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Tomasz R. Bielecki
Marek Rutkowski

Credit Risk: Modeling, Valuation and Hedging

信用风险的建模、评估和对冲

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Preface

Mathematical finance and financial engineering have been rapidly expanding fields of science over the past three decades. The main reason behind this phenomenon has been the success of sophisticated quantitative methodologies in helping professionals manage financial risks. It is expected that the newly developed credit derivatives industry will also benefit from the use of advanced mathematics. This industry has grown around the need to handle credit risk, which is one of the fundamental factors of financial risk. In recent years, we have witnessed a tremendous acceleration in research efforts aimed at better comprehending, modeling and hedging this kind of risk.

Although in the first chapter we provide a brief overview of issues related to credit risk, our goal was to introduce the basic concepts and related notation, rather than to describe the financial and economical aspects of this important sector of financial market. The interested reader may consult, for instance, Francis et al. (1999) or Nelken (1999) for a much more exhaustive description of the credit derivatives industry.

The main objective of this monograph is to present a comprehensive survey of the past developments in the area of credit risk research, as well as to put forth the most recent advancements in this field. An important aspect of this text is that it attempts to bridge the gap between the mathematical theory of credit risk and financial practice, which serves as the motivation for the mathematical modeling studied in this book. Mathematical developments are presented in a thorough manner and cover the structural (value-of-the-firm) and the reduced-form (intensity-based) approaches to credit risk modeling, applied both to single and to multiple defaults. In particular, this book offers a detailed study of various arbitrage-free models of defaultable term structures of interest rates with several rating grades.

This book is divided into three parts. Part I, consisting of Chapters 1-3, is mainly devoted to the classic *value-of-the-firm approach* to the valuation and hedging of corporate debt. The starting point is the modeling of the dynamics of the total value of the firm's assets (combined value of the firm's debt and equity) and the specification of the capital structure of the assets of the firm. For this reason, the name *structural approach* is frequently attributed to this approach. For the sake of brevity, we have chosen to follow the latter convention throughout this text.

Modern financial contracts, which are either traded between financial institutions or offered over-the-counter to investors, are typically rather complex and they involve risks of several kinds. One of them, commonly referred to as a *market risk* (such as, for instance, the interest rate risk) is relatively well understood nowadays. Both theoretical and practical methods dealing with this kind of risk are presented in detail, and at various levels of mathematical sophistication, in several textbooks and monographs. For this reason, we shall pay relatively little attention to the market risk involved in a given contract, and instead we shall focus on the credit risk component.

As mentioned already, Chapter 1 provides an introduction to the basic concepts that underlie the area of credit risk valuation and management. We introduce the terminology and notation related to defaultable claims, and we give an overview of basic market instruments associated with credit risk. We provide an introductory description of the three types of credit-risk sensitive instruments that are subsequently analyzed using mathematical tools presented later in the text. These instruments are: corporate bonds, vulnerable claims and credit derivatives. So far, most analyses of credit risk have been conducted with direct reference to corporate debt. In this context, the contract-selling party is typically referred to as the borrower or the obligor, and the purchasing party is usually termed the creditor or the lender. However, methodologies developed in order to value corporate debt are also applicable to vulnerable claims and credit derivatives.

To value and to hedge credit risk in a consistent way, one needs to develop a quantitative model. Existing academic models of credit risk fall into two broad categories: the *structural models* and the *reduced-form models*, also known as the *intensity-based models*. Our main purpose is to give a thorough analysis of both approaches and to provide a sound mathematical basis for credit risk modeling. It is essential to make a clear distinction between stochastic models of credit risk and the less sophisticated models developed by commercial companies for the purpose of measuring and managing the credit risk. The latter approaches are not covered in detail in this text.

The subsequent two chapters are devoted to the so-called *structural approach*. In Chapter 2, we offer a detailed study of the classic Merton (1974) approach and its variants due to, among others, Geske (1977), Mason and Bhattacharya (1981), Shimko et al. (1993), Zhou (1996), and Buffet (2000). This method is sometimes referred to as the *option-theoretic approach*, since it was directly inspired by the Black-Scholes-Merton methodology for valuation of financial options. Subsequently, in Chapter 3, a detailed study of the Black and Cox (1976) ideas is presented. We also discuss some generalizations of their approach that are due to, among others, Brennan and Schwartz (1977, 1980), Kim et al. (1993a), Nielsen et al. (1993), Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Cathcart and El-Jahel (1998). Due to the way in which the default time is specified, the models worked out in the references quoted above are referred to as the *first-passage-time models*.

