for allowing us to continue to use their outstanding projects, which they developed during their ten years of work on this text. Their support, encouragement, and friendship have meant a lot to me.

I sincerely thank the following reviewers for their careful analyses and constructive suggestions:

Third Edition Reviewers and Class Testers

David Austin, *Grand Valley State University*
G. Barbanson, *University of Texas at Austin*
Kenneth Brown, *Cornell University*
David Carlson, *San Diego State University*
Greg Conner, *Brigham Young University*
Casey T. Cremins, *University of Maryland*
Sylvie DesJardins, *Okanagan University College*
Daniel Flath, *University of South Alabama*
Yuval Flicker, *Ohio State University*
Scott Fulton, *Clarkson University*
Herman Gollwitzer, *Drexel University*
Jeremy Haefner, *University of Colorado at Colorado Springs*
William Hager, *University of Florida*
John Hagood, *Northern Arizona University*
Willy Hereman, *Colorado School of Mines*
Alexander Hulpke, *Colorado State University*
Doug Hundley, *Whitman College*
James F. Hurley, *University of Connecticut*
Jurgen Hurrelbrink, *Louisiana State University*
Jerry G. Ianni, *La Guardia Community College (CUNY)*
Hank Kuiper, *Arizona State University*
Ashok Kumar, *Valdosta State University*
Earl Kymala, *California State University, Sacramento*
Kathryn Lenz, *University of Minnesota–Duluth*
Jaques Lewin, *Syracuse University*
En-Bing Lin, *University of Toledo*
Andrei Maltsev, *University of Maryland*
Abraham Mantell, *Nassau Community College*
Madhu Nayakkankuppam, *University of Maryland–Baltimore County*
Lei Ni, *Stanford University*

Gleb Novitchkov, *Penn State University*
Ralph Oberste-Vorth, *University of South Florida*
Dev Sinha, *Brown University*
Wasin So, *San Jose State University*
Ron Solomon, *Ohio State University*
Eugene Spiegel, *University of Connecticut*
Alan Stein, *University of Connecticut*
James Thomas, *Colorado State University*
Brian Turnquist, *Bethel College*
Michael Ward, *Western Oregon University*
Bruno Welfert, *Arizona State University*
Jack Xin, *University of Texas at Austin*

I am deeply grateful for the assistance of Thomas Polaski, of Winthrop University. He wrote the case studies and projects for the web, helped with exercise answers and solutions, handled the technology for the HP-48G, and was always available for advice about various decisions that had to be made. I also appreciate the mathematical assistance provided by Thomas Wegleitner, Deanna Richmond, and Paul Lorczak, who checked the accuracy of calculations in the text, and by Georgia K. Mederer, who proofread the page proofs for mathematical errors. Another person who helped to polish the final manuscript was Jane Hoover, of Lifland et al., Bookmakers. She supervised the copyediting and various stages of proofreading and coordinated with the typesetter. I greatly appreciate her help.

Finally, I sincerely thank the staff at Addison-Wesley for all their help with the development and production of the Third Edition. Rachel S. Reeve, project manager, was the key person for this edition, managing the more than fifty people who worked on various parts of the project, frequently adjusting schedules, and calmly helping me find a way to get my part done. Other important members of the AW team were: Stefanie Borge, assistant editor; Beth Anderson, photo researcher; Karen Wernholm, production supervisor; Michael Boezi, marketing manager; Weslie Lewis, marketing coordinator; and Marlene Thom and Jennifer Kerber, media producers. Saved for last are two special friends who have guided the development of the book nearly from the beginning, giving wise counsel and encouragement and solving every problem that arose along the way—Greg Tobin, publisher, and Laurie Rosatone, sponsoring editor. Thank you both so much.

David C. Lay

A Note to Students

This course is potentially the most interesting and worthwhile undergraduate mathematics course you will complete. In fact, some students have written or spoken to me after graduation and said that they still use this text occasionally as a reference in their careers at several major corporations and engineering graduate schools. The following remarks offer some practical advice and information to help you master the material and enjoy the course.

In linear algebra, the *concepts* are as important as the *computations.* The simple numerical exercises that begin each exercise set only help you check your understanding of basic procedures. Later in your career, computers will do the calculations, but you will have to choose the calculations, know how to interpret the results, and then explain the results to other people. For this reason, many exercises in the text ask you to explain or justify your calculations. A written explanation is often required as part of the answer. For odd-numbered exercises, you will find either the desired explanation or at least a good hint. You must avoid the temptation to look at such answers until you have tried to write out the solution yourself. Otherwise, you are likely to think you understand something when in fact you do not.

To master the concepts of linear algebra, you will have to read and reread the text carefully. New terms are in boldface type, sometimes enclosed in a definition box. A glossary of terms is included at the end of the text. Important facts are stated as theorems or are enclosed in tinted boxes, for easy reference. I encourage you to read the first four pages of the Preface to learn more about the structure of this text. This will give you a framework for understanding how the course may proceed.

In a practical sense, linear algebra is a language. You must learn this language the same way you would a foreign language—with daily work. Material presented in one section is not easily understood unless you have thoroughly studied the text and worked the exercises for the preceding sections. Keeping up with the course will save you lots of time and distress!

