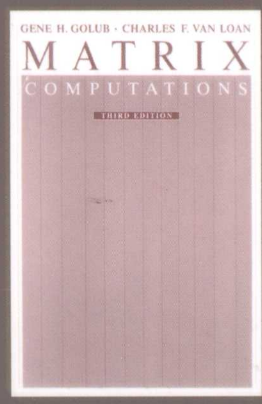
TURING 图灵原版数学·统计学系列 33

Matrix Computations

矩阵计算

（英文版·第3版）

[美] Gene H. Golub
Charles F. Van Loan 著



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内容提要

本书是数值计算领域的名著，系统介绍了矩阵计算的基本理论和方法. 内容包括：矩阵乘法、矩阵分析、线性方程组、正交化和最小二乘法、特征值问题、Lanczos方法、矩阵函数及专题讨论等. 书中的许多算法都有现成的软件包实现，每节后还附有习题，并有注释和大量参考文献.

本书可作为高等学校数学系高年级本科生和研究生的教材，亦可作为计算数学和工程技术人员的参考用书.

图灵原版数学 · 统计学系列

矩阵计算（英文版 · 第3版）

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DEDICATED TO

ALSTON S. HOUSEHOLDER

AND

JAMES H. WILKINSON

Preface to the Third Edition

The field of matrix computations continues to grow and mature. In the *Third Edition* we have added over 300 new references and 100 new problems. The LINPACK and EISPACK citations have been replaced with appropriate pointers to LAPACK with key codes tabulated at the *beginning* of appropriate chapters.

In the *First Edition* and *Second Edition* we identified a small number of global references: Wilkinson (1965), Forsythe and Moler (1967), Stewart (1973), Hanson and Lawson (1974) and Parlett (1980). These volumes are as important as ever to the research landscape, but there are some magnificent new textbooks and monographs on the scene. See *The Literature* section that follows.

We continue as before with the practice of giving references at the end of each section and a master bibliography at the end of the book.[①]

The earlier editions suffered from a large number of typographical errors and we are obliged to the dozens of readers who have brought these to our attention. Many corrections and clarifications have been made.

Here are some specific highlights of the new edition. Chapter 1 (Matrix Multiplication Problems) and Chapter 6 (Parallel Matrix Computations) have been completely rewritten with less formality. We think that this facilitates the building of intuition for high performance computing and draws a better line between algorithm and implementation on the printed page.

In Chapter 2 (Matrix Analysis) we expanded the treatment of CS decomposition and included a proof. The overview of floating point arithmetic has been brought up to date. In Chapter 4 (Special Linear Systems) we embellished the Toeplitz section with connections to circulant matrices and the fast Fourier transform. A subsection on equilibrium systems has been included in our treatment of indefinite systems.

A more accurate rendition of the modified Gram-Schmidt process is offered in Chapter 5 (Orthogonalization and Least Squares). Chapter 8 (The Symmetric Eigenproblem) has been extensively rewritten and rearranged so as to minimize its dependence upon Chapter 7 (The Unsymmetric Eigenproblem). Indeed, the coupling between these two chapters is now so minimal that it is possible to read either one first.

In Chapter 9 (Lanczos Methods) we have expanded the discussion of

① 为了节省篇幅，影印版省略了原书末的总参考文献.——编者注

the unsymmetric Lanczos process and the Arnoldi iteration. The "unsymmetric component" of Chapter 10 (Iterative Methods for Linear Systems) has likewise been broadened with a whole new section devoted to various Krylov space methods designed to handle the sparse unsymmetric linear system problem.

In §12.5 (Updating Orthogonal Decompositions) we included a new subsection on ULV updating. Toeplitz matrix eigenproblems and orthogonal matrix eigenproblems are discussed in §12.6.

Both of us look forward to continuing the dialog with our readers. As we said in the Preface to the *Second Edition*, "It has been a pleasure to deal with such an interested and friendly readership."

Many individuals made valuable *Third Edition* suggestions, but Greg Ammar, Mike Heath, Nick Trefethen, and Steve Vavasis deserve special thanks.

Finally, we would like to acknowledge the support of Cindy Robinson at Cornell. A dedicated assistant makes a big difference.