Within the framework of the structural approach, the default time is defined as the first crossing time of the value process through a default triggering barrier. Both the value process and the default triggering barrier are the model's primitives. Consequently, the main issue is the joint modeling of the firm's value and the barrier process that is usually specified in relation to the value of the firm's debt. Since the default time is defined in terms of the model's primitives, it is common to state that it is given *endogenously* within the model. Another important ingredient in both structural and reduced-form models is the amount of the promised cash flows recovered in case of default, typically specified in terms of the so-called *recovery rate* at default or, equivalently, in terms of the *loss-given-default*. Formally, it is thus possible to single out the *recovery risk* as a specific part of the credit risk; needless to say, the spread, the default and the recovery risks are intertwined both in practice and in most existing models of credit risk. Let us finally mention that econometric studies of recovery rates of corporate bond are rather scarce; the interested reader may consult, for instance, the studies by Altman and Kishore (1996) or Carty and Lieberman (1996).

The original Merton model focuses on the case of defaultable debt instruments with finite maturity, and it postulates that the default may occur only at the debt's maturity date. By contrast, the first-passage-time technique not only allows valuation of debt instruments with both a finite and an infinite maturity, but, more importantly, it allows for the default to arrive during the entire life-time of the reference debt instrument or entity.

The structural approach is attractive from the economic point of view as it directly links default events to the evolution of the firm's capital structure, and thus it refers to market fundamentals. Another appealing feature of this set-up is that the derivation of hedging strategies for defaultable claims is straightforward. An important aspect of this method is that it allows for a study of the optimal capital structure of the firm. In particular, one can study the most favorable timing for the decision to declare bankruptcy as a dynamic optimization problem. This line of research was originated by Black and Cox (1976), and it was subsequently continued by Leland (1994), Anderson and Sundaresan (1996), Anderson, Sundaresan and Tychon (1996), Leland and Toft (1996), Fan and Sundaresan, (1997), Mella-Barral and Perraudin (1997), Mella-Barral and Tychon (1999), Ericsson (2000), Anderson, Pan and Sundaresan (2000), Anderson and Sundaresan (2000).

Some authors use this methodology to forecast default events; however, this issue is not discussed in much detail in this text. Let us notice that the structural approach leads to modeling of default times in a way which does not provide any elements of surprise -- in the sense that the resulting random times are predictable with respect to the underlying filtrations. This feature is the source of the observed discrepancy between the credit spreads for short maturities predicted by structural models and the market data.

In Part II, we provide a systematic exposition of technical tools that are needed for an alternative approach to credit risk modeling – the reduced-form approach that allows for modeling of unpredictable random times of defaults or other credit events. The main objective of Part II is to work out various mathematical results underlying the reduced-form approach. Much attention is paid to characterization of random times in terms of hazard functions, hazard processes, and martingale hazard processes, as well as to evaluating relevant (conditional) probabilities and (conditional) expectations in terms of these functions and processes. In this part, the reader will find various pertinent versions of Girsanov's theorem and the martingale representation theorem. Finally, we present a comprehensive study of the problems related to the modeling of several random times within the framework of the intensity-based approach.

The majority of results presented in this part were already known; however, it is not possible to quote all relevant references here. The following works deserve a special mention: Dellacherie (1970, 1972), Chou and Meyer (1975), Dellacherie and Meyer (1978a, 1978b), Davis (1976), Elliott (1977), Jeulin and Yor (1978), Mazziotto and Szpirglas (1979), Jeulin (1980), Brémaud (1981), Artzner and Delbaen (1995), Duffie et al. (1996), Duffie (1998b), Lando (1998), Kusuoka (1999), Elliott et al. (2000), Bélanger et al. (2001), and Israel et al. (2001). Let us emphasize that the exposition in Part II is adapted from papers by Jeanblanc and Rutkowski (2000a, 2000b, 2002).

Part III is dedicated to an investigation of diverse aspects of the *reduced-form approach,* also commonly referred to as the *intensity-based approach.* To the best of our knowledge, this approach was initiated by Pye (1974) and Litterman and Iben (1991), and then formalized independently by Lando (1994), Jarrow and Turnbull (1995), and Madan and Unal (1998). Further developments of this approach can be found in papers by, among others, Hull and White (1995), Das and Tufano (1996), Duffie et al. (1996), Schönbucher (1996), Lando (1997, 1998), Monkkonen (1997), Lotz (1998, 1999), and Collin-Dufresne and Solnik (2001).

