NONLINEAR
PHYSICAL
SCIENCE

Abdul-Majid Wazwaz

Partial Differential Equations and Solitary Waves Theory

偏微分方程与孤波理论

高等教育出版社
HIGHER EDUCATION PRESS

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With 14 figures

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NONLINEAR PHYSICAL SCIENCE
非线性物理科学

NONLINEAR PHYSICAL SCIENCE

Nonlinear Physical Science focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

Topics of interest in *Nonlinear Physical Science* include but are not limited to:

- New findings and discoveries in nonlinear physics and mathematics
- Nonlinearity, complexity and mathematical structures in nonlinear physics
- Nonlinear phenomena and observations in nature and engineering
- Computational methods and theories in complex systems
- Lie group analysis, new theories and principles in mathematical modeling
- Stability, bifurcation, chaos and fractals in physical science and engineering
- Nonlinear chemical and biological physics
- Discontinuity, synchronization and natural complexity in the physical sciences

This book is dedicated to my wife, our son, and our three daughters for supporting me in all my endeavors

Preface

Partial Differential Equations and Solitary Waves Theory is designed to serve as a text and a reference. The book is designed to be accessible to advanced undergraduate and beginning graduate students as well as research monograph to researchers in applied mathematics, science and engineering. This text is different from other texts in that it explains classical methods in a non abstract way and it introduces and explains how the newly developed methods provide more concise methods to provide efficient results.

Partial Differential Equations and Solitary Waves Theory is designed to focus readers' attentions on these recently developed valuable techniques that have proven their effectiveness and reliability over existing classical methods. Moreover, this text also explains the necessary classical methods because the aim is that new methods would complement the traditional methods in order to improve the understanding of the material.

The book avoids approaching the subject through the compact and classical methods that make the material impossible to be grasped, especially by students who do not have the background in these abstract concepts. Compact theorems and abstract handling of the material are not presented in this text.

The book was developed as a result of many years of experience in teaching partial differential equations and conducting research work in this field. The author has taken account on his teaching experience, research work as well as valuable suggestions received from students and scholars from a wide variety of audience. Numerous examples and exercises, ranging in level from easy to difficult, but consistent with the material, are given in each section to give the reader the knowledge, practice and skill in partial differential equations and solitary waves theory. There is plenty of material in this text to be covered in two semesters for senior undergraduates and beginning graduates of Mathematics, Science, and Engineering.

The content of the book is divided into two distinct parts, each is a self-contained and practical part. Part I contains eleven chapters that handle the partial differential equations by using the newly developed methods, namely, *Adomian decomposition method* and *Variational Iteration Method*. Some of the traditional methods are used in this part. With a diverse readership and interdisciplinary audience of applied mathematics, science, and engineering, attempts are made so that part I presents both analytical and numerical approaches in a clear and systematic fashion to make this book accessible to many who work in this field.

Part II contains seven chapters devoted to thoroughly examine solitary waves theory. Since the discovery of solitons in 1965, mathematicians, engineers, and physicists have been intrigued by the rich mathematical structure of solitons. Solitons play a prevalent role in propagation of light in fibers, surface waves in nonlinear dielectrics, optical bistability, optical switching in slab wave guides, and many other phenomena in plasma and fluid dynamics.