Software

LAPACK

Many of the algorithms in this book are implemented in the software package LAPACK:

E. Anderson, Z. Bai, C. Bischof, J. Demmel, J. Dongarra, J. DuCroz, A. Greenbaum, S. Hammarling, A. McKenney, S. Ostrouchov, and D. Sorensen (1995). *LAPACK Users' Guide, Release 2.0, 2nd ed.*, SIAM Publications, Philadelphia.

Pointers to some of the more important routines in this package are given at the *beginning* of selected chapters:

Chapter 1.	Level-1, Level-2, Level-3 BLAS
Chapter 3.	General Linear Systems
Chapter 4.	Positive Definite and Band Systems
Chapter 5.	Orthogonalization and Least Squares Problems
Chapter 7.	The Unsymmetric Eigenvalue Problem
Chapter 8.	The Symmetric Eigenvalue Problem

Our LAPACK references are spare in detail but rich enough to "get you started." Thus, when we say that `_TRSV` can be used to solve a triangular system $Ax = b$, we leave it to you to discover through the LAPACK manual that A can be either upper or lower triangular and that the transposed system $A^T x = b$ can be handled as well. Moreover, the underscore is a placeholder whose mission is to designate type (single, double, complex, etc).

LAPACK stands on the shoulders of two other packages that are milestones in the history of software development. EISPACK was developed in the early 1970s and is dedicated to solving symmetric, unsymmetric, and generalized eigenproblems:

B.T. Smith, J.M. Boyle, Y. Ikebe, V.C. Klema, and C.B. Moler (1970). *Matrix Eigensystem Routines: EISPACK Guide, 2nd ed.*, Lecture Notes in Computer Science, Volume 6, Springer-Verlag, New York.

B.S. Garbow, J.M. Boyle, J.J. Dongarra, and C.B. Moler (1972). *Matrix Eigensystem Routines: EISPACK Guide Extension,* Lecture Notes in Computer Science, Volume 51, Springer-Verlag, New York.

LINPACK was developed in the late 1970s for linear equations and least squares problems:

EISPACK and LINPACK have their roots in sequence of papers that feature Algol implementations of some of the key matrix factorizations. These papers are collected in

J.H. Wilkinson and C. Reinsch, eds. (1971). *Handbook for Automatic Computation, Vol. 2, Linear Algebra,* Springer-Verlag, New York.

NETLIB

A wide range of software including LAPACK, EISPACK, and LINPACK is available electronically via Netlib:

World Wide Web: `http://www.netlib.org/index.html`
Anonymous ftp: `ftp://ftp.netlib.org`

Via email, send a one-line message:

```
mail netlib@ornl.gov
send index
```

to get started.

MATLAB®

Complementing LAPACK and defining a very popular matrix computation environment is MATLAB:

MATLAB *User's Guide*, The MathWorks Inc., Natick, Massachusetts.

M. Marcus (1993). *Matrices and* MATLAB*: A Tutorial*, Prentice Hall, Upper Saddle River, NJ.

R. Pratap (1995). *Getting Started with* MATLAB, Saunders College Publishing, Fort Worth, TX.

Many of the problems in *Matrix Computations* are best posed to students as MATLAB problems. We make extensive use of MATLAB notation in the presentation of algorithms.

Selected References

Each section in the book concludes with an annotated list of references. A master bibliography is given at the end of the text.

Useful books that collectively cover the field, are cited below. Chapter titles are included if appropriate but do not infer too much from the level of detail because one author's chapter may be another's subsection. The citations are classified as follows:

Pre-1970 Classics. Early volumes that set the stage.
Introductory (General). Suitable for the undergraduate classroom.
Advanced (General). Best for practitioners and graduate students.
Analytical. For the supporting mathematics.
Linear Equation Problems. $Ax = b$.
Linear Fitting Problems. $Ax \approx b$.
Eigenvalue Problems. $Ax = \lambda x$.
High Performance. Parallel/vector issues.
Edited Volumes. Useful, thematic collections.

Within each group the entries are specified in chronological order.

Pre-1970 Classics

V.N. Faddeeva (1959). *Computational Methods of Linear Algebra*, Dover, New York.

Basic Material from Linear Algebra. Systems of Linear Equations. The Proper Numbers and Proper Vectors of a Matrix.

E. Bodewig (1959). *Matrix Calculus*, North Holland, Amsterdam.

Matrix Calculus. Direct Methods for Linear Equations. Indirect Methods for Linear Equations. Inversion of Matrices. Geodetic Matrices. Eigenproblems.