In many respects, Part III, where we illustrate the developed theory through examples of real-life credit derivatives and we describe market methods related to risk management, is the most practical part of the book. In Chapter 8, we discuss the most fundamental issues regarding the intensity-based valuation and hedging of defaultable claims in case of single reference credit. From the mathematical perspective, the intensity-based modeling of random times hinges on the techniques of modeling random times developed in the reliability theory. The key concept in this methodology is the survival probability of a reference instrument or entity, or, more specifically, the hazard rate that represents the intensity of default. In the most simple version of the intensity-based approach, nothing is assumed about the factors generating this hazard rate. More sophisticated versions additionally include factor processes that possibly impact the dynamics of the credit spreads.

Important modeling aspects include: the choice of the underlying probability measure (real-world or risk-neutral – depending on the particular application), the goal of modeling (risk management or valuation of derivatives), and the source of intensities. In a typical case, the value of the firm is not included in the model; the specification of intensities is based either on the model's calibration to market data or on the estimation based on historical observations. In this sense, the default time is *exogenously* specified. It is worth noting that in the reduced-form approach the default time is not a predictable stopping time with respect to the underlying information flow. In contrast to the structural approach, the reduced-form methodology thus allows for an element of surprise, which is in this context a practically appealing feature. Also, there is no need to specify the priority structure of the firm's liabilities, as it is often the case within the structural approach. However, in the so-called hybrid approach, the value of the firm process, or some other processes representing the economic fundamentals, are used to model the hazard rate of default, and thus they are used indirectly to define the default time.

Chapters 9 and 10 deal with the case of several reference credit entities. The main goal is to value basket derivatives and to study default correlations. In case of conditionally independent random times, the closed-form solutions for typical basket derivatives are derived. We also give some formulae related to default correlations and conditional expectations. In a more general situation of mutually dependent intensities of default, we show that the problem of quasi-explicit valuation of defaultable bonds is solvable. This should be contrasted with the previous results obtained, in particular, by Kusuoka (1999) and Jarrow and Yu (2001), who seemed to suggest that the valuation problem is intractable through the standard approach, without certain additional restrictions.

In view of the important role played in the modeling of credit migrations by the methodologies based on the theory of Markov chains, in Chapter 11 we offer a presentation of the relevant aspects of this theory.

In Chapter 12, we examine various aspects of credit risk models with multiple ratings. Both in case of credit risk management and in case of valuation of credit derivatives, the possibility of migrations of underlying credit name between different rating grades may need to be accounted for. This reflects the basic feature of the real-life market of credit risk sensitive instruments (corporate bonds and loans). In practice, credit ratings are the natural attributes of credit names. Most authors were approaching the issue of modeling of the credit migrations from the Markovian perspective. Chapter 12 is mainly devoted to a methodical survey of Markov models developed by, among others, Das and Tufano (1996), Jarrow et al. (1997), Nakazato (1997), Duffie and Singleton (1998a), Arvanitis et al. (1998), Kijima (1998), Kijima and Komoribayashi (1998), Thomas et al. (1998), Lando (2000a), Wei (2000), and Lando and Skødeberg (2002).

The topics touched upon in Chapter 12 are continued and further developed in Chapter 13. Following, in particular, Bielecki and Rutkowski (1999, 2000a, 2000b, 2001a) and Schönbucher (2000), we present the most recent developments, which combine the HJM methodology of modeling of instantaneous forward rates with a conditionally Markov model of credit migrations. Probabilistic interpretation of the market price of interest rate risk and the market price of the credit risk is highlighted. The latter is used as the motivation for our mathematical developments, based on martingale methods combined with the analysis of random times and the theory of time-inhomogeneous conditionally Markov chains and jump processes.

As is well known, there are several alternative approaches to the modeling of the default-free term structure of interest rates, based on the short-term rate, instantaneous forward rates, or the so-called market rates (such as, LIBOR rates or swap rates). As we have mentioned above, a model of defaultable term structure based on the instantaneous forward rates is presented in Chapter 13. In Chapters 14 and 15, which in a sense complement the content of Chapter 13, various typical examples of defaultable forward contracts and the associated types of defaultable market rates are introduced. We conclude by presenting the BGM model of forward LIBOR rates, Jamshidian's model of forward swap rates, as well as some ideas related to the modeling of defaultable LIBOR and swap rates.