Chapter 1 provides the basic definitions and introductory concepts. Initial value problems and boundary value problems are discussed. In Chapter 2, the first order partial differential equations are handled by the newly developed methods, namely, the *Adomian decomposition method* (ADM) and the *variational iteration method* (VIM). The method of characteristics is introduced and explained in detail. Chapter 3 deals with the one-dimensional heat flow where homogeneous and inhomogeneous initial-boundary value problems are approached by using the decomposition method, the variational iteration method and the method of separation of variables. Chapter 4 is entirely devoted to the two-dimensional and three-dimensional heat flow. Chapter 5 provides the reader with a comprehensive discussion of the literature related to the one-dimensional wave equation. The decomposition method and the variational iteration method are used in handling the wave equations in a bounded and an unbounded domain. Moreover, the method of separation of variables and the D'Alembert method are also used. Chapter 6 presents a comprehensive study on wave equations in two-dimensional and three-dimensional spaces. Chapter 7 is devoted to the Laplace's equation in two- and three-dimensional rectangular coordinates and in polar coordinates. Moreover, the Laplace's equation in annulus form is also investigated by using the decomposition method and the separation of variables method. Chapter 8 introduces a comprehensive study on nonlinear partial differential equations. Even though the subject is considered difficult and mostly addressed in distinct books independent of linear PDEs, but it will be handled successfully and elegantly by using the newly developed decomposition method and the variational iteration method. Chapter 9 provides the reader with a variety of linear and nonlinear applications selected from mathematical physics, population growth models and evolution concepts. The useful concept of solitons and the recently developed concept of *Compactons* are thoroughly examined by using both traditional and new methods. Chapter 10 is concerned with the numerical techniques. Emphasis in this chapter will be on combining the decomposition series solution, the variational iteration method, and the Padé approximants to provide a promising tool that can be applied for further applications. Chapter 11 is concerned with the concepts of solitons and compactons. In this chapter, the solitons and compactons are determined by using prescribed conditions, a necessary condition for the applicability of the decomposition method.

Part II of this book gives a self-contained, practical and realistic approach to solitary wave theory. The dissipation and the dispersion effects are thoroughly investigated. Solitons play a prevalent role in many scientific and engineering phenomena.

The newly discovered *compactons*: solitons with a compact support are also studied. Part II of this book is devoted to use mainly the Hirota's bilinear method, combined with simplified version developed by Hereman and the tanh-coth method. Chapter 12 presents discussions about the dissipation and dispersion effects, analytic and nonanalytic solutions, conservation laws and multiple-soliton solutions, tanh-coth method, and Hirota's bilinear method combined with the Hereman's simplified form of the Hirota's method. In Chapter 13, the family of the KdV equations is studied. Multi-soliton solutions are obtained for only completely integrable equations of this family. Compactons solutions are also examined. Chapter 14 is concerned with KdV and mKdV equations of higher orders. The single solitons and the multiple-soliton solutions for completely integrable equations are addressed by using the Hirota's bilinear method. In addition, the Hirota-Satsuma equations and the generalized short wave equations were investigated for multiple-soliton solutions.

Chapter 15 investigates many KdV-type of equations where soliton solutions and multi-soliton solutions are obtained by using tanh-coth method and Hirota's method respectively. Chapter 16 is entirely devoted to study a family of well-known physical models for solitons and multi-soliton solutions as well. Some of these equations are Boussinesq equation, Klein-Gordon equation, Liouville equation, sine-Gordon equation, DBM equation, and others. Chapter 17 provides the reader with a comprehensive discussion of the literature related to Burgers, Fisher, Huxley, FitzHugh-Nagumo equations and related equations. Most of these equations are characterized by the dissipation phenomena that give kinks solutions. Chapter 18 presents a comprehensive study on two distinct types of equations that appear in solitary wave theory. The family of Camassa-Holm equations is examined to obtain the nonanalytic solution of peakons. On the other hand, the Schrodinger and Ginzburg-Landau equations of different orders are studied in this chapter.

The book concludes with six useful appendices. Moreover, the book introduces the traditional methods in the same amount of concern to provide the reader with the knowledge needed to make a comparison.

I deeply acknowledge Professor Louis Pennisi who made very valuable suggestions that helped a great deal in directing this book towards its main goal. I also deeply acknowledge Professor Masaaki Ito and Professor Willy Hereman for many helpful discussions and useful remarks. I owe them my deepest thanks.

I am deeply indebted to my wife, my son and my daughters who provided me with their continued encouragement, patience and support during the long days of preparing this book.

The author would highly appreciate any note concerning any constructive suggestion.

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