R.S. Varga (1962). *Matrix Iterative Analysis*, Prentice-Hall, Englewood Cliffs, NJ.

Matrix Properties and Concepts. Nonnegative Matrices. Basic Iterative Methods and Comparison Theorems. Successive Overrelaxation Iterative Methods. Semi-Iterative Methods. Derivation and Solution of Elliptic Difference Equations. Alternating Direction Implicit Iterative Methods. Matrix Methods for Parabolic Partial Differential Equations. Estimation of Acceleration Parameters.

J.H. Wilkinson (1963). *Rounding Errors in Algebraic Processes*, Prentice-Hall, Englewood Cliffs, NJ.

The Fundamental Arithmetic Operations. Computations Involving Polynomials. Matrix Computations.

A.S. Householder (1964). *Theory of Matrices in Numerical Analysis*, Blaisdell, New York. Reprinted in 1974 by Dover, New York.

Some Basic Identities and Inequalities. Norms, Bounds, and Convergence. Localization Theorems and Other Inequalities. The Solution of Linear Systems: Methods of Successive Approximation. Direct Methods of Inversion. Proper Values and Vectors: Normalization and Reduction of the Matrix. Proper Values and Vectors: Successive Approximation.

L. Fox (1964). *An Introduction to Numerical Linear Algebra*, Oxford University Press, Oxford, England.

Introduction, Matrix Algebra. Elimination Methods of Gauss, Jordan, and Aitken. Compact Elimination Methods of Doolittle, Crout, Banachiewicz, and Cholesky. Orthogonalization Methods. Condition, Accuracy, and Precision. Comparison of Methods, Measure of Work. Iterative and Gradient Methods. Iterative methods for Latent Roots and Vectors. Transformation Methods for Latent Roots and Vectors. Notes on Error Analysis for Latent Roots and Vectors.

J.H. Wilkinson (1965). *The Algebraic Eigenvalue Problem,* Clarendon Press, Oxford, England.

Theoretical Background. Perturbation Theory. Error Analysis. Solution of Linear Algebraic Equations. Hermitian Matrices. Reduction of a General Matrix to Condensed Form. Eigenvalues of Matrices of Condensed Forms. The LR and QR Algorithms. Iterative Methods.

G.E. Forsythe and C. Moler (1967). *Computer Solution of Linear Algebraic Systems,* Prentice-Hall, Englewood Cliffs, NJ.

Reader's Background and Purpose of Book. Vector and Matrix Norms. Diagonal Form of a Matrix Under Orthogonal Equivalence. Proof of Diagonal Form Theorem. Types of Computational Problems in Linear Algebra. Types of Matrices encountered in Practical Problems. Sources of Computational Problems of Linear Algebra. Condition of a Linear System. Gaussian Elimination and LU Decomposition. Need for Interchanging Rows. Scaling Equations and Unknowns. The Crout and Doolittle Variants. Iterative Improvement. Computing the Determinant. Nearly Singular Matrices. Algol 60 Program. Fortran, Extended Algol, and PL/I Programs. Matrix Inversion. An Example: Hilbert Matrices. Floating Point Round-Off Analysis. Rounding Error in Gaussian Elimination. Convergence of Iterative Improvement. Positive Definite Matrices; Band Matrices. Iterative Methods for Solving Linear Systems. Nonlinear Systems of Equations.

Introductory (General)

A.R. Gourlay and G.A. Watson (1973). *Computational Methods for Matrix Eigenproblems*, John Wiley & Sons, New York.

Introduction. Background Theory. Reductions and Transformations. Methods for the Dominant Eigenvalue. Methods for the Subdominant Eigenvalue. Inverse Iteration. Jacobi's Methods. Givens and Householder's Methods. Eigensystem of a Symmetric Tridiagonal Matrix. The LR and QR Algorithms. Extensions of Jacobi's Method. Extension of Givens' and Householder's Methods. QR Algorithm for Hessenberg Matrices. generalized Eigenvalue Problems. Available Implementations.

G.W. Stewart (1973). *Introduction to Matrix Computations,* Academic Press, New York.

Preliminaries. Practicalities. The Direct Solution of Linear Systems. Norms, Limits, and Condition Numbers. The Linear Least Squares Problem. Eigenvalues and Eigenvectors. The QR Algorithm.