Numerical Notes

I hope you read the Numerical Notes in the text, even if you are not using a computer or graphic calculator with the text. In real life, most applications of linear algebra involve numerical computations that are subject to some numerical error, even though that error may be extremely small. The Numerical Notes will warn you of potential difficulties in using linear algebra later in your career, and if you study the notes now, you are more likely to remember them later.

If you enjoy reading the Numerical Notes, you may want to take a course later in numerical linear algebra. Because of the high demand for increased computing power, computer scientists and mathematicians work in numerical linear algebra to develop faster and more reliable algorithms for computations, and electrical engineers design faster and smaller computers to run the algorithms. This is an exciting field, and your first course in linear algebra will help you prepare for it.

Study Guide

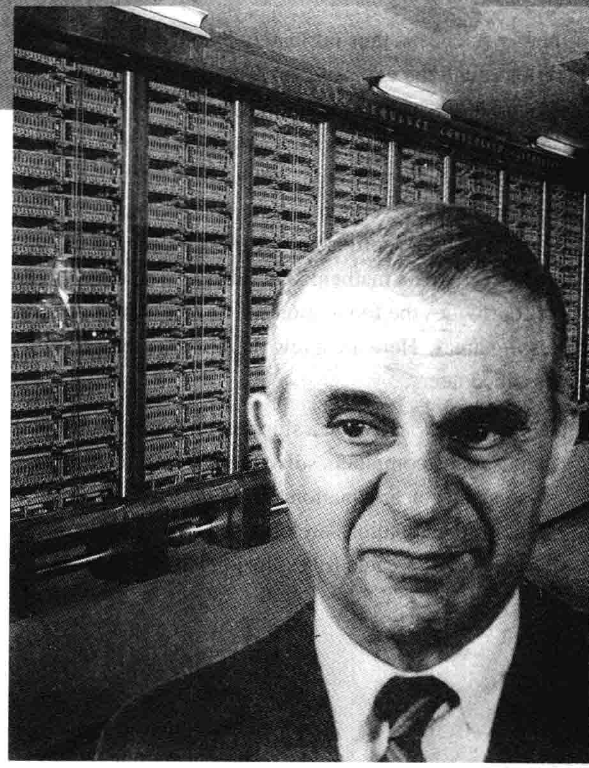
To help you succeed in this course, I have written a *Study Guide* to accompany the text. Not only will it help you learn linear algebra, but also it will show you how to study mathematics. It contains detailed solutions to every third odd-numbered exercise, plus solutions to all odd-numbered writing exercises for which only a hint is given in the Answers section. The *Study Guide* also provides warnings of common errors, helpful hints that call attention to key exercises and potential exam questions, and a separate glossary of terms for each chapter (invaluable when reviewing for an exam). Further, the *Study Guide* shows you how to use MATLAB, Maple, Mathematica, and the TI and HP graphic calculators. Use of this technology will save you many hours of homework time.

It is crucial that the *Study Guide* be separate from the text, because you must learn to write solutions by yourself, without much help. I know from years of experience that too easy access to solutions (such as in the back of the text) slows the mathematical development of most students. What you do in your first few weeks of studying this course will set your pattern for the term and determine how well you finish the course. I have placed a copy of the first chapter of the *Study Guide* on the web. This will get you started until you get your own copy. (To order it, you may need the ISBN number: 0-201-77013-X.) Please read there *How to Study Linear Algebra* as soon as possible. Then, when you get to Sections 1.4 and 1.7, follow the instructions in *Mastering Linear Algebra Concepts*. My students have found this very helpful, and I hope you will, too. Best wishes.

Contents

1

Linear Equations in Linear Algebra



INTRODUCTORY EXAMPLE

Linear Models in Economics and Engineering

It was late summer in 1949. Harvard Professor Wassily Leontief was carefully feeding the last of his punched cards into the university's Mark II computer. The cards contained economic information about the U.S. economy and represented a summary of more than 250,000 pieces of information produced by the U.S. Bureau of Labor Statistics after two years of intensive work. Leontief had divided the U.S. economy into 500 "sectors," such as the coal industry, the automotive industry, communications, and so on. For each sector, he had written a linear equation that described how the sector distributed its output to the other sectors of the economy. Because the Mark II, one of the largest computers of its day, could not handle the resulting system of 500 equations in 500 unknowns, Leontief had distilled the problem into a system of 42 equations in 42 unknowns.

Programming the Mark II computer for Leontief's 42 equations had required several months of effort, and he was anxious to see how long the computer would take to solve the problem. The Mark II hummed and blinked for 56 hours before finally producing a solution. We will discuss the nature of this solution in Sections 1.6 and 2.6.

Leontief, who was awarded the 1973 Nobel Prize in Economic Science, opened the door to a new era in mathematical modeling in economics. His efforts at Harvard in 1949 marked one of the first significant uses of computers to analyze what was then a large-scale mathematical model. Since that time, researchers in many other fields have employed computers to analyze mathematical models. Because of the massive amounts of data involved, the models are usually *linear*; that is, they are described by *systems of linear equations*.

The importance of linear algebra for applications has risen in direct proportion to the increase in computing power, with each new generation of hardware and software triggering a demand for even greater capabilities.