We hope that this book may serve as a valuable reference for the financial analysts and traders involved with credit derivatives. Some aspects of the text may also be useful for market practitioners involved with managing credit-risk sensitive portfolios. Graduate students and researchers in areas such as finance theory, mathematical finance, financial engineering and probability theory will also benefit from this book. Although it provides a comprehensive treatment of most issues relevant to the theory and practice of credit risk, some aspects are not examined at all or are treated only very succinctly; these include: liquidity risk, credit portfolio management and econometric studies.

Let us once more stress that the main purpose of models presented in this text is the valuation of credit-risk-sensitive financial derivatives. For this reason, we focus on the arbitrage-free (or *martingale*) approach to the modeling of credit risk. Although *hedging* appears in the title of this monograph, we were able to provide only a brief account of the theoretical results related to the problem of hedging against the credit risk. A complete and thorough treatment of this aspect would deserve a separate text.

On the technical side, readers are assumed to be familiar with graduate level probability theory, theory of stochastic processes, elements of stochastic analysis and PDEs. As already mentioned, a systematic exposition of mathematical techniques underlying the intensity-based approach is provided in Part II of the text.

For the mathematical background, including the most fundamental definitions and concepts from the theory of stochastic process and the stochastic analysis based on the Itô integral, the reader may consult, for instance, Dellacherie (1972), Elliott (1977), Dellacherie and Meyer (1978a), Brémaud (1981), Jacod and Shiryaev (1987), Ikeda and Watanabe (1989), Protter (1990), Karatzas and Shreve (1991), Revuz and Yor (1991), Williams (1991), He et al. (1992), Davis (1993), Krylov (1995), Neftci (1996), Øksendal (1998), Rolski et al. (1998), Rogers and Williams (2000) or Steele (2000). In particular, for the definition and properties of the standard Brownian motion, we refer to Chap. 1 in Itô and McKean (1965), Chap. 2 in Karatzas and Shreve (1991) or Chap. II in Krylov (1995).

Some acquaintance with arbitrage pricing theory and fundamentals on financial derivatives is also expected. For an exhaustive treatment of arbitrage pricing theory, modeling of the term structure of interest rates and other relevant aspects of financial engineering, we refer to the numerous monographs available; to mention a few: Baxter and Rennie (1996), Duffie (1996), Lamberton and Lapeyre (1996), Neftci (1996), Musiela and Rutkowski (1997a), Pliska (1997), Bingham and Kiesel (1998), Björk (1998), Karatzas and Shreve (1998), Shiryaev (1998), Elliott and Kopp (1999), Mel'nikov (1999), Hunt and Kennedy (2000), James and Webber (2000), Jarrow and Turnbull (2000a), Pelsser (2000), Brigo and Mercurio (2001), and Martellini and Priaulet (2001). More specific issues related to credit risk derivatives and management of credit risk are discussed in Duffee and Zhou (1996), Das (1998a, 1998b), Caouette et al. (1998), Tavakoli (1998), Cossin and Pirotte (2000), Ammann (1999, 2001), and Duffie and Singleton (2003).

It is essential to stress that we make, without further mention, the common standard technical assumptions:

- all reference probability spaces are assumed to be complete (with respect to the reference probability measure),
- all filtrations satisfy the *usual conditions* of right-continuity and completeness (see Page 20 in Karatzas and Shreve (1991)),
- the sample paths of all stochastic processes are right-continuous functions, with finite left-limits, with probability one; in other words, all stochastic processes are assumed to be RCLL (i.e., càdlàg),
- all random variables and stochastic processes satisfy suitable integrability conditions, which ensure the existence of considered conditional expectations, deterministic or stochastic integrals, etc. For the sake of expositional simplicity, we frequently postulate the boundedness of relevant random variables and stochastic processes.

As a rule, we adopt the notation and terminology from the monograph by Musiela and Rutkowski (1997a). For the sake of the reader's convenience, an index of the most frequently used symbols is also provided. Although we have made an effort to use uniform notation throughout the text, in some places an ad hoc notation was also used.

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Chicago — Tomasz R. Bielecki
Warszawa — Marek Rutkowski

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