R.J. Goult, R.F. Hoskins, J.A. Milner and M.J. Pratt (1974). *Computational Methods in Linear Algebra*, John Wiley and Sons, New York.

Eigenvalues and Eigenvectors. Error Analysis. The Solution of Linear Equations by Elimination and Decomposition Methods. The Solution of Linear Systems of Equations by Iterative Methods. Errors in the Solution Sets of Equations. Computation of Eigenvalues and Eigenvectors. Errors in Eigenvalues and Eigenvectors. Appendix – A Survey of Essential Results from Linear Algebra.

T.F. Coleman and C.F. Van Loan (1988). *Handbook for Matrix Computations*, SIAM Publications, Philadelphia, PA.

Fortran 77, The Basic Linear Algebra Subprograms, Linpack, MATLAB.

W.W. Hager (1988). *Applied Numerical Linear Algebra,* Prentice-Hall, Englewood Cliffs, NJ.

Introduction. Elimination Schemes. Conditioning. Nonlinear Systems. Least Squares. Eigenproblems. Iterative Methods.

P.G. Ciarlet (1989). *Introduction to Numerical Linear Algebra and Optimisation*, Cambridge University Press.

A Summary of Results on Matrices. General Results in the Numerical Analysis of Matrices. Sources of Problems in the Numerical Analysis of Matrices. Direct Methods for the Solution of Linear Systems. Iterative Methods for the Solution of Linear Systems. Methods for the Calculation of Eigenvalues and Eigenvectors. A Review of Differential Calculus. Some Applications. General Results on Optimization. Some Algorithms. Introduction to Nonlinear Programming. Linear Programming.

D.S. Watkins (1991). *Fundamentals of Matrix Computations*, John Wiley and Sons, New York.

Gaussian Elimination and Its Variants. Sensitivity of Linear Systems; Effects of Roundoff Errors. Orthogonal Matrices and the Least-Squares Problem. Eigenvalues and Eigenvectors I. Eigenvalues and Eigenvectors II. Other Methods for the Symmetric Eigenvalue Problem. The Singular Value Decomposition.

P. Gill, W. Murray, and M.H. Wright (1991). *Numerical Linear Algebra and Optimization, Vol. 1*, Addison-Wesley, Reading, MA.

Introduction. Linear Algebra Background. Computation and Condition. Linear Equations. Compatible Systems. Linear Least Squares. Linear Constraints I: Linear Programming. The Simplex Method.

A. Jennings and J.J. McKeowen (1992). *Matrix Computation (2nd ed)*, John Wiley and Sons, New York.

Basic Algebraic and Numerical Concepts. Some Matrix Problems. Computer Implementation. Elimination Methods for Linear Equations. Sparse Matrix Elimination. Some Matrix Eigenvalue Problems. Transformation Methods for Eigenvalue Problems. Sturm Sequence Methods. Vector Iterative Methods for Partial Eigensolution. Orthogonalization and Re-Solution Techniques for Linear Equations. Iterative Methods for Linear Equations. Non-linear Equations. Parallel and Vector Computing.

B.N. Datta (1995). *Numerical Linear Algebra and Applications.* Brooks/Cole Publishing Company, Pacific Grove, California.

Review of Required Linear Algebra Concepts. Floating Point Numbers and Errors in Computations. Stability of Algorithms and Conditioning of Problems. Numerically Effective Algorithms and Mathematical Software. Some Useful Transformations in Numerical Linear Algebra and Their Applications. Numerical Matrix Eigenvalue Problems. The Generalized Eigenvalue Problem. The Singular Value Decomposition. A Taste of Roundoff Error Analysis.

M.T. Heath (1997). *Scientific Computing: An Introductory Survey*, McGraw-Hill, New York.

Scientific Computing. Systems of Linear Equations. Linear Least Squares. Eigenvalues and Singular Values. Nonlinear Equations. Optimization. Interpolation. Numerical Integration and Differentiation. Initial Value Problems for ODEs. Boundary Value Problems for ODEs. Partial Differential Equations. Fast Fourier Transform. Random Numbers and Simulation.

C.F. Van Loan (1997). *Introduction to Scientific Computing: A Matrix-Vector Approach Using Matlab*, Prentice Hall, Upper Saddle River, NJ.

Power Tools of the Trade. Polynomial Interpolation. Piecewise Polynomial Interpolation. Numerical Integration. Matrix Computations. Linear Systems. The QR and Cholesky Factorizations. Nonlinear Equations and Optimization. The Initial Value Problem.

Advanced (General)

N.J. Higham (1996). *Accuracy and Stability of Numerical Algorithms*, SIAM Publications, Philadelphia, PA.

Principles of Finite Precision Computation. Floating Point Arithmetic. Basics. Summation. Polynomials. Norms. Perturbation Theory for Linear Systems. Triangular Systems. LU Factorization and Linear Equations. Cholesky Factorization. Iterative Refinement. Block LU Factorization. Matrix Inversion. Condition Number Estimation. The Sylvester Equation. Stationary Iterative Methods. Matrix Powers. QR Factorization. The Least Squares Problem. Underdetermined Systems. Vandermonde Systems. Fast Matrix Multiplication. The Fast Fourier Transform and Applications. Automatic Error Analysis. Software Issues in Floating Point Arithmetic. A Gallery of Test Matrices.

J.W. Demmel (1996). *Numerical Linear Algebra*, SIAM Publications, Philadelphia, PA.

Introduction. Linear Equation Solving. Linear Least Squares Problems. Nonsymmetric Eigenvalue Problems. The Symmetric Eigenproblem and Singular Value Decomposition. Iterative Methods for Linear Systems and Eigenvalue Problems. Iterative Algorithms for Eigenvalue Problems.

L.N. Trefethen and D. Bau III (1997). *Numerical Linear Algebra*, SIAM Publications, Philadelphia, PA.

Matrix-Vector Multiplication. Orthogonal Vectors and Matrices. Norms. The Singular Value Decomposition. More on the SVD. Projectors. QR Factorization. Gram-Schmidt Orthogonalization. MATLAB. Householder Triangularization. Least-Squares Problems. Conditioning and Condition Numbers. Floating Point Arithmetic. Stability. More on Stability. Stability of Householder Triangularization. Stability of Back Substitution. Conditioning of Least-Squares Problems. Stability of Least-Squares Algorithms. Gaussian Elimination. Pivoting. Stability of Gaussian Elimination. Cholesky Factorization. Eigenvalue Problems. Overview of Eigenvalue Algorithms. Reduction to Hessenberg/Tridiagonal Form. Rayleigh Quotient, Inverse Iteration. QR Algorithm Without Shifts. QR Algorithm With Shifts. Other Eigenvalue Algorithms. Computing the SVD. Overview of Iterative Methods. The Arnoldi Iteration. How Arnoldi Locates Eigenvalues. GMRES. The Lanczos Iteration. Orthogonal Polynomials and Gauss Quadrature. Conjugate Gradients. Biorthogonalization Methods. Preconditioning. The Definition of Numerical Analysis.

Analytical

F.R. Gantmacher (1959). *The Theory of Matrices Vol. 1*, Chelsea, New York.

Matrices and Operations on Matrices. The Algorithm of Gauss and Some of its Applications. Linear Operators in an n-dimensional Vector Space. The Characteristic Polynomial and the Minimum Polynomial of a Matrix. Functions of Matrices, Equivalent Transformations of Polynomial Matrices, Analytic Theory of Elementary Divisors. The Structure of a Linear Operator in an n-dimensional Space. Matrix Equations. Linear Operators in a Unitary Space. Quadratic and Hermitian Forms.

F.R. Gantmacher (1959). *The Theory of Matrices Vol. 2*, Chelsea, New York.

Complex Symmetric, Skew-Symmetric, and Orthogonal Matrices. Singular Pencils of Matrices. Matrices with Nonnegative Elements. Application of the Theory of Matrices to the Investigation of Systems of Linear Differential Equations. The Problem of Routh-Hurwitz and Related Questions.

A. Berman and R.J. Plemmons (1979). *Nonnegative Matrices in the Mathematical Sciences*, Academic Press, New York. Reprinted with additions in 1994 by SIAM Publications, Philadelphia, PA.

Matrices Which Leave a Cone Invariant. Nonnegative Matrices. Semigroups of Nonnegative Matrices. Symmetric Nonnegative Matrices. Generalized Inverse-Positivity. M-Matrices. Iterative Methods for Linear Systems. Finite Markov Chains. Input-Output Analysis in Economics. The Linear Complementarity Problem.

G.W. Stewart and J. Sun (1990). *Matrix Perturbation Theory*, Academic Press, San Diego.

Preliminaries. Norms and Metrics. Linear Systems and Least Squares Problems. The Perturbation of Eigenvalues. Invariant Subspaces. Generalized Eigenvalue Problems.

R. Horn and C. Johnson (1985). *Matrix Analysis*, Cambridge University Press, New York.

Review and Miscellanea. Eigenvalues, Eigenvectors, and Similarity. Unitary Equivalence and Normal Matrices. Canonical Forms. Hermitian and Symmetric Matrices. Norms for Vectors and Matrices. Location and Perturbation of Eigenvalues. Positive Definite Matrices.

R. Horn and C. Johnson (1991). *Topics in Matrix Analysis*, Cambridge University Press, New York.

The Field of Values. Stable Matrices and Inertia. Singular Value Inequalities. Matrix Equations and the Kronecker Product. The Hadamard Product. Matrices and Functions.

Linear Equation Problems

D.M. Young (1971). *Iterative Solution of Large Linear Systems*, Academic Press, New York.

Introduction. Matrix Preliminaries. Linear Stationary Iterative Methods. Convergence of the Basic Iterative Methods. Eigenvalues of the SOR Method for Consistently Ordered Matrices. Determination of the Optimum Relaxation Parameter. Norms of the SOR Method. The Modified SOR Method: Fixed Parameters. Nonstationary Linear Iterative Methods. The Modified SOR Method: Variable Parameters. Semi-Iterative Methods. Extensions of the SOR Theory; Stieltjes Matrices. Generalized Consistently Ordered Matrices. Group Iterative Methods. Symmetric SOR Method and Related Methods. Second Degree Methods. Alternating Direction Implicit Methods. Selection of an Iterative Method.

L.A. Hageman and D.M. Young (1981). *Applied Iterative Methods*, Academic Press, New York.

Background on Linear Algebra and Related Topics. Background on Basic Iterative Methods. Polynomial Acceleration. Chebyshev Acceleration. An Adaptive Chebyshev Procedure Using Special Norms. Adaptive Chebyshev Acceleration. Conjugate Gradient Acceleration. Special Methods for Red/Black Partitionings. Adaptive Procedures for Successive Overrelaxation Method. The Use of Iterative Methods in the Solution of Partial Differential Equations. Case Studies. The Nonsymmetrizable Case.

A. George and J. W-H. Liu (1981). *Computer Solution of Large Sparse Positive Definite Systems.* Prentice-Hall Inc., Englewood Cliffs, New Jersey.

Introduction. Fundamentals. Some Graph Theory Notation and Its Use in the Study of Sparse Symmetric Matrices. BAnd and Envelope Methods. General Sparse Methods. Quotient Tree Methods for Finite Element and Finite Difference Problems. One-Way Dissection Methods for Finite Element Problems. Nested Dissection Methods. Numerical Experiments.

S. Pissanetsky (1984). *Sparse Matrix Technology,* Academic Press, New York.

Fundamentals. Linear Algebraic Equations. Numerical Errors in Gaussian Elimination. Ordering for Gauss Elimination: Symmetric Matrices. Ordering for Gauss Elimination: General Matrices. Sparse Eigenanalysis. Sparse Matrix Algebra. Connectivity and Nodal Assembly. General Purpose Algorithms.

I.S. Duff, A.M. Erisman, and J.K. Reid (1986). *Direct Methods for Sparse Matrices,* Oxford University Press, New York.

Introduction. Sparse Matrices:Storage Schemes and Simple Operations. Gaussian Elimination for Dense Matrices: The Algebraic Problem. Gaussian Elimination for Dense Matrices: Numerical Considerations. Gaussian Elimination for Sparse Matrices: An Introduction. Reduction to Block Triangular Form. Local Pivotal Strategies for Sparse Matrices. Ordering Sparse Matrices to Special Forms. Implementing Gaussian Elimination: Analyse with Numerical Values. Implementing Gaussian Elimination with Symbolic Analyse. Partitioning, Matrix Modification, and Tearing. Other Sparsity-Oriented Issues.

R. Barrett, M. Berry, T.F. Chan, J. Demmel, J. Donato, J. Dongarra, V. Eijkhout, R. Pozo, C. Romine, H. van der Vorst (1993). *Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods,* SIAM Publications, Philadelphia, PA.

Introduction. Why Use Templates? What Methods are Covered? Iterative Methods. Stationary Methods. Nonstationary Iterative Methods. Survey of Recent Krylov Methods. Jacobi, Incomplete, SSOR, and Polynomial Preconditioners. Complex Systems. Stopping Criteria. Data Structures. Parallelism. The Lanczos Connection. Block Iterative Methods. Reduced System Preconditioning. Domain Decomposition Methods. Multigrid Methods. Row Projection Methods.

W. Hackbusch (1994). *Iterative Solution of Large Sparse Systems of Equations,* Springer-Verlag, New York.

Introduction. Recapitulation of Linear Algebra. Iterative Methods. Methods of Jacobi and Gauss-Seidel and SOR Iteration in the Positive Definite Case. Analysis in the 2-Cyclic Case. Analysis for M-Matrices. Semi-Iterative Methods. Transformations, Secondary Iterations, Incomplete Triangular Decompositions. Conjugate Gradient Methods. Multi-Grid Methods. Domain Decomposition Methods.

O. Axelsson (1994). *Iterative Solution Methods*, Cambridge University Press.

Direct Solution Methods. Theory of Matrix Eigenvalues. Positive Definite Matrices, Schur Complements, and Generalized Eigenvalue Probems. Reducible and Irreducible Matrices and the Perron-Frobenious Theory for Nonnegative Matrices. Basic Iterative Methods and Their Rates of Convergence. M-Matrices, Convergent Splittings, and the SOR Method. Incomplete Factorization Preconditioning Methods. Approximate Matrix Inverses and Corresponding Preconditioning Methods. Block Diagonal and Schur Complement Preconditionngs. Estimates of Eigenvalues and Condition Numbers for Preconditioned Matrices. Conjugate Gradient and Lanczos-Type Methods. Generalized Conjugate Gradient Methods. The Rate of Convergence of the Conjugate Gradient Method.

Y. Saad (1996). *Iterative Methods for Sparse Linear Systems*, PWS Publishing Co., Boston.

Background in Linear Algebra. Discretization of PDEs. Sparse Matrices. Basic Iterative Methods. Projection Methods. Krylov Subspace Methods – Part I. Krylov Subspace Methods – Part II. Methods Related to the Normal Equations. Preconditioned Iterations. Preconditioning Techniques. Parallel Implementations. Parallel Preconditioners. Domain Decomposition Methods.

Linear Fitting Problems

C.L. Lawson and R.J. Hanson (1974). *Solving Least Squares Problems*, Prentice-Hall, Englewood Cliffs, NJ. Reprinted with a detailed "new developments" appendix in 1996 by SIAM Publications, Philadelphia, PA.

Introduction. Analysis of the Least Squares Problem. Orthogonal Decomposition by Certain Elementary Transformations. Orthogonal Decomposition by Singular Value Decomposition. Perturbation Theorems for Singular Values. Bounds for the Condition Number of a Triangular Matrix. The Pseudoinverse. Perturbation Bounds for the Pseudoinverse. Perturbation Bounds for the Solution of Problem LS. Numerical Computations Using Elementary Orthogonal Transformations. Computing the Solution for the Overdetermined or Exactly Determined Full Rank Problem. Computation of the Covariance Matrix of the Solution Parameters. Computing the Solution for the Underdetermined Full Rank Problem. Computing the Solution for Problem LS with Possibly Deficient Pseudorank. Analysis of Computing Errors for Householder Transformations. Analysis of Computing Errors for the Problem LS. Analysis of Computing Errors for the Problem LS Using Mixed Precision Arithmetic. Computation of the Singular Value Decomposition and the Solution of Problem LS. Other Methods for Least Squares Problems. Linear Least Squares with Linear Equality Constraints Using a Basis of the Null Space. Linear Least Squares with Linear Equality Constraints by Direct Elimination. Linear Least Squares with Linear Equality Constraints by Weighting. Linear least Squares with Linear Inequality Constraints. Modifying a QR Decomposition to Add or Remove Column Vectors. Practical Analysis of Least Squares Problems. Examples of Some Methods of Analyzing a Least Squares Problem. Modifying a QR Decomposition to Add or Remove Row Vectors with Application to Sequential Processing of Problems Having a Large or Banded Coefficient Matrix.